Stochastic Effects in Polarisation Switching VCSELs : A Rate Equation Approach

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We analytically and numerically study simple rate equations for a semiconductor laser with two nearly degenerate polarisation modes to explain the peculiar polarisation behaviour of VCSELs. The rate equations we put forth have a current dependent gain that saturates with increasing optical power. Steady state analysis reveals that switching occurs when the net gains equalise. A small region of bistability exists due to saturation effects. The introduction of stochastic terms allows us to explain the experimentally observed mode-hopping as a first passage time problem in a double potential well.

Introduction

As opposed to edge-emitting lasers, Vertical-Cavity Surface-Emitting Laser exhibit a peculiar polarisation behavior in their fundamental mode regime. In general VCSELs emit light in two orthogonal linearly polarised modes and current driven switching between these modes has been observed [1], [2], [3]. Typically, the VCSEL starts lasing in a certain polarisation, and switches to the orthogonal one at a certain current (see Fig.1.a). This switching current is very sensitive to stress and temperature and can be tuned in this way [4].

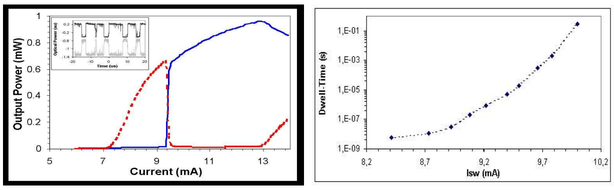

Figure 1: (a) Experimental polarisation resolved intensity versus current curve, the inset showing the mode hopping at the polarisation switching point. (b) Experimental dwell time as a function on the switching current

Stochastic phenomena clearly play an important role : at the switching current, random hops between the two modes occur (see inset Fig. 1.a). There are two characteristic times to distinguish : the dwell time is the average time the lasers stays in one mode while the

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switching time is the time it takes to switch from one mode to the other once the VCSEL has 'decided' to switch. The dwell time scales over eight orders of magnitude with the switching current [3], [5], [6] (see Fig. 1.b), while the switching time is always of the order of $10ns$. In this paper we explain these stochastic effects, using simple intensity rate equations with stochastic noise terms.

The stochastic Rate Equations

To describe the polarisation behaviour of VCSELs we use rate equations with two nearly degenerate modes [7]. The gain coefficients are both current dependent and saturate with increasing optical power. In each equation we add a stochastic term to describe the noise.

$$\left\{\begin{array}{l} \frac{dP_x}{dT} = v_{gx}\Gamma_x a_x(I)\left(N-N_{tr}\right)\left(1-E'_{sx}P_x-E'_{xy}P_y\right)P_x - \frac{P_x}{\tau_{px}} + \beta_{sp,x}N + \tilde{F}'_x \\ \frac{dP_y}{dT} = v_{gy}\Gamma_y a_y(I)\left(N-N_{tr}\right)\left(1-E'_{sy}P_y-E'_{yx}P_x\right)P_y - \frac{P_y}{\tau_{py}} + \beta_{sp,y}N + \tilde{F}'_y \\ \frac{dN}{dT} = \frac{I}{q_eV} - \frac{N}{\tau_c} - v_{gx}a_x(I)\left(N-N_{tr}\right)\left(1-E'_{sx}P_x-E'_{xy}P_y\right)P_x \\ \qquad -v_{gy}a_y(I)\left(N-N_{tr}\right)\left(1-E'_{sy}P_y-E'_{yx}P_x\right)P_y + \tilde{F}'_N \end{array}\right. \tag{1}$$

In these equations $P_{x,y}$ are the photon densities of the two polarisation modes, N is the carrier density in the active region, N_{tr} is the transparency carrier density, T is the time, $E'_{sx,sy,xy,yx}$ are the saturation coefficients, $v_{gx,y}$ are the group velocities in the laser cavity, $\Gamma_{x,y}$ are the confinement factors, $a_{x,y}(I)$ are the current dependent differential gain coefficients, $\tau_{px,y}$ are the photon life times, I is the injected current, q_e is the charge of the carriers, V is the volume of the active region, $\beta_{sp,x,y}$ describes the average spontaneous emission coupled into each mode and $\tilde{F}'_{x,y,n}$ describes the stochastic noise.
We first reduce these equations, taking advantage of the different time scales present in the model and of the fact that the modes are nearly degenerate and have nearly equal parameters [7]. We only look at times of the order of ns and longer, as this is the time scale of the polarisation switching. We get [7] :

$$\left\{\begin{array}{l} \frac{dp_x}{dt} = p_x\left[\eta-\varepsilon_{sx}p_x-\varepsilon_{xy}p_y\right]+R_{sp}+\tilde{F}_x \\ \frac{dp_y}{dt} = p_y\left[\eta+G(J)-\varepsilon_{sy}p_y-\varepsilon_{yx}p_x\right]+R_{sp}+\tilde{F}_y \\ \frac{d\eta}{dt} = \frac{J-p_x-p_y}{\rho} - \eta - p_x\left[\eta-\varepsilon_{sx}p_x-\varepsilon_{xy}p_y\right] - p_y\left[\eta-\varepsilon_{sy}p_y-\varepsilon_{yx}p_x\right] + \frac{\tilde{F}_n}{\rho} \end{array}\right. \tag{2}$$

The time t is reduced with respect to the carrier life time (i.e. $\sim 1ns$), J is the reduced current above threshold, η is the deviation of the carrier density from its clamped value, $G(J)$ is the reduced current dependent difference between the gain coefficients of the two modes and ρ is the ratio of the carrier and the photon life time (i.e. $\sim 10^{-3}$). The stationary solutions -when neglecting the noise terms- of these equation can be found in [7], [8] . The analysis shows that switching between two pure mode solutions (where only one of the modes is lasing) occurs when $G(J)$ crosses zero, with a region of bistability around this point if :

$$\Delta = \varepsilon_{sx}+\varepsilon_{sy}-\varepsilon_{xy}-\varepsilon_{yx} < 0 \tag{3}$$

Considering the order zero in ρ of the carrier equation of (2) we can derive a conservation relation:

$$p_x + p_y = J + \rho\tilde{F}_N \tag{4}$$

We see that when we neglect the carrier noise, the total intensity equals the reduced current above threshold. This means that the photon densities in the two polarisation modes are anti-correlated, on the time scale of the reduction (and longer). Using (4) we can reduce (2) to one single equation that governs the total dynamics of the polarisation switch.

$$\begin{aligned} \dot{p}_y &= \frac{\Delta}{J} p_y^3 + \left(2\varepsilon_{yx} - 2\varepsilon_{sx} + \varepsilon_{xy} - \varepsilon_{sy} - \frac{G}{J}\right) p_y^2 \\ &\quad + \left[(\varepsilon_{sx} - \varepsilon_{yx}) J + G - \frac{2R_{sp}}{J}\right] p_y + R_{sp} + \widetilde{F}_y - \frac{\widetilde{F}_x + \widetilde{F}_y}{J} p_y \end{aligned} \tag{5}$$

It is important to note that we do not perform an adiabatic elimination here, but use instead a multiple timescale analysis which is valid for every current (above threshold) and for all times equal or larger than the carrier lifetime. Using (5) we can derive the (deterministic) switching time, which is inversely proportional to the net gain difference between the two modes, and has the value of $10ns$ for reaslistic values of the net gain difference [7].

First Passage Times and Mode Hopping

If we do not consider the noise terms and the spontaneous emission (i.e. $\tilde{F}_{x,y} = 0$ and $R_{sp} = 0$), (5) has two stable stationary pure mode solutions in the bistable region ($p_y = 0$ and $p_y = J$) and one unstable mixed mode solution. Due to the noise term, random hops between these two stable solutions occur. Such transitions between two stable solutions can be treated as a First Passage Time problem over a potential barrier [5], [9]. Denoting p_y as p, the potential takes the following functional form, shown in Fig. 2.a.

$$U(p) = -\frac{\Delta}{4J} p^4 - \frac{1}{3}\left(2\varepsilon_{yx} - 2\varepsilon_{sx} + \varepsilon_{xy} - \varepsilon_{sy} - \frac{G}{J}\right) p^3 - \frac{1}{2}\left[(\varepsilon_{sx} - \varepsilon_{yx}) J + G - \frac{2R_{sp}}{J}\right] p^2 - R_{sp} p \tag{6}$$

One can then derive the mean time the laser takes to hop from one mode to the other [9].

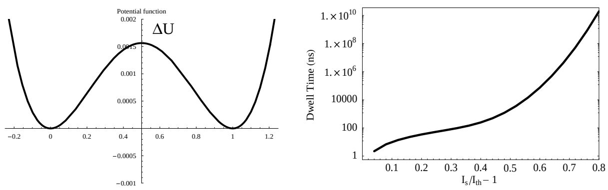

Figure 2: (a) Potential in the bistable case and (b) dwell time as a function of the switching current

We make the additional assumption that $\varepsilon_{sx} = \varepsilon_{sy}$ and $\varepsilon_{xy} = \varepsilon_{yx}$, we neglect R_{sp} and we combine all the noise terms of (5) in one term $\tilde{F}$. This dwell time becomes :

$$T_{dwell} = T_c \exp\left[\frac{K_p}{D'}\left(\frac{I_{sw}}{I_{th}} - 1\right)^3\right] \tag{7}$$

where K_p and T_c are functions of the parameters and T_c has a weak dependence on the current, I_{sw} is the current for which G equals zero and D' the covariance of the non-reduced noise terms $\tilde{F}'_x$ and $\tilde{F}'_y$. Eq.(7) and Fig.2.b show the strong dependence of the dwell times on the switching current, as is experimentally recorded [6], [5]. We note that the time it takes for the laser to actually perform the switch is given by the deterministic analysis in [7]. Indeed, when the laser has reached the maximum of the potential barrier, it will fall back quickly to the other stable state. This fast process will be mostly due to the drift terms in (5), and can thus be solved deterministically.

Conclusion

We have used simple stochastic rate equations to explain the polarisation behaviour of VCSELs. The rate equations we use have current dependent gain coefficients and saturate with increasing optical power. The steady state analysis of these equations has revealed that switching between the two pure mode solutions occurs when the net gain of the modes equalise, with a region of bistability due to the saturation [7]. To explain the mode hopping we have introduced noise terms, to account for the stochastic behaviour of the spontaneous emission. We have taken advantage of the fact that the two modes are nearly degenerate and have nearly equal parameters and we have exploited the different time scales present in the model. This enabled us to reduce the rate equation to one dynamical stochastic equation, which revealed that the photon densities are anti-correlated on a ns time scale. Mode hopping between the two stable states occurs, and the dwell time of this hopping phenomenon scales over eight orders of magnitude with the switching current, as is experimentally recorded [6], [5].

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