Full Rate-equation Description for Multi-mode Semiconductor Lasers

Daan Lenstra[1,*], Mirvais Yousefi[2]

[1]Photonic Integration Group, Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands
[2]Photonic Sensing Solutions, Amsterdam, The Netherlands
*d.lenstra@tue.nl

A set of rate equations is presented describing the deterministic multi-mode dynamics of a semiconductor laser. Mutual interactions among the lasing modes, induced by high frequency modulations of the carrier distribution, are described by carrier-inversion moments and lead to special spectral content of each spatial mode. The Bogatov effect of asymmetric gain suppression in semiconductor lasers will be derived. We will explicitly discuss the nontrivial relationship between the modes of the nonlinear cavity and the optical spectrum of the laser output and illustrate this for a two and three-mode laser.

Introduction

Many phenomena in multi-mode semiconductor diode lasers have their origin in the nonlinear dynamical evolution of the lasing modes and certain hole-burning (grating) effects in the population inversion. The numerical analysis of the resulting, often complicated, dynamics would benefit greatly from a set of coupled rate equations capable of describing the full deterministic dynamics of the lasing modes as well as their mutual interactions. Such description in terms of first-order ordinary differential equations (ODEs) is thus perfectly suited for an efficient bifurcation analysis using existing standard tools. Moreover, the inclusion of spontaneous-emission noise in the modes and recombination noise in the carriers in such formalism is straightforward and could easily be carried through.

In this paper we report on the derivation of multi-mode rate equations for a semiconductor laser in terms of ODE's for the complex modal field amplitudes, the overall population inversion and the various lasing-induced population-inversion gratings (spatial hole burning). This is a big advantage over existing theories, which are based on complicated partial-differential and/or integral equations [1-3]. We take into account two mode-mode interaction mechanisms: (i) carrier sharing and (ii) mutual coherent optical injection. Different modes are identified by their spatial profiles in the laser cavity and described by their complex amplitudes. The full electron-hole density in the active medium is expanded in a complete set of base functions, which allow the introduction of inversion moments describing not only the formation of gratings burned by the modal fields, but also the mutual injection of modal fields. Thus, we reproduce the Bogatov-effect on asymmetric side-mode suppression [4] as well as the peculiar periodic multi-mode switching scenario reported and explained in [5] and [6]. The full derivation of the multi-mode rate equations will be published elsewhere.

Multi-mode rate equations

The rate equations for a semiconductor laser operating in M longitudinal modes are given by

$$\frac{d}{dt}E_k(t) = -\frac{1}{2}(\Gamma_k - g_k)E_k(t) + \frac{1}{2}\sum_{j=1,..M}\sum_{n} f_{kj;n}\xi_j(1+i\alpha_j)N_n E_j(t)e^{i\omega_{jk}t}; \quad (1)$$

$$\frac{dN_n}{dt} = \Delta J_n - \frac{N_n}{T_n} - \mathrm{Re}\left\{\sum_{jk} g_j\, f_{jk;n} E_j^* E_k e^{i\omega_{kj}t}\right\} - \mathrm{Re}\left\{\sum_{jkm} \xi_j(1+i\alpha_j)\, f_{jk;nm} N_m E_j^* E_k e^{i\omega_{kj}t}\right\}, \quad (2)$$

for j,k=1,…,M and m,n=0,1,2,… and where the meaning of the symbols is summarized in Table 1. Using sine functions as the spatial profiles the longitudinal modes and expanding the population inversion in cosine functions, the f-coefficients are given by:

$$f_{jk;0} = \delta_{jk};\quad f_{jk;0m} = f_{jk;m0} = f_{jk;m},\ (\forall m);\ f_{jk;m} = \frac{1}{\sqrt{2}}[\delta_{m,|k-j|} - \delta_{m,k+j}],\ (m \ge 1);$$
$$f_{jk;nm} = \frac{1}{2}[\delta_{|j-k|,|n-m|} + \delta_{|j-k|,n+m} - \delta_{j+k,n+m} - \delta_{j+k,|n-m|}],\quad (n,m \ge 1). \quad (3)$$

The formation of inversion gratings (described by the moments N_m) is regulated by the spatial diffusion of carriers in the semiconductor medium. For this reason, the diffusion of carriers has to be considered in the dynamics of multimode semiconductor lasers. The grating formation is responsible for such effects as the gain asymmetry which favours longer wavelengths over shorter ones (in case of a positive alpha parameter and the other way around for negative alpha).

Table. 1 Explanation and meaning of the various symbols in (1) and (2)

	Unit	Name	Value in Figs.1and 2
E_k	1	Field amplitude (complex) for mode k	
N_0	1	Overall population inversion w.r.t. lasing threshold	
N_m	1	m-th population inversion moment	
k,j	1	Mode number	1,2,3
m,n	1	Inversion-moment number	0,1,2
α_k	1	Linewidth parameter for mode k	3
Γ_k	1/s	Cavity loss rate for mode k	$1ps^{-1}$
g_k	1/s	Linear gain for mode k	$g_1 = 0.9996ps^{-1}, g_2 = 0.9999ps^{-1}, g_3 = 1.0ps^{-1}$
ξ_k	1/s	Differential gain coefficient for mode k	$5000\,s^{-1}$
$f_{kj;n}$	1	Coupling coefficient of modes k and j via n-th moment	see (3)
$f_{kj;nm}$	1	Coupling coefficient of moments n and m via modes j and k	see (3)
$\omega_{jk} \equiv \omega_j - \omega_k$	Rad/s	Angular mode-frequency difference	$\Delta\omega = 2\pi * 12\,GHz$
ω_k	Rad/s	Optical angular frequency of mode k	$\omega_3 > \omega_2 > \omega_1$
ΔJ_k	1/s	k-th injection-current moment w.r.t. threshold current	$\Delta J_0 = 1.0\,10^{17}\,s^{-1}$
T_m	s	lifetime for moment m	1 ns

Numerical Results

An interesting consequence of mode-mode interaction through grating formation is illustrated in Fig.1. Two modes are considered with mode spacing 80 GHz and equal losses. In the absence of hole-burning, i.e. no induced carrier grating (N_1 =0), the system would be indifferent as to which mode will be excited: the total laser power will be distributed over the two modes

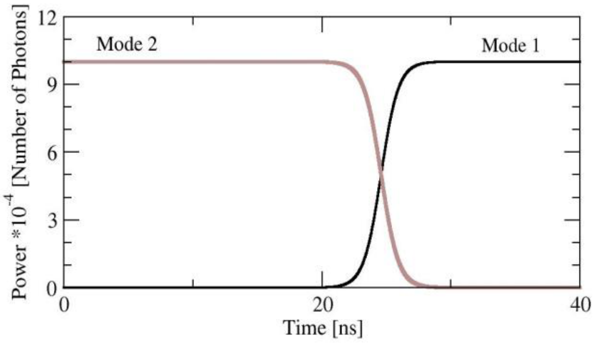

FIG. 1: Transition of lasing mode due to modal interaction: the frequency spacing between the two modes is taken as 80GHz. The system is prepared in the short wavelength mode and gradually evolves to the long-wavelength mode, because of the Bogatov effect (see text below). The time is measured in nanoseconds and the power is given in number of photons. Parameters are as in Table 1. The laser is pumped twice above threshold.

determined by initial conditions. When the carrier grating is taken into account (N_1 =0), the system becomes bi-stable, meaning that either one of the modes can be lasing, the other being off. However, only one of the modes is stable and for positive alpha this will be the long-wavelength mode. This effect, named Bogatov effect [4], is also responsible for the asymmetric side-mode suppression observed in multi-mode semiconductor lasers. It is also sometimes referred to as a four-wave mixing process. The qualitative explanation for this phenomenon is that if two modes are lasing simultaneously, their fields create an oscillating grating in the inversion (N_1). This grating causes the waves in each mode to be scattered into the other mode thus creating additional gain or loss in that other mode.

Fig.2 shows the simulation result for a 3-mode laser with mode spacing 12 GHz. The modes have slightly different linear gains as indicated in Table 1. Once there is power in mode 3 (dashed line in Fig. 2), it automatically feeds the other modes through the effective dynamical (Bogatov) gain effect (see the above-given discussion). The reddest mode (1) takes the most advantage of this gain as it is most favored by the dynamical gain and starts building up power. This decreases the amount of gain available for operation in mode 3. While the power builds up in mode 1, mode 2 experiences two effects, i.e. suppression by mode 1 and enhancement by mode 3. Apparently, mode 2 survives only during the short time interval, where mode 1 is decreasing under influence of its higher loss and mode 3 is recovering. As soon mode 1 starts to grow, it effectively suppresses mode 2. This cycle then repeats itself. We note in Fig.2 that the relaxation oscillation (~3.8 GHz) plays an important intermingling role in the above-described scenario and that the slow dynamics-induced oscillation corresponds to ~760MHz, which seems to define a period-5 limit cycle.

Another interesting aspect of these dynamics becomes evident once we look at the optical spectra in Fig.3 for the same case as Fig.2. Each mode contains spectral components of itself and other modes. Interestingly, dynamics at the frequency of the middle mode (mode 2) have been fully suppressed as can be concluded from the absence of any substantial spectral content at ~12 GHz. Clearly, mode 2 is only driven by the injection fields of the side modes and generates no frequency content at its “own” frequency. Mode 2 is used rather as ‘stepping stone’ while the energy bounces between mode 1 and mode 3. Although there are three active (spatial) modes inside the laser, a diffraction-grating-resolved spectrum taken from the laser output would show only two dominant frequencies separated by ~24 GHz (and higher harmonics).

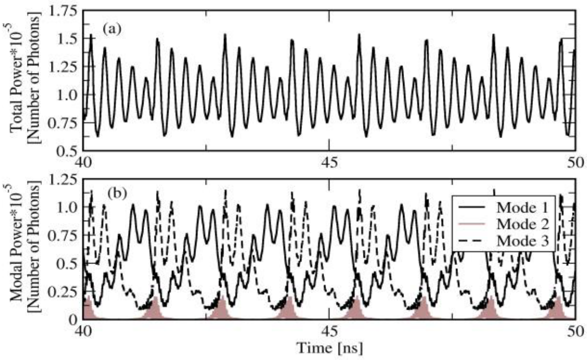

Fig.2 Sequential on-off switching of modes in the mode-resolved time series. (a) shows the total photon number and (b) the power of the individual modes. The parameters are as in the table. The mode spacing is $\Delta\omega = 2\pi * 12$ GHz. Note the period-5 dynamics.

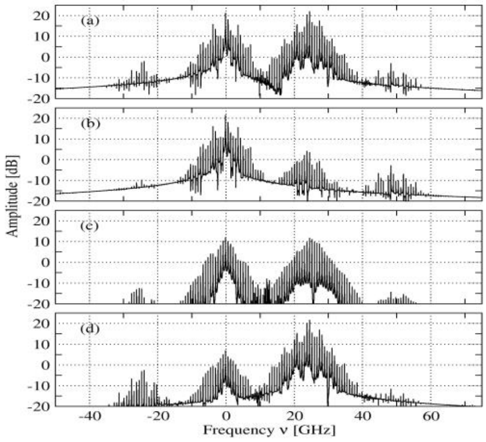

Fig.3 Optical spectra of the dynamics depicted in Fig.2. Plot (a) shows the spectrum of the total field $\Sigma E_j(t)e^{i\omega_j t}$, plots (b), (c) and (d) show modes 1, 2 and 3, respectively. The relaxation oscillation is ~3.6 GHz and the ~720 MHz peak fine structure corresponds to the period-5 oscillation in Fig.2 and is caused by the system dynamics and mode competition. Note that each mode contains spectral components of itself and other modes.

Conclusion

A new rate-equation model for a multi-mode semiconductor laser has been developed and shown to be applicable to two and three-mode lasers. For a positive value of the α-parameter, side modes with longer wavelengths compared to the dominant mode will be amplified, whereas side modes with shorter wavelengths will be suppressed (Bogatov effect).

The simulated spectrum shows that a straight correspondence between modes and externally observed spectral peaks is not possible due to the parametric interactions. The spectral contents belonging to one mode shows aspects of other modes as well. An observation of the output field will not provide full information on each modal amplitude inside the laser. This is clearly visible in the spectrum shown in Fig. 3.

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