# Spatial polarization domain-wall in colloidal semiconductor nanocrystals

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*Optical domain-walls can be formed at the boundary separating two distinguishable adjacent laser beams. They constitute the fundamental soliton associated with Berkhoer and Zakharov modulational instability. These solitons exist in defocusing Kerr-type materials providing that cross-phase modulation is higher than self-phase modulation. To date, these solitons have only been experimentally observed in the temporal domain. In this work, we investigate the formation of spatial domain-walls in colloidal semiconductor nanocrystals by numerical simulations.*

## Introduction

In anomalous nonlinear Kerr media, it is well known that bright solitons are related to modulational instability. In the same way, Haelterman *et al.* predicted the existence of a new kind of soliton related to the Berkhoer and Zakharov modulational instability that occurs in the normal dispersion regime of propagation [1]. Such structures can be formed when cross-phase modulation is higher than self-phase modulation. They consist in the transition between two domains of opposite circular polarizations, and were therefore named “polarisation domain wall solitons” (PDW) [2]. The existence of PDW has experimentally been demonstrated in the temporal domain in 1999 [3].
Up to now, the domain-wall soliton has not yet been observed in the spatial domain, probably because its observation requires the propagation of light beam in a medium exhibiting a strong negative instantaneous nonlinear refractive index ($n_2$). In this work, we propose to make use of colloidal semiconductor quantum dots (CSQD), as they can present the required strong negative $n_2$ [4]. Although the crystalline structure of semiconductors is not isotropic, the use of CSQD ensures isotropy on a macroscopic scale, which in turns implies that cross-phase modulation effects are two times higher than self-phase modulation.
The geometry of our system is as follows: two orthogonally polarized beams are copropagating side by side so that the soliton is formed at their boundary. Below we report on the numerical simulations that were performed in order to determine the experimental conditions enabling the observation of the spatial PDW soliton.

## Propagation Equations

The propagation of two orthogonal circularly polarized optical beams in an isotropic medium with nonresonant electronic instantaneous defocusing Kerr nonlinearity can be described by the normalized coupled nonlinear schrödinger equations:

$$\begin{aligned} \mathrm{i}\partial_z A_+ + \frac{1}{2}\Delta_\perp A_+ - (|A_+|^2 + 2|A_-|^2)A_+ &= 0, \\ \mathrm{i}\partial_z A_- + \frac{1}{2}\Delta_\perp A_- - (|A_-|^2 + 2|A_+|^2)A_- &= 0, \end{aligned}$$

where $A_+$ ($A_-$) is the slowly varying beam envelope of the right (left) handed circularly polarized electric field, normalized with respect to $\sqrt{3\gamma L_D}$, with $\gamma = \frac{2\omega n_2}{9c} < 0$, the nonlinear coefficient and $L_D = ka^2$ with $k$ the wave number in the material; $\Delta_\perp = \partial^2_{xx} + \partial^2_{yy}$ denotes the transverse Laplacian where $x$ and $y$ are normalized with respect to $a$; and $z$ is the longitudinal coordinate normalized with respect to $L_D$.

## Numerical results

The two beams were propagated using the well-known split-step Fourier method. Since PDW corresponds to the transition between two beams of constant amplitude, we have first analyzed how the PDW formation is affected by the finite size and the shape of the beam profile modulating the PDW. The initial condition consists of two half-supergaussian beams, right- and left-handed polarized, while the transition is an hyperbolic tangent of variable width $w_0$:

$$\begin{aligned} A_+ &= \left(\frac{1+\tanh\left(\frac{y}{w_0}\right)}{2}\right) \mathrm{e}^{-\left(\frac{x}{x_0}\right)^{2g}} \mathrm{e}^{-\left(\frac{y}{y_0}\right)^{2g}}, \\ A_- &= \left(\frac{1+\tanh\left(-\frac{y}{w_0}\right)}{2}\right) \mathrm{e}^{-\left(\frac{x}{x_0}\right)^{2g}} \mathrm{e}^{-\left(\frac{y}{y_0}\right)^{2g}}, \end{aligned}$$

where $g$ is the order of the supergaussian.

Figure 1a shows clearly that using a PDW inscribed onto a broader square-shaped ($g > 1$) beam, the carrier beam broadens as a result of diffraction, while the domain-wall propagates without distortion as a result of the interaction between diffraction and nonlinearity [5].

In a practical experiment, it would be desirable to simplify the beam shaping as much as possible. Therefore, we investigate the case were the carrier beam is Gaussian ($g = 1$). Figure 1b shows that with Gaussian beams, since the domains are not pseudo-infinite, the diffraction of the Gaussian beam induces a change of peak power that in turn alters the domain-wall, *i.e.* the change in peak power lowers the magnitude of the nonlinear effect, which results in the broadening of the domain-wall.

As the broadening of the PDW inscribed on a Gaussian beam prevents us to identify the PDW as a structure that propagates undistorted, we will define a parameter V, called “visibility”, allowing for the comparison between the linear and the nonlinear propagation regimes. This parameter is defined as the ratio $\frac{w_L}{w_{NL}}$ between the width of the domain wall (at an intensity value of $I = 0.5$) after propagation in the linear ($L$) and the nonlinear ($NL$) regime:

$$V = \frac{w_L}{w_{NL}}.$$

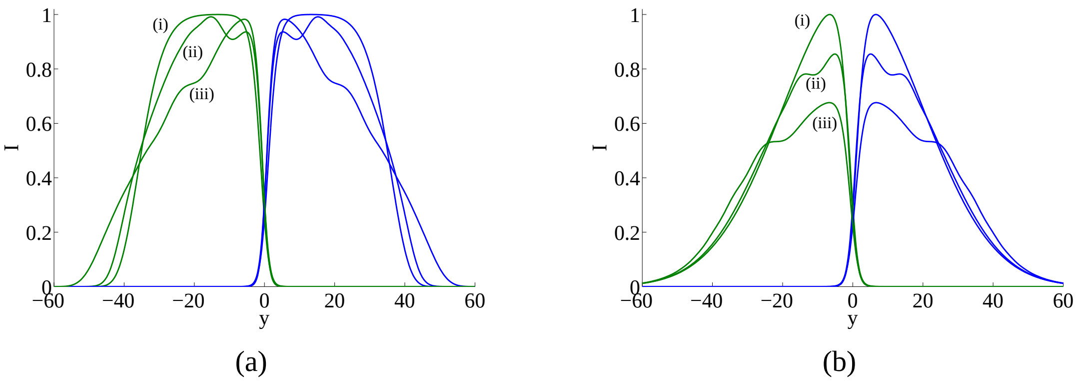


Figure 1: Evolution of the beam component intensities along the propagation: (i) $z = 0$, (ii) $z = 16$, (iii) $z = 32$, with two different input beams: (a) supergaussian ($g = 4$); and (b) Gaussian ($g = 1$), with $w_0 = 3$ and $x_0 = 40$.

In Figure 2a, the visibility $V$ is plotted as a function of the PDW width $w_0$, at $z = 10$ and for $x_0 = 30$ and $y_0 = 40$. We observe a maximum around $w_0 = 3$, which is the optimum value when the domains are not pseudo-infinite. Moreover, these results suggest that the formation of a PDW should be observed even for step-like profiles ($w_0 \to 0$). As can be seen in Figure 2b for $w_0 = 3$, the visibility monotonically increases with the input Gaussian beam width $x_0 = y_0$. However, at a value of $x_0 = y_0 \approx 30$ we already have a good visibility. By taking into account the Kerr coefficient of PbS Qdots ($n_2 \approx 10^{-11}$cm$^2$/W) a power of 80 kW is needed to form the spatial DW. Given this rather high value despite the large nonlinear coefficient $n_2$, we are considering the propagation in planar waveguide to decrease the spatial width of the beam in the $x$ direction. We expect the formation of a PDW providing that the beating length between TE and TM modes is sufficiently long in comparison with the propagation length.

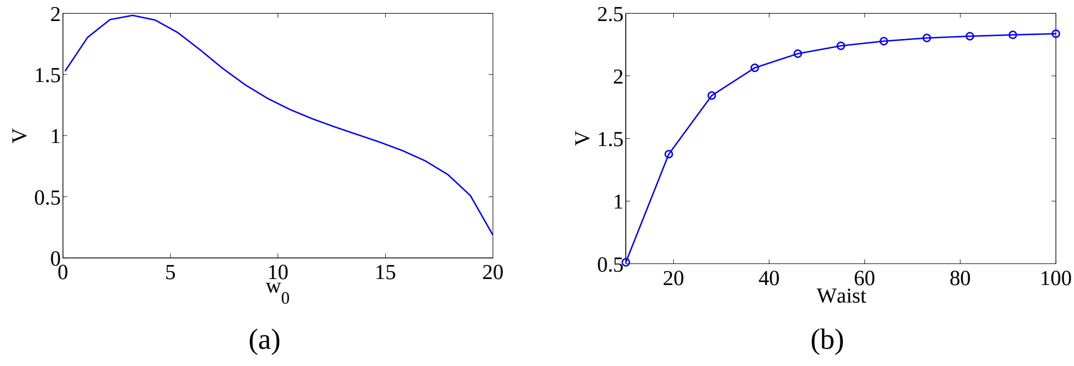


Figure 2: Visibility as a function of the parameters of the input beams : (a) visibility with respect to PDW width $w_0$; (b) visibility with respect to $x_0 = y_0$.

## Experimental set-up

The experimental setup currently built for the observation of the spatial PDW is depicted in Fig. 3. The beam at the output of a pulsed laser source is first split in two linearly polarized components $E_x$ and $E_y$.The vertically polarized beam passes through a π-phase-step plate in such a way that after the recombination of the two beams, we end up with opposite circularly polarized adjacent components $E_{\pm} = \frac{E_x \pm iE_y}{\sqrt{2}}$. The recombined beam in then focused on the colloidal nanocrystals cell and at the output the profile of both polarized components are recorded.

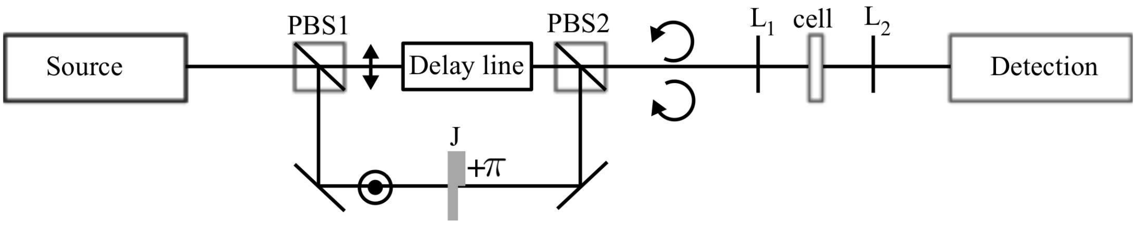


Figure 3: Experimental principle set-up

## Conclusion

PDW solitons have been predicted two decades ago but are still only experimentally demonstrated in the temporal domain. In this work, we have shown that the formation of spatial PDW is possible even with initial condition far from ideal, i.e. with a sharp initial transition between the two polarization components and Gaussian shape for the beam the domain wall is inscribe on. Since PDWs are transitions between pseudo-infinite domains, the overall extend of the beam should be sufficiently large with respect to the width of the DW. This result in required beam peak powers of the order of 80 kW to observe spatial PDW in CSQD, despite their large nonlinear Kerr coefficient in comparison with other conventional nonlinear defocusing Kerr media. A possible solution to decrease the required power is to resort to planar waveguide made up of CSQD.

## References

[1] M. Haelterman and A. P. Sheppard. Extended modulational instability and new type of solitary wave in coupled nonlinear Schrödinger equations. *Physics Letters A*, 185(3):265–272, 1994.

[2] M. Haelterman and A. P. Sheppard. Bifurcations of the dark soliton and polarization domain walls in nonlinear dispersive media. *Physical Review E*, 49(5):4512–4518, 1994.

[3] S. Pitois, G. Millot, P. Grelu, and M. Haelterman. Generation of optical domain-wall structures from modulational instability in a bimodal fiber. *Physical Review E - Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics*, 60(1):994–1000, 1999.

[4] A. Omari, I. Moreels, F. Masia, W. Langbein, P. Borri, D. Van Thourhout, P. Kockaert, and Z. Hens. Role of interband and photoinduced absorption in the nonlinear refraction and absorption of resonantly excited pbs quantum dots around 1550 nm. *Physical Review B - Condensed Matter and Materials Physics*, 85(11), 2012.

[5] M. Haelterman and A. P. Sheppard. Polarization domain walls in diffractive or dispersive kerr media. *Optics Letters*, 19(2):96–98, 1994.