Metamaterials Transforming the Frequency of Optical Pulses

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Transformation optics allows for the implementation of general relativistic effects inside optical systems. We present a time-dependent metamaterial device that mimics the cosmological redshift. Theoretically, this linear medium requires a monotonically time-varying permittivity and permeability. Our recent results, however, indicate that it is possible to alter the frequency of optical pulses in a medium with solely a modulated permittivity. Furthermore, it is shown that the overall frequency shift does not depend on the actual variation of the permitivity or the inclusion of material losses. These robust features suggest a feasible technological implementation for frequency conversion of optical pulses.

Transformation optics

Transformation optics [1, 2] is a novel framework in electromagnetism that is based on the analogy between the macroscopic Maxwell's equations in complex materials and the free-space Maxwell's equations on the background of an arbitrary metric. It allows to design optical devices with nontrivial parameters from the perspective of a topological deformation of the background metric. The intuitive geometrical approach of transformation optics already generated various novel applications within the fields of invisibility cloaking [1, 2], electromagnetic beam manipulation [3], optical information storage [4] and imaging [5].
Nevertheless, the framework of transformation optics is not limited to three-dimensional deformations and can be extended to four-dimensional metric transformations, which allow for the implementation of metrics that occur in general relativistic or cosmological models [6]. This enables, for example, the implementation of black hole phenomena inside dielectrics with exotic material parameters [7, 8, 9].

The cosmological redshift

According to several cosmological measurements, the distance between distant points of the universe increases as a function of time. Consequently, a photon is frequency shifted as it is travelling from one point in spacetime to another. If it is emitted at a frequency $\omega_{\rm em}$ at the instance $t_{\rm em}$, it will be measured at a different frequency $\omega_{\rm obs}$ at the time $t_{\rm obs}$. These two frequencies are related by:

$$\frac{\omega_{\rm obs}}{\omega_{\rm em}} = \frac{a(t_{\rm em})}{a(t_{\rm obs})}. \tag{1}$$

In the case of an expanding universe $a(t_{\text{obs}})$ will be bigger than $a(t_{\text{em}})$ and so the observed frequency will be lower—redshifted—than the emitted frequency. This effect has been labelled with the name *cosmological redshift*.
A well-known cosmological model of such an expanding universe is the Robertson-Walker metric, in which one postulates that the universe is spatially homogeneous and isotropic but can be evolving in time. The spatially flat solution is given by

$$ds^2 = -c^2 \mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + r^2\, d\Omega^2\right], \tag{2}$$

where the dimensionless scale factor $a(t)$ encodes the evolution of this universe. It is related to the famous Hubble parameter $H = \dot{a}/a$, where $\dot{a}$ refers to the derivative with respect to time.

The transformation-optical analogue of the cosmological redshift

One can now try to mimic the cosmological redshift inside a metamaterial by using the equivalence relations of transformation optics applied to the Robertson-Walker metric [10]. It can then be found that the permittivity and permeability tensors have to satisfy

$$\varepsilon^i{}_j = \mu^i{}_j = a(t)\delta^i_j. \tag{3}$$

The Robertson-Walker metric without spatial curvature is thus electromagnetically equivalent to an isotropic homogeneous dielectric whose permittivity and permeability equal the scale factor $a(t)$ at all times.
By combining the Maxwell's equations with the constitutive parameters into a generalized wave equation, one can demonstrate that the electromagnetic wave solutions of this “Robertson-Walker-medium” are waves with time-dependent amplitude propagating along an arbitrary direction labelled by the unit vector $\mathbf{1}_{\mathrm{k}}$:

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{a(t)} G\left(\mathbf{r}\cdot\mathbf{1}_{\mathrm{k}} - c\int_{t_0}^{t} \frac{\mathrm{d}\tilde{t}}{a(\tilde{t})}\right)\mathbf{1}_{\mathrm{E}}. \tag{4}$$

In the preceding expression $G(x)$ is an arbitrary differentiable function. By defining the instantaneous frequency as the derivative of the phase with respect to time, it can be shown that the instantaneous frequencies at two different times t_1 and t_2 satisfy $\omega_{\text{inst}}(t_2)/\omega_{\text{inst}}(t_1) = a(t_1)/a(t_2)$, which is in agreement with the redshift formula of Eq. (1).

Frequency conversion using realistic time-dependent metamaterials

We now demonstrate that the frequency shift also occurs in more realistic devices, considering the fact that practical devices have a finite extent and bounded material parameters. Firstly, we show that a device of finite length offers a novel approach to shift the frequency of an incident electromagnetic wave. We therefore consider the setup as shown in Fig. 1.
The dielectric is illuminated from the left in region (I) with a monochromatic plane wave of frequency $\omega_0 = |\mathbf{k}_0|c$. To calculate the wave that is emitted in region (III), we calculate the general electromagnetic wave solutions inside each region and combine them with the appropriate boundary conditions. By imposing the continuity of the tangential component of the electric field (since the material is impedance matched, the magnetic boundary

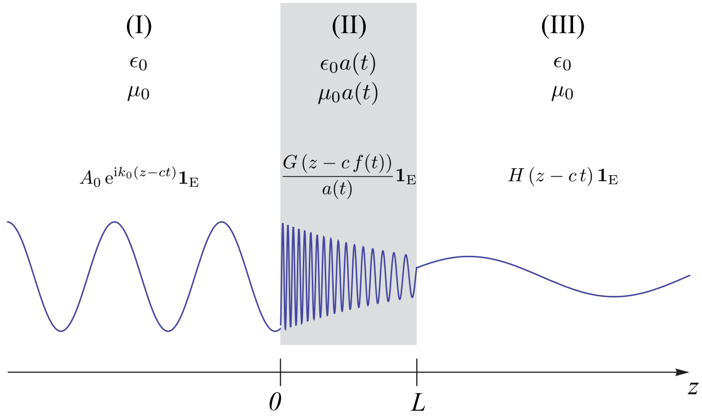

Figure 1: We consider a finite Robertson-Walker device, which is illuminated at the leftmost boundary by a monochromatic wave of frequency ω_0 and amplitude A_0. We show that the output undergoes a frequency shift according to the cosmological redshift formula.

condition will not differ for the electric one) at $z = 0$, we find for the electric field in region (II)

$$\mathbf{E}_{\mathrm{II}}(L,t) = \frac{a\left(f^{-1}\left(f(t)-\frac{L}{c}\right)\right)}{a(t)} A_0 e^{-ik_0 c f^{-1}\left(f(t)-\frac{L}{c}\right)}\mathbf{1}_{\mathrm{E}}, \tag{5}$$

where we have introduced the function

$$f(t) = \int_{t_0}^{t} \frac{d\tilde{t}}{a(\tilde{t})} \tag{6}$$

to simplify the notation.

The instantaneous frequency of the electromagnetic wave emitted in the vacuum region (III) behind the device ($z = L$) can be written as

$$\omega_{\mathrm{out}} = \omega_0 \frac{a\left(f^{-1}\left(f(t)-\frac{L}{c}\right)\right)}{a(t)}. \tag{7}$$

We know that $L = \int_{t_1}^{t_2} c/a(t)\,dt = c\,(f(t_2)-f(t_1))$, where t_1 and t_2 indicate the time of incidence and departure of a wave front. If we are observing at time t at the rightmost boundary, then $f^{-1}\,(f(t)-L/c)$ corresponds to the time when the wave front was at the leftmost boundary. Hence, we retrieve the cosmological redshift formula, $\frac{\omega_{\mathrm{inst}}(t_2)}{\omega_{\mathrm{inst}}(t_1)} = \frac{a(t_1)}{a(t_2)}$. Subsequently, we have performed numerical simulations to calculate the effect of bounded material parameters. Indeed, the material parameters that implement an "ideal Robertson-Walker dielectric," Eq. (3), are continuously growing as a function of time. We therefore investigated in these simulations what would happen if the material parameters follow a saw tooth shaped evolution. Obviously, the width of the rising slope is determined by the width of the incident pulses, as can be seen in Fig. 2. Secondly, the overall frequency shift will be limited by the range over which the material parameters can vary, as can be inferred from Eq. (1). The numerical simulations, however, clearly indicate that despite the two preceding considerations, the overall performance of the finite frequency converter

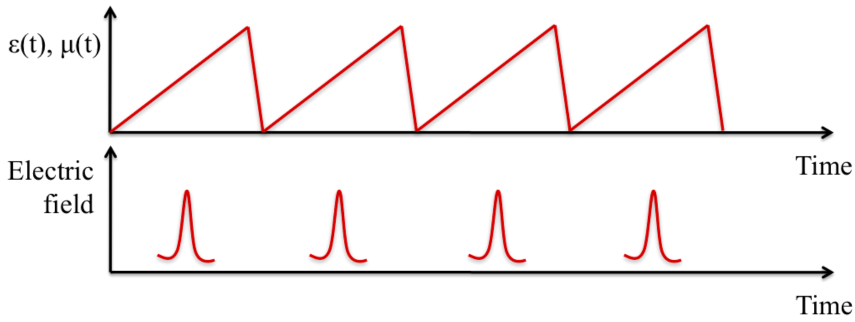

Figure 2: Setup of the time-dependent frequency converter for optical pulses. The material parameters rise linearly when the pulse is inside the converter and fall back in between two incident pulses. As a result, they follow a saw tooth shaped evolution as a function of time.

is not affected by introducing the saw tooth evolution, even when considerable losses are introduced. These results open up the possibility to fabricate this frequency converting device with currently available metamaterials.

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