

Engineering Goos-Hänchen Shifts

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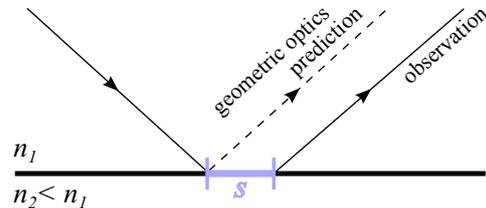
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Transformation optics allows for the design of advanced optical components like beam benders and invisibility cloaks without making use of the ray approximation. Although this technique was initially developed for the manipulation of light beam trajectories, we show here that it can also be used to understand and design optical phenomena at material interfaces. We illustrate how the Goos-Hänchen shift—a lateral shift of a light beam upon total internal reflection—can be related to the geometric properties of the reflecting material. We can use this relationship to dramatically enhance Goos-Hänchen shifts, opening a route towards efficient beam manipulation.

The Goos-Hänchen Effect

When an electromagnetic beam with finite cross section totally reflects at the interface between two media, the reflected beam experiences a small shift s along the interface—known as the Goos-Hänchen shift—with respect to the beam predicted by geometric optics [Fig. 1]. This effect was observed experimentally for the first time by Goos and Hänchen in 1947, employing multiple total internal reflections in a glass slab, hereby significantly increasing the extremely small shift associated with a single reflection [1].

Fig. 1: Electromagnetic beams with finite cross section display a lateral shift s , known as the Goos-Hänchen shift, when passing from an optically dense medium to an optically less dense medium.



A theoretical explanation of this reflection phenomenon was originally proposed by Artmann in 1948 using a stationary phase method [2-3]. Considering the incident beam as the superposition of plane waves that encounter different phase shifts upon reflection, it can be shown that the superposition of these slightly phase-shifted plane waves result in a reflected beam that is laterally displaced along the interface. If ϕ_r denotes the phase of the complex reflection coefficient r , the Goos-Hänchen shift s is given by

$$s = -\frac{\partial \phi_r}{\partial k_{in}^{\parallel}} = -\frac{1}{k_{in} \cos \theta_{in}} \frac{d\phi_r}{d\theta_{in}},$$

where k_{in}^{\parallel} represents component parallel to the material interface of the incident wave vector and θ_{in} is the angle of incidence.

Over the past few decades, the Goos-Hänchen effect has been extensively investigated among all kinds of optical media, such as dielectrics [4-6], metals [7-8], photonic crystals [9-10] and metamaterials [11-12]. An open question in this field is finding a scheme to enhance the magnitude of the Goos-Hänchen effect. In this contribution, we explore a new route towards Goos-Hänchen shift tuning using the geometric paradigm of transformation optics.

Transformation Optics

Transformation optics is a novel paradigm in electromagnetism that uses the invariance of Maxwell's equations under coordinate transformations in a brilliant way. In 2006, Pendry and Leonhardt shrewdly realized that an equivalence relation can be established between Maxwell's equations describing the propagation of light in empty space, expressed on the background of a curved coordinate system, and Maxwell's equations describing light propagation in a specific material, expressed in a traditional (Cartesian) coordinate system [13-14]. This equivalence can then be used to design macroscopic metamaterials that allow for enhanced manipulation of light trajectories, based on geometrical deformations.

The design procedure usually consists of three steps, as visualized in Fig. 2. We start from light propagation in an empty space expressed in a Cartesian coordinate system [Fig. 2(a)]. Subsequently, we transform the path of the light ray in a desired way, by changing the coordinate system as shown in Fig. 2(b). Using the equivalence relations of transformation optics, we then obtain the electromagnetic parameters of the specific (meta)material in which light propagates along this desired path [Fig. 2(c)].

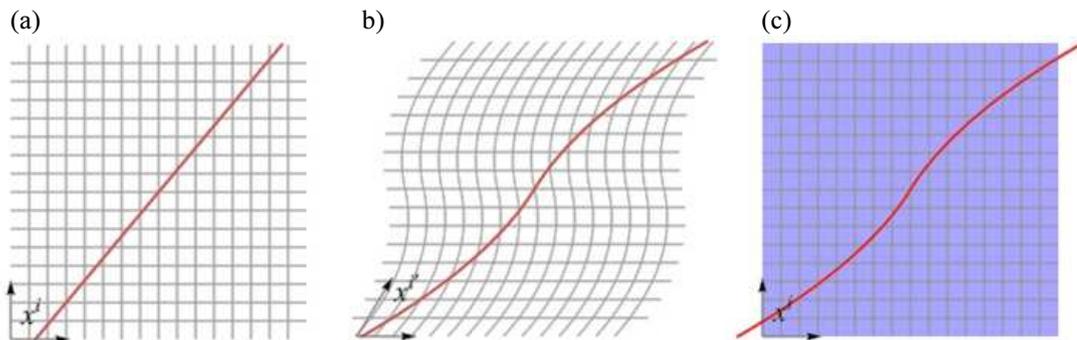


Fig. 2: The design procedure of transformation optics: (a) the path of a light ray in empty space expressed in Cartesian coordinates, (b) is transformed under a coordinate transformation. (c) Using the equivalence relations, the effect of this coordinate transformation is implemented in a specific material.

So far, this elegant technique has been successfully used to design various optical devices, such as beam benders and beam concentrators without making use of the ray approximation, i.e., without introducing aberrations. The development of the invisibility cloak is probably the most celebrated application of this technique [15-17]. Transformation optics continues to push the boundaries in recent developments where this design technique is applied for the manipulation of electromagnetic fields going far beyond the mere manipulation of light trajectories [18].

Engineering Goos-Hänchen Shifts at Metamaterial Interfaces

Transformation optics has been extensively used to manipulate the propagation of electromagnetic beams through continuous media. In our setup, however, we will use this technique to describe the physics that arises at the interface between a vacuum region that remains untransformed, and a transformed vacuum region. Applying a first-principles approach using Maxwell's equations in combination with the appropriate boundary conditions, we found analytical expressions for the (complex) reflection coefficients describing the scattering at the interface between an (untransformed) vacuum region and a metamaterial region implementing an isotropic stretching of the

coordinates. Using this analytical model, different reflective regimes can be distinguished, depending on the stretching parameter a of the coordinate transformation and on the angle of incidence θ_{in} . The derived expressions for the reflection and transmission coefficients were verified numerically by full-wave simulations using a finite-elements solver, both for TE and TM polarization. Figure 3 shows the analytical model (blue lines) for the magnitude (a,c) and the phase angle (b,d) of the reflection coefficient for TE polarized light, together with the numerical verifications (red circles), as a function of the angle of incidence θ_{in} for a fixed stretching parameter a (a,b), and vice versa (c,d).

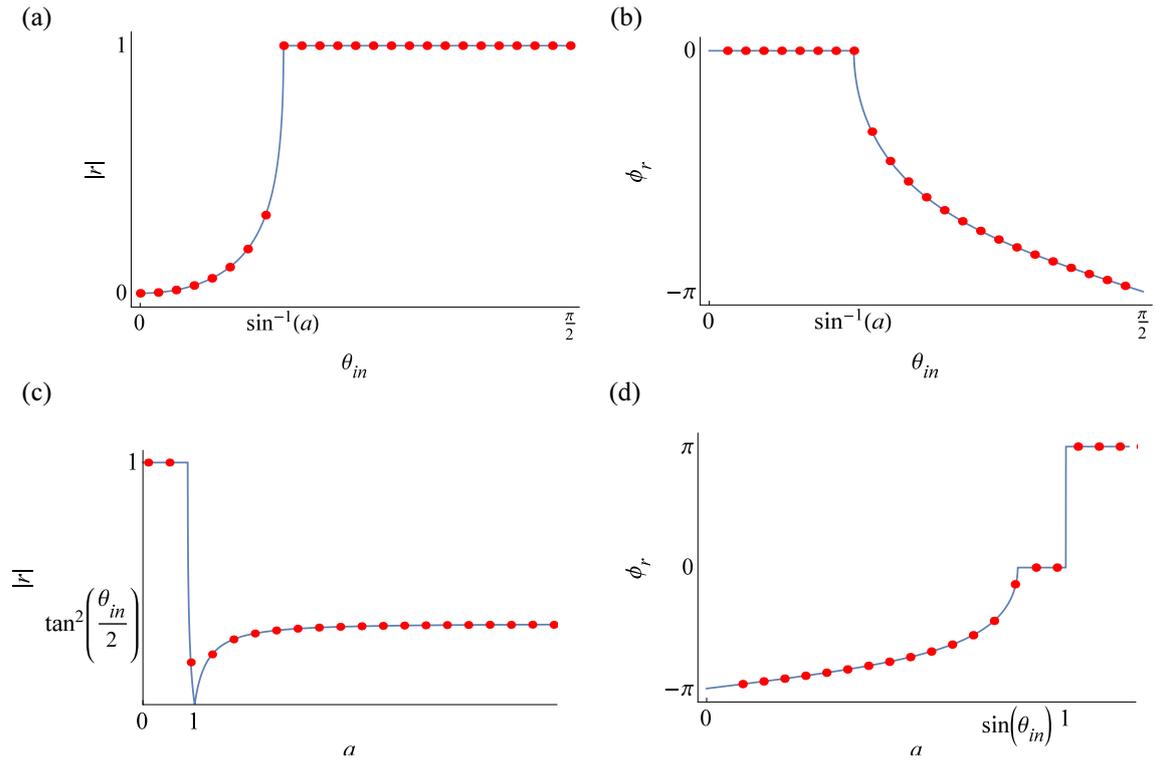


Fig. 1: Magnitude (a-c) and phase angle (b-d) of the reflection coefficient r for a TE-polarized wave, incident on an interface between a vacuum region and a material implementation of an isotropic coordinate stretching as a function of the angle of incidence θ_{in} (a-b) and as a function of the stretching parameter a (c-d).

The steep variation of the phase of the reflection coefficient with respect to the angle of incidence apparent in the regime of total reflection, i.e. for stretching parameters $a < 1$, implies the presence of a non trivial Goos-Hänchen effect. From the analytical model, one can obtain expressions for the Goos-Hänchen shift s , depending on the angle of incidence θ_{in} for a stretching of the coordinates by a . Fig. 4(a) shows the analytical prediction for s as a function of θ_{in} for different stretching parameters a . Asymptotic behaviour can be distinguished near the critical angle and near grazing incidence. The evolution of the minimum shift s_{min} obtained for increasing stretching parameters a is shown in Fig 4(b). For stretching parameters $0.7 < a < 1$, the minimal Goos-Hänchen shift s_{min} that can be obtained already exceeds one wavelength, opening up a route towards dramatic Goos-Hänchen shift enhancement.

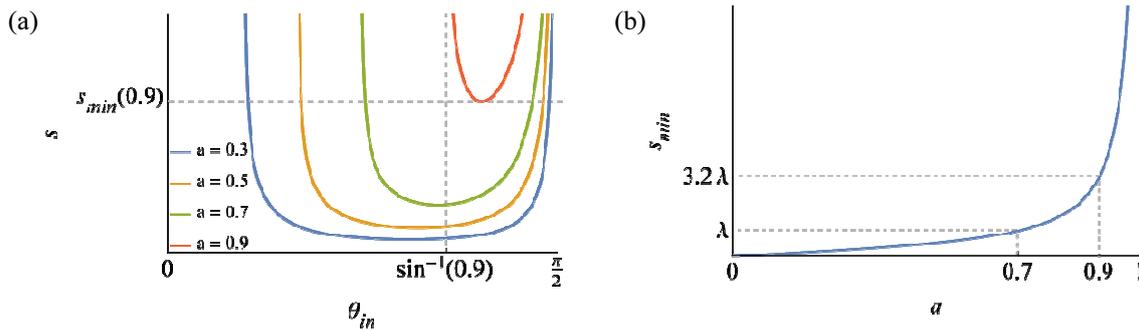


Fig. 4: (a) The Goos-Hänchen shift s as a function of the angle of incidence θ_{in} for different stretching parameters a . (b) The evolution of the minimum shift s_{min} obtained for a certain stretching parameter a is shown for increasing values of a .

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