Optical injection in semiconductor ring lasers

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In this contribution, we theoretically investigate optical injection in semiconductor ring lasers. Unexpected dynamical regimes are disclosed, which to the best of our knowledge have not yet been observed in any other optically injected semiconductor laser system. Through numerical simulations and bifurcation continuation, it is shown that for some values of the detuning the basin of attraction of the injection-locked solution does not cover the whole phase space, as it usually does. Two parameter regions in which two different injection-locked solutions coexist are disclosed, in addition to a region in which a stable limit cycle coexists with an injection-locked solution.

Introduction

Optically injected laser systems generally consist of a master laser whose output is coupled into the cavity of a second slave laser. These relatively simple systems nevertheless exhibit a wealth of dynamical behavior, which has been widely studied for different types of lasers [1, 2, 3].

An important class of semiconductor lasers for which optical injection has not yet been investigated are the semiconductor ring lasers or SRLs. A SRL is a semiconductor laser in which the light is confined in a circular waveguide structure. As a result, SRLs generate light in two opposite directions, referred to as the clockwise (CW) and the counterclockwise (CCW). SRLs have received increasing attention in recent years, because they are very suitable candidates as key components in photonic integrated circuits [4]. The bistable character of their directional mode operation allows them to be used in systems for all-optical switching and as all-optical memories [5, 6].

Optical injection can be particularly important in SRLs, as a control mechanism for the dynamics in switching applications or when a holding beam is used to enforce unidirectional operation in the injected direction. However, optical injection can also give rise to very intricate dynamics, which may obstruct the desired dynamical behavior.

We will focus on unidirectional optical injection, i.e. optical injection in one of the two counterpropagating modes. This will also facilitate comparison of our results with future experiments. The counterpropagating modes will be referred to as the clockwise (CW) and the counterclockwise (CCW) mode, and we will suppose that we are injecting the CW mode. We further assume that the SRL is biased in the bistable unidirectional regime.

We will perform a bifurcation analysis using the software continuation package AUTO [7], complemented by numerically solving the rate equation model. This approach allows us to compute the bifurcation diagrams for optically injected SRLs and to reveal the stability of the invariant structures present in the system.

Formulation of the model

Following Refs. [8, 9] with a straightforward modification to account for the optical injection as in [10], we can write the following rate equations for an optically injected SRL; assuming single-mode operation, neglecting spatial variations within the laser and adiabatically eliminating the polarization dynamics:

$$\frac{dE_1}{dt} = \kappa(1+i\alpha)\left[N\left(1-s|E_1|^2-c|E_2|^2\right)-1\right]E_1 - ke^{i\phi_k}E_2 - i\Delta E_1 + \frac{1}{\tau_{in}}E_i \tag{1a}$$

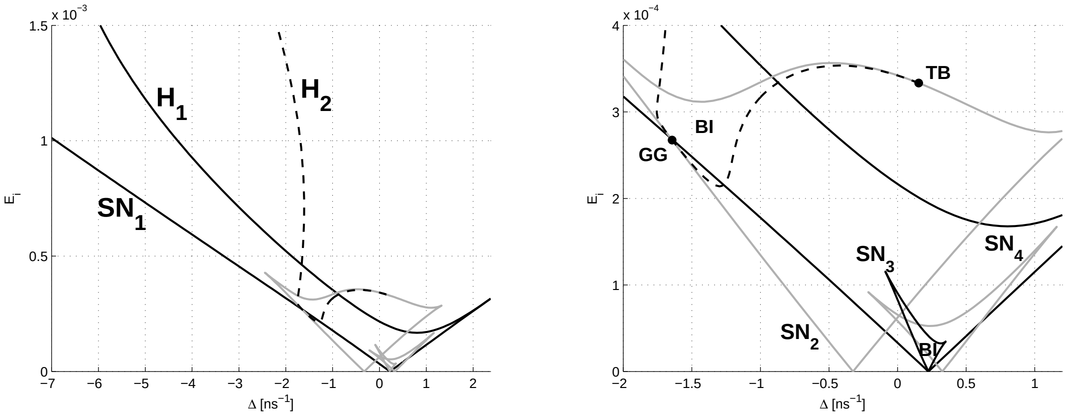
$$\frac{dE_2}{dt} = \kappa(1+i\alpha)\left[N\left(1-s|E_2|^2-c|E_1|^2\right)-1\right]E_2 - ke^{i\phi_k}E_1 - i\Delta E_2 \tag{1b}$$

$$\frac{dN}{dt} = \gamma\Big[\mu - N - N(1-s|E_1|^2-c|E_2|^2)|E_1|^2 - N(1-s|E_2|^2-c|E_1|^2)|E_2|^2\Big] \tag{1c}$$

Here t is time, E_1 and E_2 are the complex slowly varying envelopes of the counterpropagating waves, N is the carrier population inversion, μ is the renormalized injection current ($\mu \approx 0$ at transparency and $\mu \approx 1$ at lasing threshold), κ is the field decay rate, γ is the carrier decay rate, α is the linewidth enhancement factor and τ_{in} is the cavity round-trip time. The self and cross-saturation of the gains is phenomenologically modeled by s and c. Note that for a SRL, cross-saturation is always stronger than self-saturation ($c \approx 2s$). Furthermore, backscattering on defects inside the cavity results in a linear coupling between the two fields, modeled by an amplitude k and a phase shift ϕ_k. The two control parameters are the injected power E_i^2 and its detuning Δ from the longitudinal mode frequency of the SRL. A detuning $\Delta > 0$ corresponds to a higher master than slave frequency.
In a typical experimental setup, the photon lifetime $\tau_p = \kappa^{-1}$ and the carrier lifetime $\tau_c = \gamma^{-1}$ are respectively of the orders 10 ps and 5 ns, yielding two different time scales in the system. The other parameters are fixed to the realistic values $\alpha = 3.5$, $s = 0.005$, $c = 0.01$, $k = 0.4412$ ns^{-1}, $\phi_k = 1.4966$ and $\tau_{in} = 0.6$ ps [8]. The bias current μ is chosen to be 1.704, such that the SRL operates in the bistable unidirectional regime, but still relatively close to the alternate oscillation regime [11]. The detuning is varied up to 1 GHz, while the values used for E_i span several orders of magnitude, ranging from $O(10^{-7})$ up to $O(10^{-2})$.

Bifurcation analysis

The behavior of the solutions of (1) will generally vary for different values of the injected field amplitude E_i and the detuning Δ. Qualitative changes in the dynamical behavior of the system, so-called bifurcations, can be numerically detected and continued in this two-dimensional parameter space using for example the bifurcation package AUTO [7]. Figure 1 shows different regions in the (Δ, E_i)-plane bounded by bifurcation lines, with each region corresponding to different dynamical behavior.
Two of these bifurcation lines, SN_1 and H_1, are familiar. They arise in many other optically injected lasers for small injection amplitude and detuning [1, 2, 12, 13, 14]. The



(a) (b)

Figure 1: Bifurcation diagram in the (Δ, E_i)-plane, generated using the rate equations (1). Figure (b) is a blow-up of Figure (a). The curves SN_2 and SN_4 have their origin symmetrically around $\Delta = 0$, while the SN_3 curve has the same origin as the SN_1 curve in (a). The parameter regions in which two injection-locked solutions coexist are denoted by BI.

region in parameter space confined between these two lines contains a stable injection-locked solution, where the SRL is phase-locked to the injected signal. However, the bifurcation diagram in Figure 1 reveals the presence of bifurcation lines which are not present for a regular semiconductor laser. The H_2 line is a different Hopf bifurcation; while the SN_2, SN_3 and SN_4 lines in Figure 1(b) are saddle-node bifurcations, which to the best of our knowledge are all novel in optically injected laser systems. The four saddle-node bifurcations can each be related to one of the steady state solutions in the solitary SRL. The H_2 Hopf bifurcation is supercritical and the associated stable periodic solution disappears when crossing the H_2-line upwards, where it turns into an injection-locked solution. This injection-locked solution s_{bi} is different from the injection-locked solution s_{cw} which is generated at the saddle-node bifurcation SN_1. Both solutions are phase-locked to the master laser, but their directional suppression ratio differs; s_{cw} has the optical power concentrated in the mode in which we optically inject (CW), while s_{bi} has approximately equal powers in both modes. Below that region, the injection-locked solution s_{cw} coexists with the stable periodic solution (i.e. oscillating output intensities), associated to the H_2 bifurcation. The presence of the SN_3 line—which is related to the CCW unidirectional solution of the solitary SRL—confirms the intuitive reasoning that due to the symmetry of the solitary SRL (stable CW and CCW states), two separate injection-locked states should originate from the CW and the CCW solution at low injection power. When crossing the SN_3 line from below, an injection-locked solution s_{ccw} appears near the original CCW solution through a saddle-node bifurcation. It is the CCW equivalent of the s_{cw} solution associated to the SN_1 line.

Conclusion

In this paper, we have theoretically investigated optical injection in SRLs. Starting from a rate equation model, we used numerical simulations and a bifurcation analysis to reveal all the relevant dynamical regimes that will unfold for different parameter values. We have shown that the bistability of the SRL leads to different parameter regions in which two injection-locked states coexist. One region contains two unidirectional injection-locked solutions in which either the CW or the CCW mode dominates. The second region contains one unidirectional solution (CW), and one bidirectional solution where both counterpropagating modes have approximately equal powers.

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