Guided waves in hyperbolic media

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Nowadays there is a lot of scientific interest in optical hyperbolic media. This class of media supports propagating waves which are not restricted in spatial frequency by the diffraction limit. Furthermore, in waveguides consisting of hyperbolic media there can be infinitely many guided modes. This is interesting for both optical communication and optical sensing. In this research we discuss the properties of the guided waves in hyperbolic media and explain transmission properties of hyperbolic media in terms of the guided modes.

Introduction

The propagation of electromagnetic waves through matter is a topic that is very important to understand. In normal media, whether isotropic or anisotropic, the diffraction spot size is limited by the diffraction limit. Surpassing this diffraction limit is a very hot topic. The diffraction limited resolution is proportional to the wavelength and inversely proportional to the Numerical Aperture of the optical system ($\frac{\lambda}{\mathrm{NA}}$), which in turn depends linearly on the index of refraction of the medium in which the spot is focussed. An example from industry where the diffraction limit is extremely important is in optical lithography for fabricating chips. To be able to make small chips, they keep trying to reduce the smallest feature size of the structures. For this, the wavelength is reduced and/or the numerical aperture is enhanced. A prototype lithographic machine which uses light of wavelength 13.5 nm has been realized recently. Another way to get a better resolution is by changing the index of refraction, hence the effective wavelength in a medium ($\lambda = \lambda_0/n$, with λ_0 being the vacuum wavelength). By increasing the (effective) index of refraction of the medium, we could also improve the resolution. This is the principle behind hyperbolic media. We will show in this Letter that the effective index of refraction of a hyperbolic medium is higher than the index of refraction of the individual layers it consists of.

Not only imaging is a nice application of hyperbolic media, but optical waveguiding is as well. Optical waveguiding is getting more and more important. One field of interest is in Photonic Integrated Circuits (PIC). In order to be able to continue Moore's law [3], which states that every year the optical components density on a chip gets doubled, we need to switch from electronics to optics, as there is less heat produced and dimensions can be reduced. For optical communication, we need to guide the light by optical waveguides. Optical waveguides are known to guide a discrete and finite number of modes. This limits the amount of information which can be transported. It turns out that waveguides made of lossless hyperbolic media are able to support an **infinite amount of discrete modes**.

In this Letter we will propose an explanation of transmission properties of hyperbolic media in terms of guided modes and some guiding properties of hyperbolic media are reviewed.

Hyperbolic Media

A hyperbolic medium is a relatively new class of medium [6] which is by definition uniaxial anisotropic, hence the material has a unique optical axis. To illustrate what hyperbolic means in this case, we will assume to have a lossless uniaxial crystal (i.e. $\mathrm{Im}\,\varepsilon = 0$). The reference frame is chosen such that the z-axis corresponds to the optical axis. Then, the permittivity tensor can be written as:

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{bmatrix}. \tag{1}$$

In anisotropic media, both $\varepsilon_\perp$ and $\varepsilon_\parallel$ are positive, hence the tensor is positive definite. However, the key property of hyperbolic media is that the permittivity tensor is <u>not</u> positive definite, i.e. $\varepsilon_\perp \cdot \varepsilon_\parallel < 0$. The consequence of this is that the isofrequency surface for TM-polarized light in reciprocal k-space is hyperbolic instead of elliptic. We define this as hyperbolic spatial dispersion in this Letter[1]. Consider a plane wave with wavevector $\vec{k} = \vec{k}_\perp + k_z\hat{z}$, then the Fresnel's equation of wave normals is:

$$\left(\frac{k_\perp^2 + k_z^2}{\varepsilon_\perp} - k_0^2\right) \cdot \left(\frac{k_\perp^2}{\varepsilon_\parallel} + \frac{k_z^2}{\varepsilon_\perp} - k_0^2\right) = 0. \tag{2}$$

The first factor corresponds to TE-polarization and clearly refracts ordinarily at an interface between an isotropic medium and the hyperbolic medium of which the normal is parallel to the optical (z-) axis. The second factor corresponds to TM-polarization and describes an ellipse when both $\varepsilon_\perp$ and $\varepsilon_\parallel$ are positive. However, for hyperbolic media the signs are opposite, which means that the isofrequency surface is a hyperboloid. This is the origin of the name hyperbolic media. A TM-polarized wave then exhibits negative refraction at the mentioned interface.

What is important to mention, is that for hyperbolic media with $\varepsilon_\perp > 0$ and $\varepsilon_\parallel < 0$, there do not exist evanescent waves for TM-polarized light, when the interface lies in the (x,y)-plane, i.e. whatever the spatial frequency $k_\perp$, the corresponding k_z is always real and hence the wave is always propagating:

$$k_z = \pm\sqrt{\varepsilon_\perp k_0^2 + \frac{\varepsilon_\perp}{|\varepsilon_\parallel|} k_\perp^2}. \tag{3}$$

Transmission properties

In the fields of for example imaging or sensing, we are often limited by the diffraction limit. This imposes a lowerbound on the smallest dimension which can be imaged or sensed. However, in hyperbolic media we do not have this constraint and ideally we can have a high transmission for all TM-polarized spatial modes. This can be seen in Figure 1.

[1] Normally the term spatial dispersion is differently used, namely for waves in non-local media.

The Fresnel transmission coefficient of TM-polarized light through a 150nm thick layer of hyperbolic medium has been plotted. The input and output media are both air and k_0 is the wavenumber in air. Both angle of incidence and wavelength have been changed. For a more thorough discussion on this figure and a nice review paper on hyperbolic media, please refer to [7].

A question which arises is about the origin of so many transmission peaks for each wavelength? According to a review paper [7] the occurence of the transmission peaks for the long wavelength part of the visible spectrum can be understood as excitations of coupled surface plasmon polariton waves. In this Letter, we propose a different explanation for the origin of the transmission peaks, namely guided waves.

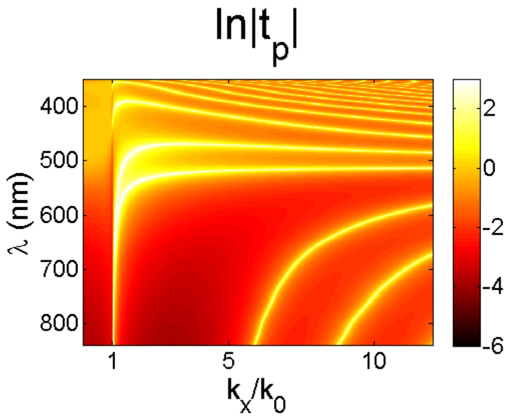


Figure 1: The natural logarithm of the transmission through a 150nm thick slab of hyperbolic medium. Dispersion has been taken into account and the angle has been changed (k_x/k_0). Losses have been neglected.

Origin of transmission peaks

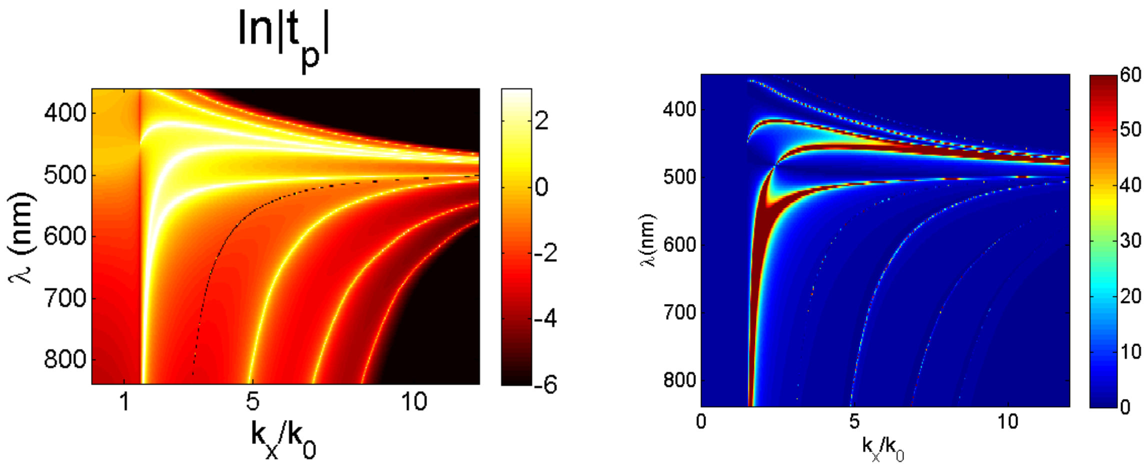
In the visible part of the electromagnetic spectrum, no hyperbolic media have been found sofar. This means that we need to mimic spatial dispersion. As an example, this can be done by making a multilayer stack of a dielectric (Re $\varepsilon > 0$, in our simulations titaniumdioxide) and a metal (Re $\varepsilon < 0$, in our simulations silver). Each layer is about 15 nm thick. Effective Medium Theory can be applied when the thicknesses of the layers are subwavelength and it can be shown that the effective permittivity tensor is of a hyperbolic medium, i.e. $\varepsilon_\perp \cdot \varepsilon_\| < 0$. We calculated rigorously the transmission and reflection coefficients $t_\downarrow$, $t_\uparrow$, $r_\downarrow$, $r_\uparrow$, where the arrows indicate the direction fo the incident beam [8]. We searched for a guided mode inside the multilayer stack, i.e. a field distribution, which does not need to have any input field. This corresponds to finding the poles of $t_\uparrow t_\downarrow - r_\uparrow r_\downarrow$.

For our assertion, both figures in Figure 2 look very similar, which is the reason that the excitation of guided modes makes a large field around the edges of the multilayer stack.

Properties of guided modes

A hyperbolic medium has some peculiar guiding properties. Some differences compared to common waveguides are:

- There are infinitely many guided modes.
- The mode with the smallest propagation constant has the fewest oscillations in the direction of confinement. Because k_z increases with increasing propagation constant, the number of oscillations in the confinement direction keeps increasing.



(a)

(b) Plot of: $t_{\uparrow}t_{\downarrow} - r_{\uparrow}r_{\downarrow}$

Figure 2: Comparison between the transmission plot and $t_{\uparrow}t_{\downarrow} - r_{\uparrow}r_{\downarrow}$. Material dispersion has been included, but we neglected losses in order to show the guided modes clearly.

Conclusion

In hyperbolic media, the wavevector behaves differently we are used to. When we consider a slab waveguide, consisting of a hyperbolic material, then for TM-polarized guided waves, the wavevector component in the confinement direction increases, while the propagation constant increases. This only occurs in hyperbolic media. Hyperbolic dispersion can be mimicked by a multilayer stack made of a metal and a dielectric with layerthicknesses (much) smaller than the wavelength. The high transmission of plane waves with high spatial frequencies which is obtained can be explained by the excitation of guided modes. In lossless, homogeneous hyperbolic media, there are infinitely many guided modes possible. This is in contrast with normal waveguides, where a cut-off propagation constant exists.

References

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