

# Cost-effective, compact and high-speed integrable multi-mode interference modulator

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*The transmission through a symmetric 1x1 multi-mode interference (MMI) waveguide can be substantially suppressed by introducing a small asymmetry in the core refractive index. This can be achieved by applying a voltage in reversed bias to half of the waveguide in the lateral direction, so as to involve the anti-symmetric lateral mode, which in the unbiased situation would not participate. Typical plots will be shown for the calculated transmission as a function of the refractive index change.*

## Introduction

Many applications require single-mode semiconductor lasers that can be modulated with high modulation speeds above 25 GHz. Direct modulation is possible, but without expensive tricks the modulation speed is restricted to ~10 to 15 GHz as a consequence of the relaxation oscillation. Hence for low-budget applications a different solution must be sought. One possible solution is to use a multi-mode interference device as external modulator in which half of the waveguide can be phase modulated so as to mimic a Mach-Zehnder modulator. Such a device can be integrated with a single-mode semiconductor laser on a single chip with footprint < 1 mm<sup>2</sup>.

We have developed a simple theory for such an MMI-modulator based on a three-mode model for the lateral modes in the MMI and calculated the modulation properties of the MMI, in terms of the refractive index variations induced by an electric field applied to half of the waveguide.

## Theory and model

Consider a model for the 1x1 multi-mode interference (MMI) wave guide as sketched in Fig.1. It is assumed that the reversed voltage applied to the grey area induces a refractive index change  $\Delta n$  in the guiding layer region which overlaps with the grey area. The unperturbed wave equation ( $\Delta n = 0$ ) describing the lateral modes can be written as

$$H_0 \Psi_v^{(0)} = E_v^{(0)} \Psi_v^{(0)}, \quad (1)$$

where [1]

$$H_0 = -\partial_y^2; E_v^{(0)} = \frac{(v+1)^2 \pi^2}{W^2}. \quad (2)$$

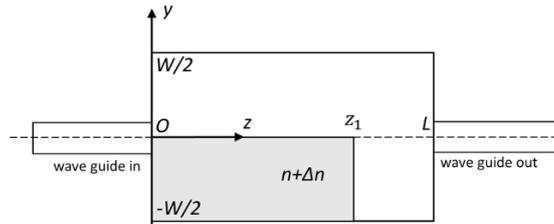


Fig.1 Sketch of the 1x1 MMI transmitter with region where the index of refraction can be modulated by applying a (reversed) voltage (grey area).

The perturbed index  $\Delta n$  gives rise to an effective potential change

$$\Delta V = \frac{8\pi^2 n}{\lambda^2} \Delta n, \quad (3)$$

where  $\lambda$  is the wavelength in vacuum. The unperturbed transverse modes are taken as

$$\Psi_0^{(0)}(y) = \sqrt{\frac{2}{W}} \cos\left(\frac{\pi y}{W}\right), \quad \Psi_1^{(0)}(y) = \sqrt{\frac{2}{W}} \sin\left(\frac{2\pi y}{W}\right), \quad \Psi_2^{(0)}(y) = \sqrt{\frac{2}{W}} \cos\left(\frac{3\pi y}{W}\right). \quad (4)$$

It is assumed that the influence of  $\Delta n \neq 0$  on the lateral modes can be adequately described in the function space spanned by the three functions (4).

The perturbed problem is given by

$$(H_0 + V)\Psi = E\Psi, \quad (5)$$

with

$$V(y) = \Delta V \left(0 < y < \frac{W}{2}\right); \\ = 0 \text{ (elsewhere)}. \quad (6)$$

Expanding the perturbed modes as linear combinations of the unperturbed modes,

$$\Psi = \sum_{j=0}^2 c_j \Psi_j^{(0)}, \quad (7)$$

yields upon substitution in (5)

$$\sum_{j=0}^2 c_j (E_j^{(0)} - E + V) \Psi_j^{(0)}. \quad (8)$$

Since the unperturbed functions (4) are orthonormal we can derive the set of three linear equations

$$\sum_{j=0}^2 c_j \{V_{kj} + (E_j^{(0)} - E)\delta_{kj}\} = 0, \quad (9)$$

for  $k=0,1,2$  and where the matrix elements  $V_{kj}$  are given by

$$V_{kj} = \Delta V \int_0^{\frac{W}{2}} dy \Psi_k^{(0)}(y) \Psi_j^{(0)}(y). \quad (10)$$

Using (4), (5), (6) and (8) this yields

$$V_{kk} = \frac{1}{2} \Delta V, \quad (k=0,1,2), \quad V_{01} = V_{10} = \frac{4\Delta V}{3\pi}, \quad V_{12} = V_{21} = -\frac{4\Delta V}{5\pi}, \quad V_{02} = 0. \quad (11)$$

The perturbed eigenvalues  $E_0, E_1$  and  $E_2$  are the three roots of the determinant equation

$$\text{Det}\{V_{kj} + (E_j^{(0)} - E)\delta_{kj}\} = 0. \quad (12)$$

The corresponding perturbed orthonormal transverse modes can now be determined, expressed in the unperturbed  $\{\Psi_v^{(0)}\}$ . After propagation from  $z=0$  to  $z=z_1$  the perturbed modes evolve to

$$\Psi(E, z_1) = \Psi(E) e^{i\beta(E)z_1}, \quad (13)$$

where the propagating constants  $\beta(E)$  for the respective modes are (see [1])

$$\beta(E)^2 = k_0^2 n^2 - E, \quad (16)$$

or, since  $E \ll k_0^2 n^2$ ,

$$\beta(E) \sim k_0 n - \frac{E\lambda_0}{4\pi n}. \quad (17)$$

with  $E = E_\nu$ ; ( $\nu = 0,1,2$ ). The inverse relations, expressing the unperturbed modes  $\Psi_v^{(0)}$  in the perturbed modes  $\Psi(E)$ , are given by

$$\Psi_v^{(0)} = \sum_{E=E_0}^{E_2} c_{E\nu}^* \Psi(E). \quad (18)$$

The input field corresponding to symmetric excitation at  $z=0$  is taken as

$$\theta_0 = \frac{1}{3} \sqrt{6} \Psi_0^{(0)} + \frac{1}{3} \sqrt{3} \Psi_2^{(0)}, \quad (19)$$

This can be expressed in the perturbed modes, using (18), and after propagation to  $z_1$  ( $c_{E\nu}$  is real; for details see [4]),

$$\theta_0(z_1) = \frac{1}{3}\sqrt{3} \sum_{E=E_0}^{E_2} (\sqrt{2}c_{E0} + c_{E2}) \Psi(E) e^{i\beta(E)z_1}, \quad (20)$$

with  $\beta(E)$  given by (17).

Using (9) we can express (20) in the unperturbed modes  $\Psi_\nu^{(0)}$  as

$$\theta_0(z_1) = \frac{1}{3}\sqrt{3} \sum_{\nu=0}^2 \sum_{E=E_0}^{E_2} (\sqrt{2}c_{E0} + c_{E2}) c_{E\nu} e^{i\beta(E)z_1} \Psi_\nu^{(0)}, \quad (21)$$

which, after propagation from  $z_1$  to  $L$  can be expressed as

$$\theta_0(L) = \frac{1}{3}\sqrt{3} \sum_{\nu=0}^2 \sum_{E=E_0}^{E_2} (\sqrt{2}c_{E0} + c_{E2}) c_{E\nu} e^{i\beta(E)z_1 + i\beta(E_\nu^{(0)})(L-z_1)} \Psi_\nu^{(0)}. \quad (22)$$

The projection of (22) on the center mode (19) yields for the transmission coefficient  $t$

$$t = \sqrt{\frac{2}{9}} \sum_{E=E_0}^{E_2} (\sqrt{2}c_{E0} + c_{E2}) c_{E0} e^{i\beta(E)z_1 + i\beta(E_0^{(0)})(L-z_1)} + \sqrt{\frac{1}{9}} \sum_{E=E_0}^{E_2} (\sqrt{2}c_{E0} + c_{E2}) c_{E2} e^{i\beta(E)z_1 + i\beta(E_2^{(0)})(L-z_1)}. \quad (23)$$

Taking  $z_1 = L$ , we can express the transmission coefficient as

$$|t| = \frac{1}{3} |1 + e^{-\frac{i\lambda_0}{4\pi n}(E_1-E_0)L} + e^{-\frac{i\lambda_0}{4\pi n}(E_2-E_0)L}|. \quad (24)$$

The shortest length for which the unperturbed MMI has transmission one is given by

$$L_1 = \frac{nW^2}{\lambda_0}. \quad (25)$$

## Numerical results

The numerical results for the transmission coefficient  $|t|$  are obtained for various values of  $L$  and  $W$ . Fig.2 shows the transmission  $|t|^2$  in dB versus the reversed-voltage induced index change  $\Delta n$ , for  $W = 15 \mu\text{m}$  and  $L = L_1 = 472 \mu\text{m}$ . The 10 dB modulation depth occurs at  $\Delta n \sim 0.00145$ . To demonstrate the sensitivity of this result on the precise device width, Fig.3 shows how the transmission at  $\Delta n = 0.00145$  changes upon changing  $W$  at  $L = 472 \mu\text{m}$  (upper curve). It must be realized that when changing  $W$ , also the transmission at  $\Delta n = 0$  reduces (lower curve), but the net modulation depth remains below 10 dB (middle curve). A much shorter device length leads to much smaller modulation depths, as illustrated in Fig.4 for  $W = 7 \mu\text{m}$  and  $L = L_1 = 103 \mu\text{m}$ . A longer device length, however, yields much higher modulation strengths, as illustrated in Fig.5 for  $W = 20 \mu\text{m}$  and  $L = L_1 = 839 \mu\text{m}$ .

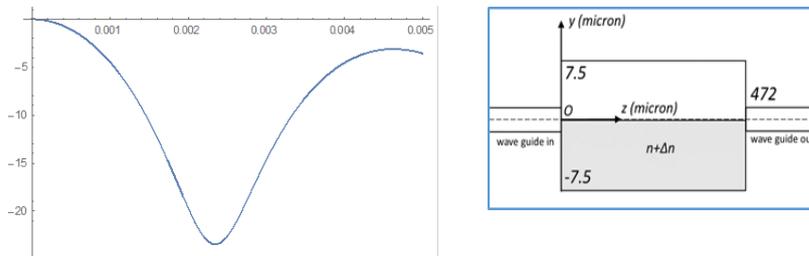


Fig.2 Left: Transmission (vertical; in dB) versus refractive index change  $\Delta n$  (horizontal) for a 15 micron wide MMI waveguide of length  $L = L_1 = 472 \mu\text{m}$  (see right-hand side sketch).

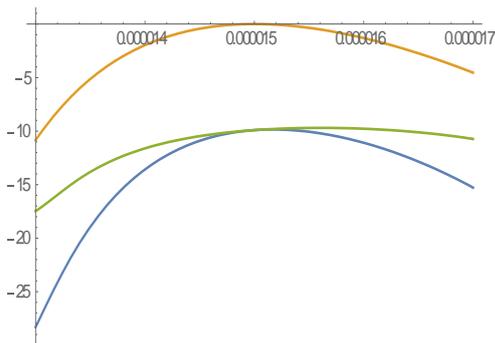


Fig.3 Upper curve: Transmission (vertical; in dB) vs.  $W$  (horizontal; in  $\mu m$ ) at  $\Delta n = 0.00145$  for  $L = 472 \mu m$ . Lower curve: Transmission (vertical; in dB) vs.  $W$  (horizontal; in  $\mu m$ ) at  $\Delta n = 0$  for  $L = 472 \mu m$ . Middle curve: modulation depth (lower minus upper curve).

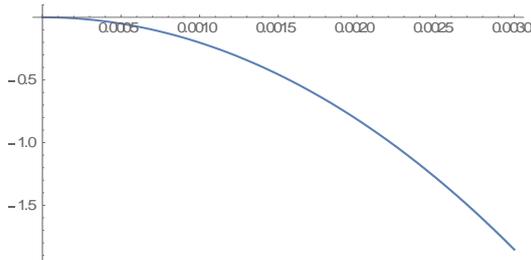


Fig.4 Transmission (vertical; in dB) vs  $\Delta n$  (horizontal) for  $W = 7 \mu m$  and  $L = L_1 = 103 \mu m$ .

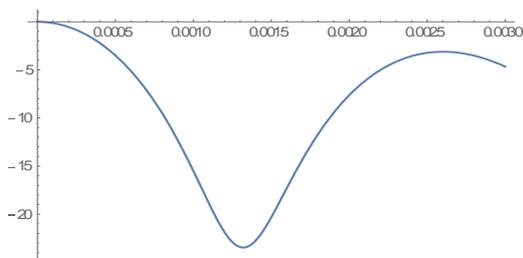


Fig.5 Transmission (vertical; in dB) vs  $\Delta n$  (horizontal) for  $W = 20 \mu m$  and  $L = L_1 = 839 \mu m$ .

## Conclusions

Using a simple model with the lowest three lateral modes in a MMI waveguide, we have shown that for a  $15 \mu m$  wide waveguide of length  $472 \mu m$  a modulation depth of 10 dB can be reached for 0.00145 index change. Since in theory a reversed bias-induced electro-optic index change of  $\sim 10^{-4} V^{-1}$  can be achieved [2], this means that  $\sim 10$  dB extinction ratio is feasible. As to the modulation speed, estimating the capacity of this modulator  $\sim 0.7$  pF and the electric resistance  $< 8$  Ohm, the cut-off frequency will be  $> 25$  GHz. The modulator can easily be integrated with a single-mode (f.i. DFB) laser, which in case of a multi-project wafer run would be very cost affordable [3]. Compared to electro-absorption modulators, the MMI modulator here proposed has less insertion loss as it does not inherently absorb light, but operates on interference.

## References

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