Novel approach for frequency conversion of optical signals using time-dependent metamaterials

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It is nowadays quite difficult to achieve accurate frequency conversion of electromagnetic signals without generating unwanted sidebands. In this contribution, we propose a novel mechanism to shift the frequency of optical signals. We apply the framework of transformation optics to mimic the cosmological redshift inside metamaterials. We therefore calculate the material properties that implement the Robertson-Walker metric. We discuss how such a medium with well-defined time-dependent material parameters can change the optical frequency even though it supports linear electromagnetic waves and we demonstrate perfect redshifting or blueshifting of an electromagnetic wave without the creation of sidebands.

Transformation Optics

Transformation optics, first proposed independently by Pendry [1] and by Leonhardt [2], provides a new way of looking at the interaction between light and matter. This recently developed method in electromagnetism is based on the analogy between the macroscopic Maxwell's equations in complex materials and the free-space Maxwell's equations on the background of an arbitrary metric. It describes how a general topological deformation of reality can be implemented using materials with nontrivial constitutive parameters. The design of an invisibility cloak, which bends the electromagnetic reality around a hole that turns invisible, is probably the most famous example of this technique [1, 2]. Other spatial transformations have been applied to the design of electromagnetic concentrators, beam bends, beam shifters, polarisation rotators, and subwavelength cavities, amongst others.

Four-Dimensional Transformation Optics

Transformation optics, however, applies to a general four-dimensional metric, and thus extends beyond the contemporary applications. Electromagnetic effects that occur due to the background of a general relativistic metric can thus be generated within dielectrics. Two intriguing examples of space-time transformational media have been demonstrated using this approach: an optical analogue of an event horizon and an optical analogue of the Aharonov-Bohm effect [3]. Both examples—involving the study of light propagation in moving media—have a fundamental theoretical importance. Hitherto, there has been little attention for space-time transformational media in practical applications.

The Robertson-Walker Metric

In this contribution, we demonstrate how the four-dimensional Robertson-Walker metric from general relativity can be implemented within dielectric metamaterials [4]. To study the evolution and structure of the universe in its totality, we appeal to the cosmological models. One of these models is the Robertson-Walker metric, in which one postulates that the universe is spatially homogeneous and isotropic but can be evolving in time. In its most general appearance, this solution is given by

$$ds^2 = -c^2 \mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2\, d\Omega^2\right]. \tag{1}$$

The dimensionless scale factor $a(t)$ encodes the evolution of this universe and is related to the famous Hubble parameter $H = \dot{a}/a$, where $\dot{a}$ refers to the derivative with respect to time. The curvature parameter κ is a measure of the spatial curvature. It can take on any real value: a negative value is refered to as an open universe, a positive value is called closed, and $\kappa = 0$ is flat.
A photon emitted at a frequency ω_{em} at the instance t_{em} will be measured at a different frequency ω_{obs} at the time t_{obs}. The relation between these two frequencies is given by:

$$\frac{\omega_{\mathrm{obs}}}{\omega_{\mathrm{em}}} = \frac{a(t_{\mathrm{em}})}{a(t_{\mathrm{obs}})}. \tag{2}$$

In the case of an expanding universe $a(t_{\mathrm{obs}})$ will be bigger then $a(t_{\mathrm{em}})$ and so the observed frequency will be lower—redshifted—than the emitted frequency. This effect has been labeled with the name *cosmological redshift*.

Optical Analogue of the Cosmological Redshift

One can implement this metric, Eq. (1), inside a metamaterial using the equivalence relations of transformation optics [1, 2]. The cosmological redshift also occurs in the spatially flat implementation of the metric ($\kappa = 0$) and so we can restrict ourselves to this case. In cartesian coordinates, this metric can be written as:

$$ds^2 = -c^2 \mathrm{d}t^2 + a^2(t)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]. \tag{3}$$

The permittivity and permeability therefore have to staisfy

$$\varepsilon^i{}_j = \mu^i{}_j = a(t)\delta^i_j. \tag{4}$$

The Robertson-Walker metric without spatial curvature can thus be translated into an isotropic homogeneous dielectric whose permittivity and permeability equal the scale factor $a(t)$ at all times.
The electromagnetic wave solutions inside such a medium are found by solving Maxwell's equations in the absence of free sources and currents. In combination with Eq. (4), Maxwell's equations combine into a generalized wave equation,

$$\Delta\mathbf{E} - \frac{1}{c^2}\frac{\partial}{\partial t}\left(a(t)\frac{\partial}{\partial t}(a(t)\mathbf{E})\right) = 0, \tag{5}$$

where c is the speed of light in vacuum. Its solutions are waves with time-dependent amplitude propagating along an arbitrary direction labeled by the unit vector $\mathbf{1}_\mathrm{k}$:

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{a(t)} G\left(\mathbf{r}\cdot\mathbf{1}_\mathrm{k} - c\int_{t_0}^{t} \frac{\mathrm{d}\tilde{t}}{a(\tilde{t})}\right)\mathbf{1}_\mathrm{E}, \tag{6}$$

where $G(x)$ is an arbitrary differentiable function. A straightforward calculation of the instantaneous frequency $\omega_\mathrm{inst}(t) = -\partial_t(\mathbf{k}\cdot\mathbf{r} - |\mathbf{k}|c\int_{t_0}^{t}\mathrm{d}\tilde{t}/a(\tilde{t}))$ reveals that the frequencies at two different times t_1 and t_2 satisfy $\omega_\mathrm{inst}(t_2)/\omega_\mathrm{inst}(t_1) = a(t_1)/a(t_2)$, in agreement with the redshift formula of Eq. (2).

Frequency conversion of optical signals

In this section, we demonstrate that a device of finite length offers a novel approach to shift the frequency of an electromagnetic wave using linear time-dependent materials. We therefore consider the setup as shown in Fig. 1. To simplify the notation, we introduce the function

$$f(t) = \int_{t_0}^{t} \frac{\mathrm{d}\tilde{t}}{a(\tilde{t})}. \tag{7}$$

We illuminate the dielectric from the left in region (I) with a monochromatic plane wave of frequency $\omega_0 = |\mathbf{k}_0|c$. To calculate the wave that is emitted in region (III), we calculate the general electromagnetic wave solutions inside each region and combine them with the appropriate boundary conditions. In general, each interface would give rise to transmitted and reflected waves, and one would explicitly impose continuity of the tangential components of both $\mathbf{E}$ and $\mathbf{H}$. However, since the impedance $\eta = (\mu/\varepsilon)^{1/2}$ is the same on both sides of each interface, we can anticipate that there are no reflected waves. We therefore restrict our attention to the electric field $\mathbf{E}$, considering purely right-moving waves and imposing continuity at each interface.

By imposing the continuity of the tangential component of the electric field at $z = 0$, we find for the electric field in region (II)

$$\mathbf{E}_\mathrm{II}(L,t) = \frac{a\left(f^{-1}\left(f(t)-\frac{L}{c}\right)\right)}{a(t)} A_0 \mathrm{e}^{-\mathrm{i}k_0 c f^{-1}\left(f(t)-\frac{L}{c}\right)}\mathbf{1}_\mathrm{E}, \tag{8}$$

where we have introduced the inverse function f^{-1}. To retrieve a general expression for the electric field in region (III), we have to express the continuity of the electric field at the rightmost boundary ($z = L$) and we ultimately find that

$$\mathbf{E}_\mathrm{III}(z,t) = \mathbf{E}_\mathrm{II}\left(L, t - \frac{z-L}{c}\right). \tag{9}$$

The instantaneous frequency of the electromagnetic wave emitted in the vacuum region (III) behind the device ($z = L$) can be written as

$$\omega_\mathrm{out} = \omega_0 \frac{a\left(f^{-1}\left(f(t)-\frac{L}{c}\right)\right)}{a(t)}. \tag{10}$$

We know that $L = \int_{t_1}^{t_2} c/a(t)\,\mathrm{d}t = c\,(f(t_2) - f(t_1))$, where t_1 and t_2 indicate the time of incidence and departure of a wave front. If we are observing at time t at the rightmost

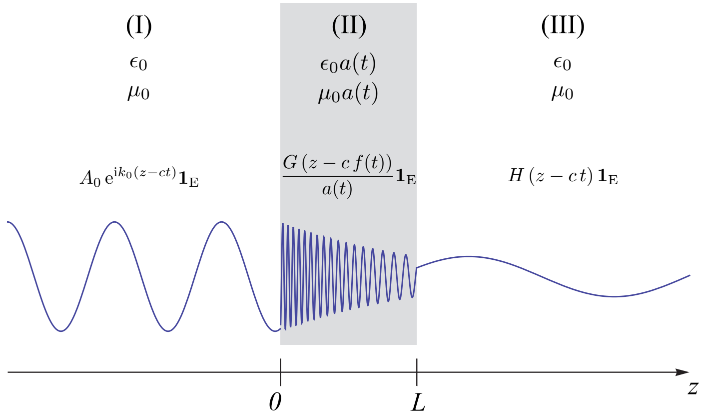

Figure 1: We consider a finite Robertson-Walker device, which is illuminated at the leftmost boundary by a monochromatic wave of frequency ω_0 and amplitude A_0. We show that the output undergoes a frequency shift according to the cosmological redshift formula.

boundary, then $f^{-1}(f(t) - L/c)$ corresponds to the time when the wave front was at the leftmost boundary. Hence, we retrieve the cosmological redshift formula, $\frac{\omega_{\text{inst}}(t_2)}{\omega_{\text{inst}}(t_1)} = \frac{a(t_1)}{a(t_2)}$. To conclude, we discuss how the material parameters, as given by Eq. (4), can be implemented by metamaterials. We can generate the permittivity variation by low-frequency electro-optic modulation. The most difficult part is the time evolution of the permeability. This can be achieved by introducing an array of split-ring resonators in the electro-optic material. Operating far below resonance, one can approximate the permeability by $\mu(\omega) \approx 1 + F\omega^2 c^2 l^2 \varepsilon_C/(dw)$, with l, d and w the geometrical parameters of the split rings. By properly choosing the electro-optic material inside the gap of the split rings, we can thus vary the permeability and permittivity according to Eq. (4) with electro-optics to establish a frequency shift determined by Eq. (2).

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