Staggered delay tuning algorithms for ring resonators in optical beam forming networks

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A tuning algorithm is needed for a previously demonstrated phased array antenna system using an integrated optical beamforming network (OBFN). The optical delays in the OBFN are realized by cascading optical ring resonators (ORRs), which can be tuned by varying their parameters. In this paper an algorithm is presented that converts a set of desired delay values into the set of ORR parameters that minimizes the phase error.

1 Introduction

The major benefits of phased array antenna systems are their high gain and the possibility of electronic beam steering and shaping, which is together called beam forming. A phased array antenna system consists of multiple antenna elements with corresponding tunable phase shifters or delay elements, and some splitting/combining circuitry. Delay elements should be used instead of phase shifters when it is applied to broadband beam forming. The steering resolution for the system then depends on the tuning resolution of the delay elements.

In the considered system the delay elements are realized in the optical domain. This has several advantages. The system becomes immune to electromagnetic interference, has a high bandwidth and low loss. It is also compact and has a low weight when it is implemented using integrated optics. The complete system is explained elsewhere in these proceedings [1].

2 Problem definition

The optical beam forming network (OBFN) in this system contains splitters, combiners, Mach-Zehnder interferometers (MZIs) and optical ring resonators (ORRs) [2].

Especially the ORRs are interesting, since they generate the required delays for constructive interference between signals from different antenna elements. An ORR behaves as an all-pass filter with a periodic bell-shaped group delay response. This is illustrated by the dashed lines in the figure below, which are the individual group delays of the three cascaded ORRs shown in the inset.

Fig. 1. Theoretical group delay response of three cascaded ORR sections with resonance frequencies \( f_i \). (Inset: three ORR sections in series, with round-trip time \( T \), round-trip phase shift \( \phi_i \), and coupling coefficient \( \kappa_i \).)

The resonance frequencies of the ORRs, \( f_1, f_2 \) and \( f_3 \), depend on the round-trip times \( T \) and the extra phase shifts \( \phi_1, \phi_2 \) and \( \phi_3 \), and the group delays at the resonance frequencies depend on the coupling coefficients \( \kappa_1, \kappa_2 \) and \( \kappa_3 \). Both the \( \kappa \)'s and \( \phi \)'s can be tuned using chromium heaters, based on the thermo-optical effect.
By cascading multiple ORRs a broadband delay element can be created, as illustrated in Fig. 1. The total group delay response (the solid line in Fig. 1) then follows by summing the group delays of the individual ORRs, resulting in

$$\tau_{\text{total}}(f) = \sum_l \tau_l(f) = \sum_l \frac{\kappa_l T}{2 - \kappa_l - 2 \sqrt{1 - \kappa_l} \cos(2 \pi f T + \phi_l)},$$

in which $l$ is used to distinguish between the different ORRs. Obviously, the response is not perfectly flat, so, to get a response with a minimal ripple, the $\kappa$s and $\phi$s should be tuned to an optimal setting. In the measurements that we have presented so far [1],[2], this has been done manually. This paper presents an algorithm to derive the tuning settings that result in a minimal delay ripple for a given delay and bandwidth. (The latter are normalized with respect to $T$, to make the algorithm applicable for any system which uses ORRs as delay element, with arbitrary value of $T$.) From the complexity of (1) it should be obvious that this optimization is not simply a matter of analytical inversion.

### 3 Optimization approach

To optimize the group delay responses, one should first know how optimality is defined. Therefore this section will derive three criteria to optimize for, together with associated constraints. The first criterion is directly based on (1), and the result can be compared with an ideal flat response with a normalized delay of $D$. This comparison should be carried out in the bandwidth of importance, so integrating the square of the error over that frequency bandwidth would be a good measure of optimality:

$$\mu = \int_{f_{\text{min}}}^{f_{\text{max}}} \left[ \tau_{\text{total}}(f) - D \right]^2 \, df$$

The integral of this squared error function results into a metric $\mu$, which should be minimized in order to get an optimal result. The method of minimizing is Minimum Mean Square Error (MMSE). This method is chosen because there are many channels in the spectrum which should all have an acceptable signal level. A result where some channels are perfect whereas others are very bad is inherently avoided, since MMSE punishes larger errors harder than small ones. The nature of the ORR responses with relatively smooth curves in the considered regions does not require a higher order error function than a quadratic one. The maximum error in the considered set of frequencies could also be taken as criterion, but that criterion would result into mathematical issues, since the second order derivative would not be a continuous function anymore. However the analytical evaluation of the integral in (2) would produce a complicated expression, resulting into enormous calculation times for the minimization. Therefore it would be better to approximate the integral with a Riemann sum:

$$\mu = \sum_k \left[ \tau_{\text{total}}(f_k) - D \right]^2,$$

in which $f_k$ is the set of frequencies over which the summation has to be carried out. Since the response functions are quite smooth, the magnitude response and group delay responses can be assumed constant within one channel. This justifies the replacement of the integral over $f$ by a sum over the channel frequencies. The approximation can be made more accurate when more points are chosen.

The second criterion is based on the phase response. It can be derived from the delay response by integrating with respect to $f$ and multiplication with $-2\pi$:

$$\psi_{\text{total}}(f) = \sum_l \left[ \arctan\left( \frac{\sin(2 \pi f T + \phi_l)}{\sqrt{1 - \kappa_l} - \cos(2 \pi f T + \phi_l)} \right) - \arctan\left( \frac{\sqrt{1 - \kappa_l} \sin(2 \pi f T + \phi_l)}{1 - \sqrt{1 - \kappa_l} \cos(2 \pi f T + \phi_l)} \right) \right].$$

244
After approximation with a Riemann sum the following criterion is derived:

\[ \mu = \sum_k [\psi_{\text{total}}(f_k) + 2\pi D f_k]^2. \]  

(5)

The major advantage of choosing the phase as criterion is that many side effects of improper tuning depend on the phase error. The radiation pattern will have stronger sidelobes and its nulls will be less deep when the phase error becomes larger. Therefore a minimal phase error is desired to prevent interference from other transmitters on different locations.

The last criterion is the power tuning criterion, which maximizes the total output power of the system. It should be noted that this criterion evaluates the performance of the complete OBFN block according to optimal interference of the signals coming from the different antenna elements at the input. This results into the following formula [3]:

\[ \mu = \left[ M - \sum_{m=1}^{M} \exp\{j\psi_{\text{total},m}(f_k) + j2\pi D_m f_k\} \right]^2, \]

(6)

in which the index \( m \) is used to represent the different antenna elements. It is easy to recognize the part of the phase error in the formula, since it is the same as in (5), but this time the sum over the complex exponentials of all the phase errors are taken. It can be checked easily that similar phase errors are better than small but different phase errors.

The structure with a metric to minimize is well-suited for solving with a non-linear programming (NLP) solving method. The type of solver used in this research is a semi-quadratic programming (SQP) method. More about that method can be found in [4]. Such solver generally minimizes or maximizes a certain function, subject to certain constraints for the parameters that are to be optimized. In this case the constraints are that the value of \( \kappa \) cannot be smaller than 0, and, due to limited fabrication accuracy of the Mach-Zehnder-based tunable couplers, \( \kappa \) cannot be larger than 0.995. For \( \phi \) the bounds are between \(-\pi\) and \(+\pi\), since larger phases would not make sense.

4 Results

The optimizations were carried out for one, two and three cascaded rings as well as for a complete 1×4 OBFN chip (see Fig. 1 in [1]) using all three criteria. When a reasonable initial estimate was supplied to the NLP solver (e.g. within 10% of the actual solution) all criteria gave optimal results. However in Section 3 it was already mentioned that the phase criterion also optimizes for the effect of the stronger sidelobes and the decreasing depth of the nulls. Therefore it is considered better than delay tuning. Furthermore the power tuning criterion gives insignificant advantage compared to the phase criterion for the output power, and it is more difficult to derive rules of thumb from to implement the results into a microchip. Some phase output results are presented in Fig. 2.

There are two possibilities to implement the results into a microchip. The first is to store the optimal values of the \( \kappa \)s and \( \phi \)s for any desired normalized delay and bandwidth in a look-up table. The other is to fit a second order polynomial to the curves obtained with the algorithm. For example, the curves for the optimal \( \kappa \) for one ring and optimal \( \phi \)s for two rings are shown in Fig. 3, together with their second order approximations. Note that for the \( \phi \)s a polynomial with negative exponents was needed. In both cases the approximation is quite accurate.

Another application of this optimization algorithm is that the number of required ORRs for a certain bandwidth and delay can be determined when a limit on the phase error is set. A bandwidth/delay curve is shown below in Fig. 2. More results and details about the algorithm can be found in [3].
Fig. 2. Optimized phase response for a normalized delay of 5 and 7.5 (solid) and ideal response (dashed) is shown on the left. Maximum bandwidth/delay products with error of 0.01 rad$^2$ (right).

Fig. 3. Optimal tuning for the $\kappa$ of one ORR as a function of the normalized delay (solid) and approximation (dashed) is shown on the left, and the optimal tuning for the $\phi$ of two cascaded ORRs (solid) and approximation (dashed) on the right.

5 Conclusion
The optimization problem for the ORRs in the OBFN was converted into a structure which is solvable with NLP techniques. This resulted into an algorithm which gives optimal results and was always able to reach a solution when a reasonable initial estimate is supplied. The best criterion was the phase criterion, since it took all negative side effects into account as well. To implement the results into a microchip an accurate second order approximation could be made.

References