Photo-induced stress and refractive index modulation in Bragg gratings written in Ge-doped silica fibers

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Based on the elasticity theory equations we analyze the model of the photosensitivity, which relates the changes in the refractive index in fibre Bragg gratings to structural transformations and corresponding density variations. We have found the analytical solution that describe the displacement field resulting from the inscription of a Bragg grating in a germanium-doped optical fiber. Our phenomenological approach allows calculation of the induced stresses and related index modulation without detailed understanding of the microscopic nature of UV-light-induced defects. The proposed model provides an efficient tool to analyze the impact of radiation-induced density variations on the properties of fibre Bragg gratings.

The fabrication of a Type I fiber Bragg grating (FBG) commonly relies on the change of the refractive index (RI) in Ge-doped silica glass induced by exposure to UV radiation. The photosensitivity is ascribed to local changes of the glass structure under UV exposure. One model of the photosensitivity, which is usually invoked to account for the RI change, is the "color center model" [1]. The RI change is derived from the induced absorption using the Kramers-Kronig relations. Another type of models relate the changes in the RI to structural transformations [2], which result in density variations [3] and stress increase [4] or possibly relaxation of frozen stress [2]. Densification is an experimentally established effect accompanying the grating formation in Ge-doped silica fibers, [3] which can explain the high values of the RI change via the photoelastic effect. A quantitative evaluation of the role of density changes in the photosensitivity phenomena requires an analysis of the stress distribution induced by a grating inscription.

In our paper, we present an approach which allows the analytical calculation of the stress in an optical fiber, resulting from density changes due the inscription of a grating. The structural changes are analyzed on the basis of a set of differential equations that follow from the continuum approximation of the elasticity theory. The fiber symmetry plays an important role in this analysis. We assume that the glass density change at a given point $\mathbf{r}$ is a function of the absorbed UV-light dose D($\mathbf{r}$) at that point:

$$\Delta\rho / \rho = -f(D(\mathbf{r})), \qquad (1)$$

where the function f is not specified at the moment. The density change can often be approximated by a sublinear (Type I gratings) dependence [5]. The displacement field $\mathbf{u}$ induced by the radiation defects is then described by [6]:

$$\mu\vec{\nabla}^2\mathbf{u} + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}\cdot\mathbf{u}) = K\vec{\nabla}f(D(\mathbf{r})), \quad K = \lambda + \tfrac{2}{3}\mu, \qquad (2)$$

where λ and μ are the Lamé coefficients. We use a cylindrical system of coordinates $\mathbf{r}$ = (r, φ, z) with the polar axis (z) directed along the fiber axis. We can represent $\mathbf{u} = \{u_r, u_\varphi, u_z\}$ as $\mathbf{u} = \vec{\nabla}\Phi + \mathbf{w}$, where Φ and $\mathbf{w}$ are the scalar and the vector potentials, respectively. Substitution into Eq. (2) gives

$$\vec{\nabla}\left[(\lambda+2\mu)\vec{\nabla}^2\Phi - Kf(\mathbf{r})\right] + \left\{\mu\vec{\nabla}^2\mathbf{w} + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}\cdot\mathbf{w})\right\} = \vec{0}. \qquad (3)$$

In Eq. (3) the vector and the scalar potentials are separated. This separation holds if Φ satisfies the Poisson equation and if the vector potential **w** is a solution of the equation corresponding to that part of Eq.(3) between the curly brackets. The boundary conditions for our problem can be derived from the periodicity of the FBG. Let us consider the planes $z = \pm\Lambda/2 \pm k\Lambda$, $k = 0, 1, \ldots$, which correspond to the minima of the UV-radiation intensity. Λ is the grating period. Due to the translation invariance, the *z*-component of the force acting from the left hand side and the right hand side compensate each other. Therefore, these planes can be considered as "*z*-clamped" (the first boundary condition). The second boundary condition corresponds to the absence of stresses on the free (outer) surface of the fiber.

In a step-index single mode fiber only the fiber core, of radius *R,* is photosensitive. For such a fiber, the diameter of the core is small in comparison with the inverse absorption coefficient; e.g. the natural absorption coefficient in a silica fiber core doped with 11 mol.% of GeO_2 is about 50 cm^{-1} at 240 nm. Therefore, we assume the concentration of radiation induced colour/damage centers responsible for the density perturbation to be constant across the fiber cross-section for a given axial position *z*

$$f(z) = N(z)\Theta(R - r), \qquad (4)$$

where $\Theta(x)$ is the step function and *N(z)* describes the axial distribution of the density variation. Solution of Eq.(3) for any given distribution *N(z)* results in rather complicated expressions for the potentials. However, for the particular case of sinusoidal modulation

$$N(z) = \tfrac{1}{2} N_0 (1 + b\cos(az)), \quad a = 2\pi / \Lambda, \qquad (5)$$

which corresponds to the absence of saturation, the solution can be significantly simplified. In Eq.(5), N_0 is the relative density modulation amplitude, and *b* is the fringe visibility.

Direct substitution shows that the scalar field given by

$$\Phi(r,z) = -\gamma\big(F(r) + b\cos(az)P(r)\big), \quad \gamma = N_0 K / 2(\lambda + 2\mu), \qquad (6)$$

satisfies the Poisson equation and the first boundary condition. Here

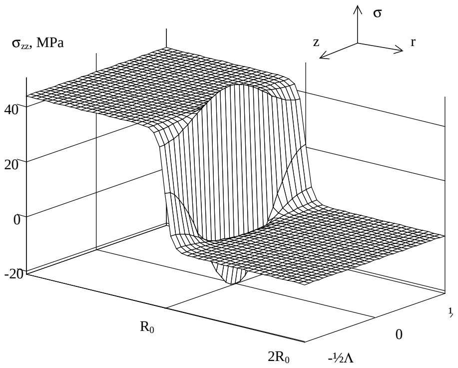
$$P(r) = \begin{cases} a^{-2} - (R/a) I_0(ar) K_1(aR), & r < R \\ (R/a) K_0(ar) I_1(aR), & r > R \end{cases} \quad \text{and} \qquad (7)$$

$$F(r) = R\int_0^\infty d\alpha\, \alpha^{-2} J_0(\alpha r) J_1(\alpha R),$$

where J_0, J_1, I_0, I_1, K_0, K_1 are the standard Bessel functions. The solution is angle independent due to the cylindrical symmetry of the density variation. This solution also shows that the scaling factor for the axial displacements is the grating period and the radial displacements scale with the core radius. For the case $b = 0$ (fringe-less UV-irradiation) only the radial dependence related to *F(r)* is important.

The stress distribution described by Eqs.(6,7) decays rapidly outside the fiber core. For a standard single-mode telecommunication fiber $R_{clad} \gg R$ and we can neglect the stresses related to the scalar potential solution on the free surface of the fiber, i.e. the trivial solution $\mathbf{w} = 0$ fits the problem and Eqs.(6,7) represent the complete solution.

An example of photo-induced stress distribution is shown in Fig. 1. The core radius R = 4.0 µm, $N_0 = -3 \cdot 10^{-3}$, and b = 1. In a boundary layer just outside the core the induced axial stress $\sigma_{zz}(r,z)$ is tensile or compressive, depending on the axial position z.



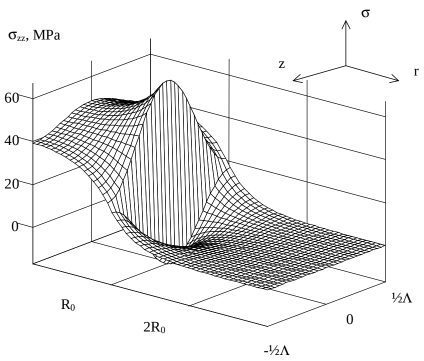


Fig.1 a) The grating period Λ = 0.4 µm, b) Λ = 4 µm

At the phenomenological level the effect of the density variation on the refractive index is a combination of the material structure transformation (inelastic) and the photo-elastic effects. A differential form of the Lorentz-Lorenz equation shows that the inelastic contribution is directly proportional to the density change:

$$\Delta n^{in} = a^{in}(\Delta\rho/\rho), \quad a^{in} = (1-\chi)(n^2-1)(n^2+2)/6n \approx 0.42\,, \tag{8}$$

where n ≈ 1.44 is the glass refractive index at 1.55 µm and the parameter $\chi \approx 0.2$ (silica) accounts for the dependence of the refractivity on the density.

The elastic contribution can be characterized by the stress-optical equations. The optical field in a single-mode fiber is described by two LP_{01}-modes that have transverse electric fields and polarization vectors parallel to an arbitrary pair of orthogonal directions in the fiber cross-section. These modes are uniformly polarized. To describe light propagation, we use a rectangular system of coordinates with the z-axis directed along the fiber axis and LP_{01}-modes being X- and Y-polarized. The stress-optical equation for the X-polarized mode has the form:

$$\Delta n_x^{el} = -B_2\sigma_{xx} - B_1\left(\sigma_{zz} + \sigma_{yy}\right), \tag{9}$$

where B_1 and B_2 are the stress-optical constants and σ_{ii}, $ii = xx, yy, zz$ are the components of the stress tensor. A similar equation holds for the Y-polarized mode.

The strain tensor ε_{ij} can be transformed from the cylindrical to the rectangular system of coordinates in the standard way and the stresses are calculated via the Duhamel-Neumann relations. We have for the photo-elastic contributions to the mean value of the refractive index

$$\Delta\bar{n}_x^{el} = -\mu\frac{K}{\lambda+2\mu}\left[3B_1+B_2\right]\frac{\Delta\bar{\rho}}{\rho} \approx -\bar{a}^{el}\frac{\Delta\bar{\rho}}{\rho}, \quad \bar{a}^{el} \approx -0.19\,, \tag{10}$$

and for the index modulation amplitude

$$\Delta\hat{n}_x^{el} \approx -\mu\frac{K}{\lambda+2\mu}\left[B_1+B_2\right]\frac{\Delta\hat{\rho}}{\rho} \approx -\hat{a}^{el}\frac{\Delta\hat{\rho}}{\rho}, \quad \hat{a}^{el} \approx -0.14\,. \tag{11}$$

Here “bar” and “hat” are used to designate the mean value and the modulation amplitude. The numerical values in Eqs.(10),(11) have been computed for silica.

For material with $\chi < 1$ and positive stress-optical constants, the inelastic, Eq.(8), and the elastic, Eqs.(10) and (11), contributions have opposite signs. The reduction of the mean index change is about 45 %. For the modulation amplitude the reduction is smaller (33%). This reduction is in agreement with the general principle that a reaction of a stable system reduces the effect of a perturbation.

An approximately linear dependence of the refractive index modulation amplitude on the average axial stress was observed experimentally [7] with the proportionality coefficient equal to $(0.8\pm0.2)\cdot10^{-11}$ $m^2\cdot N^{-1}$. In the linear theory developed in the average axial stress is proportional to a density change:

$$\bar{\sigma}_{zz} = [\mu K/(\lambda+2\mu)]\Delta\bar{\rho}/\rho\,. \qquad (12)$$

This equation is a form of Hook's law. A dilatation center with an additional volume ΔV_d gives rise to a sample volume change $\Delta V = [3K/(\lambda+2\mu)]\Delta V_d$. Taking into account that $\Delta V/V \approx 3\Delta L/L$, where L is the sample length, Eq.(6) can be rewritten as

$$\bar{\sigma}_{zz} = -\mu(\Delta L/L)\,. \qquad (13)$$

For a nearly ideal gratings $\hat{\sigma}_{zz}(r) \approx \bar{\sigma}_{zz}(r)$ and we have from Eqs.(8),(11) and (12)

$$\Delta\hat{n}_x^{el} \approx a^{in}\,\Delta\hat{\rho}/\rho - [B_1+B_2]\hat{\sigma}_{zz} \approx 0.96\cdot10^{-11}\,m^2N^{-1}\bar{\sigma}_{zz}\,, \qquad (14)$$

in a good agreement with the experimental value. The computed value is slightly bigger than the experimental one. The difference can be a result of the saturation effect. This saturation is rather weak due to a high fringe visibility achieved in the experiment [7]. Comparison with results of high resolution stress distributions measurements in UV-irradiated fibers reported recently [8] also shows a good agreement [9].

In conclusion, we have derived analytical expressions that quantify the effect of UV-light induced density changes on the glass refractive index in optical fibers. The results of our computations are in a good agreement with published experimental data. They provide an additional support for the model invoking structural changes as major contribution to UV-induced refractive index changes. We have found that for gratings written in germanium-doped silica fibers, with large refractive index modulation amplitudes, the photosensitivity is dominated by the density effect contribution.

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