

Bend-Induced Loss in Single-Mode Fibers

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To compute the bending losses of modes in optical fibers, a full-vectorial analysis of the bent fiber is performed. For this analysis, we distinguish between field solutions inside and outside the fiber. For the interior region, a coupled system of ordinary differential equations is integrated numerically from the known regular solutions at the center to the boundary. For the exterior region, triple integrals involving products of modified Bessel functions with large, complex order and argument arise. The (exact) bending losses of a step-index, single-mode fiber have been computed for various radii of curvature and set against approximate results available in literature.

1 Introduction

It is well known that radiation (bending) losses occur as soon as an optical fiber is bent. As a consequence the propagating modes attenuate which is an undesirable effect in many telecommunication applications, e.g. long-distance communications. Therefore, it is important to be able to design fibers in such a way that they are relatively insensitive to bending.

Several authors have developed theoretical expressions of the attenuation coefficient for weakly guiding fibers. The methods presented in [1], [2] and [3] are based on the scalar wave equation of the equivalent straight waveguide. In [4] an electric-current contrast source is introduced to compute the bending loss. All these methods rely on the assumption that the radius of curvature of the bend is very large with respect to the core radius of the fiber. However, in practice it turns out that these theoretical expressions are not always accurate enough.

Owing to the significant increase in the speed of computers, a full-vectorial analysis of the bent fiber can now be performed. For this analysis, a toroidal coordinate system is most convenient (see Figure 1). Since no approximations are made, the complex propagation coefficients, and consequently the bending loss, can be computed exactly. However, since the object under consideration is large in the electromagnetic sense, the problem remains numerically challenging. Moreover, in a toroidal coordinate system, Maxwell's equations do not separate. This implies that the spectral coefficients with respect to the 2-D transverse Fourier basis are coupled.

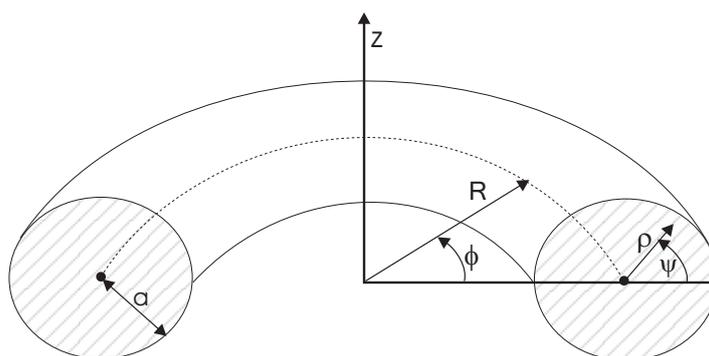


Figure 1: Bent fiber located in a toroidal coordinate system.

In our approach, we distinguish between an interior and exterior region of the fiber. For the latter region, the complexity of the resulting expressions is daunting. To keep our description lucid, we restrict ourselves to the computational bottleneck, viz., the computation of a product of (modified) Bessel functions with large, complex order and argument. For the interior region, we will discuss the computation of the field solutions at the boundary of the fiber, obtained by a numerical integration of a system of coupled ordinary differential equations. Finally, the computed bending losses of a step-index, weakly-guiding, single-mode fiber for various radii of curvature are set against the losses obtained via several approximate theoretical expressions existing in literature.

2 Theoretical considerations

We are interested in mode propagation along the ϕ -direction. A mode depends on ϕ through the exponential term $\exp(-j\kappa R\phi)$, where κ denotes the complex propagation coefficient along the torus and R is the radius of curvature. This implies that we drop the periodicity in ϕ by effectively cutting the fiber open.

In our approach, two electromagnetic states are considered. For the first state, the coupled system in the core (and possibly cladding) region is integrated numerically. For the second state, filamentary current sources with an exponential phase factor involving a trial propagation coefficient are considered. These are placed in a homogeneous medium with a relative permittivity corresponding to that of the cladding (or the coating if the numerical integration extends to the cladding region).

Subsequently, the reciprocity theorem is used to describe the interaction between these two states at the fiber surface. Only when the trial propagation coefficient coincides with the one of the actual propagation coefficients, the field mismatch corresponding to the equivalent boundary surface sources will vanish identically.

2.1 Solutions inside the fiber

To obtain a solution for the coupled system of ordinary differential equations inside the fiber, a standard numerical integration technique, viz., an adaptive Adams method, is used. This implies that the profile of the core (and possibly cladding) can be arbitrary, and consequently single- as well as multi-mode fibers can be analyzed. To construct the field solutions, we introduce a discrete truncated Fourier series in the ψ -direction, involving the harmonic constituents $\exp(-jm\psi)$ in one large field vector. Since the constituents are coupled, they need to be considered simultaneously.

We would like to integrate a system of coupled ordinary differential equations from the fiber axis at $\rho = 0$ to the exterior boundary at $\rho = a$. However, $\rho = 0$ is a singular point of the system of coordinates, where only half of the field vector solutions remain bounded. These regular solutions can be expanded about $\rho = 0$. Subsequently, we start with the numerical integration from $\rho = \rho_0$, with ρ_0 small.

2.2 Solutions outside the fiber

The actual field exterior to the fiber is a linear combination of all possible fields that can be generated by sources, with the indicated ϕ -dependence, inside the fiber. The corresponding expressions are very intricate. They involve products of (modified) Bessel functions,

which take the following form in our region of interest [5]:

$$I_\nu(x_1)K_\mu(x_2) = (\pi j/2) \exp[\pi j(\mu - \nu)/2] J_\nu(jx_1)H_\mu^{(1)}(jx_2), \quad (1)$$

with $\{x_1, x_2, \nu, \mu\} \in \mathbb{C}$, $\nu, \mu \sim R/a$ and $\text{Im}(\nu, \mu) < 0$. Due to possible large radii of curvature, the imaginary part of the trial propagation coefficient can be 10^{-9} times smaller than the real part. This implies that the computation of the Bessel functions has to be performed to a very high relative precision (10^{-13}), since some accuracy is lost due to cancelations. Asymptotic expansion techniques for large orders and/or arguments cannot be guaranteed to provide sufficient accuracy in the range of interest ($50 < \text{Re}(\nu) < 10^6$). Therefore, we have used the integral representations of the Bessel functions, i.e., [6]

$$\begin{aligned} H_\nu^{(i)}(x) &= (\pi j)^{-1} \int_{C_i} e^{-x \sinh t + \nu t} dt, \quad i=1,2; \\ J_\nu(x) &= (2\pi j)^{-1} \int_{C_3} e^{-x \sinh t + \nu t} dt. \end{aligned} \quad (2)$$

Initially, we chose to integrate Eq. (2) along the steepest descent path (SDP). Since ν, x are complex, no analytic expression for the SDP in the complex t -plane is available. Hence, we have to search for the path numerically. To perform this search a Van Wijngaarden-Dekker-Brent root-finding scheme has been applied.

After some extensive testing, it turned out that the integration can be accelerated considerably if we integrate along well-chosen piecewise linear path segments instead of along the SDP. One of these segments passes through the saddle point at a certain angle with the real axis. This angle is equal to the angle at which the SDP passes through the saddle point. Consequently, the integrand will hardly oscillate along the segment. Next, this segment is connected to other segments that run towards infinity along the asymptotes. To avoid overflow and underflow, we normalize the Bessel functions with respect to the value of the integrand at the saddle point.

3 Numerical validation

We have not found a single mathematical package which could be used as a reference to validate our code concerning the product of (modified) Bessel functions over the entire range of complex orders and arguments. Therefore, we have settled for a relatively small constant complex order $\nu = 2000 - 2j$, and varied the real part of the complex argument x , with $\text{Im}(x) = -500$ fixed. Mathematica [7] has been used to verify these results, as is shown in Figure 2. Mathematica fails to compute some of the 40 values shown in Figure 2, although we requested an accuracy up to 60-digits. In addition, it turned out that our subroutine is a time factor of 450 faster. The relative error between both results is about 10^{-14} , which is well within the set relative accuracy of 10^{-13} .

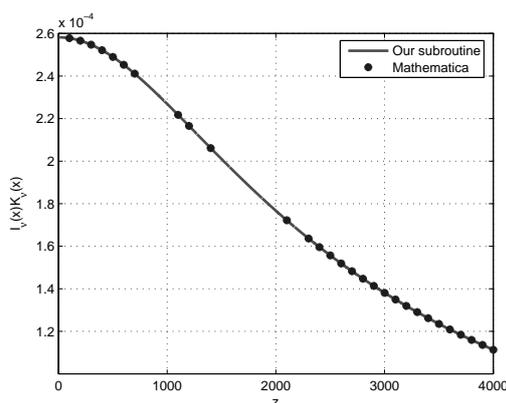


Figure 2: Product of the (modified) Bessel functions $I_\nu(x)K_\nu(x)$, where $\nu = 2000 - 2j$ and $x = z - 500j$.

To validate the computation of the bending loss, we introduce a step-index, single-mode weakly guiding fiber with an infinite cladding, although our code is not restricted to such a fiber, as discussed in Section 2.1. In Figure 3, a comparison between several theoretical expressions [2]–[4] and our full vectorial code is made. We observe that the bending losses obtained by Faustini’s expression are in quite good agreement with our full-vectorial results. However, this expression has a singularity in the integrand at $R \approx 1.8\text{mm}$ and therefore no results are available below this radius of curvature. Full-vectorial results are currently computed for smaller radii of curvature.

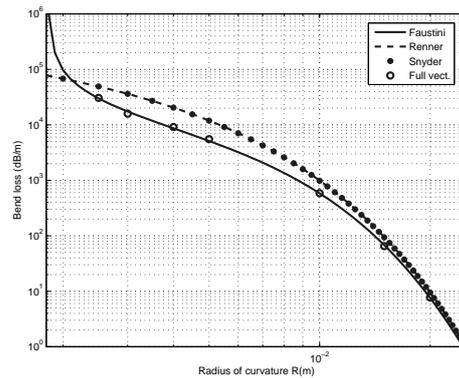


Figure 3: Bending loss of a step-index, single-mode fiber with an infinite cladding, $\lambda = 1.3\mu\text{m}$, $a = 5\mu\text{m}$, $n_{co} = 1.45$ and $n_{cl} = 1.447$. The results of our code (circles) are set against [2] (dashed line), [3] (solid line) and [4] (dots).

4 Concluding remarks

A full-vectorial analysis of the bent fiber is presented and the most important aspects of our computational scheme are discussed. We have computed the bending losses in a step-index, single-mode weakly guiding fiber with an infinite cladding for various radii of curvature and set our full-vectorial results against several approximate results existing in literature. It turns out that for large radii of curvature Faustini’s theoretical expression of the attenuation coefficient is valid.

However, a sufficiently accurate computation of the exterior field vectors and hence of the complex propagation coefficients of the modes is still very time-consuming. Moreover, the more the fiber is bent the more terms are needed in our truncated Fourier series which increases the computation time considerably. Therefore computation time becomes a crucial factor for very small radii of curvature.

References

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