

# Analytical Models for the Analysis of High Bit-Rate Lightwave Systems

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*A novel physical model for analyzing the high-frequency behavior of p-i-n photodetectors has been implemented to simulate a 10 Gb/s, 1.55  $\mu\text{m}$  IMDD optical link. The photodiode response was determined by considering both the transit time spent by carriers to cross the intrinsic region, and the RC time constant. The dependence of the carrier velocities on the electric field was also considered. As models presented in this paper are fully analytical, the computational time is virtually zero. Simulations show that when the real physical response rather than the ideal square-law detector one is considered, optimized p-i-n photodiodes reduce signal overshoots at the receiver side.*

## Introduction

In order to design and optimize lightwave communications systems, computer simulations are required. Two different philosophies are followed when writing a software for this purpose. The first consists in modeling optoelectronic components by means of equivalent electrical circuits, then in simulating the link obtained by cascading electrical building blocks into a spice-like circuit simulator [1]. Another solution consists in developing a physical model for each component, then simulating each block independently using the parameters specified by the user [2]. The latest approach presents many advantages in comparison with the first one, because the interpretation of the results obtained by using equivalent circuits is usually difficult. On the other hand, good simulators based on a physical model are generally expensive. In this paper we show a user-friendly way to determine analytically physical models for both direct modulated lasers, p-i-n photodetectors, and electrical amplifiers in order to simulate optical links by using MatLab.

## Analytical models

Currently, the most used format for fiber transmission systems is the intensity modulation direct detection (IMDD), as the dynamic range of the system can be easily improved both by increasing the laser source power and by adding optical amplifiers. Several papers on IMDD optical system performance can be found in literature [3]-[4]; however the authors neglect the impact of the photodetector frequency response on the system by assuming either a photodetector bandwidth much larger than that of the receiver or an ideal square-law device. In this paper, the analyzed lightwave link consists of a single mode laser directly modulated, a single mode optical fiber with infinite bandwidth, and an optical receiver. A perturbational approach is used to determine the transfer function for both laser and photodetector analytically. A complex sinusoidal perturbation of infinitesimal amplitude in the frequency domain is applied to each variables in order to determine a small-signal model for these components.

## A. Transmitter

The behavior of a single mode semiconductor laser above threshold is described by the three coupled rate equations [3]. When considering only the light intensity, and neglecting the gain compression factor, direct modulated laser behavior is described by the following equations:

$$\frac{dn(t)}{dt} = \frac{I(t)}{qV} - v_g g(t) s(t) - \frac{n(t)}{\tau_n} \quad (1)$$

$$\frac{ds(t)}{dt} = \Gamma v_g g(t) s(t) - \frac{s(t)}{\tau_p} + n(t) \frac{\Gamma \beta_s}{\tau_n} \quad (2)$$

where  $n$  and  $s$  are the electron and photon densities in the active layer,  $v_g$  the group velocity,  $\tau_n$  the carrier lifetime,  $\tau_p$  the photon lifetime,  $q$  the electron charge,  $V$  the active layer volume,  $I(t)$  the injected laser current,  $\Gamma$  the confinement factor, while the  $g(t)$  function is given by:

$$g(t) = a_0(n(t) - n_0) \quad (3)$$

where  $a_0$  is the active gain layer coefficient and  $n_0$  is the electron concentration at transparency. By using a perturbational approach it is possible to write two sets of equations: a DC set and an AC set. Both DC and AC equation systems have an analytical solution depending on the laser characteristics and the external bias. Laser parasitics are modeled by taking into account source resistance of the laser driver, laser series resistance and parallel capacitance, and laser diode package inductance. The laser output power is given by:

$$P = \frac{\eta h \nu V s}{2 \Gamma \tau_p} \quad (4)$$

where  $h$  is the Planck constant,  $\nu$  the optical frequency, and  $\eta$  the differential quantum efficiency. As the transmitter is an electrical to optical converter, the laser transfer function can be easily defined as the ratio between the output signal optical power and the input signal electrical current or voltage. In the same way the receiver functions as an O/E transducer.

## B. Optical fiber

As shown in [5] the optical fiber impulse response results as:

$$h_f(t) = (1 + j) (4 \pi \beta L)^{-\frac{1}{2}} e^{-j \frac{t^2}{2 \beta L}} \quad (5)$$

where  $c$  is the velocity of the light,  $L$  the fiber length, and the fiber dispersion  $D$  is equal to  $-2 \pi c \beta / \lambda^2$ . Optical losses due to the coupling between laser and fiber, and fiber and photodetector, are neglected, however they can be introduced easily. Attenuation along the fiber has been taken into account by inserting an optical attenuator block.

## C. Receiver

The receiver converts an optical input signal into an electrical signal, then amplifies it with a front-end amplifier. As amplifier we consider a trans-impedance type as it achieves a large dynamic range and a wide bandwidth. The frequency response of the amplifier

is specified by choosing the maximum value for the trans-impedance, a low-frequency zero and a high-frequency pole. A photodetector model is more complex as the detection mechanism is related to three coupled partial differential equations, which describe the carrier transport. In order to determine an analytical frequency response for the  $p-i-n$  photodiode we introduce some assumptions. By considering a photodiode junction characterized by an abrupt change of the doping profile, we assume the electric field to be constant versus the position inside the intrinsic region. Moreover, we set the doping concentration of the depletion region to zero. These hypotheses allow us to decouple the transport equation and to apply the perturbational approach we used in determining the laser transfer function. Although both recombination and diffusion are neglected, the photodiode transfer function we determined offers a realistic description of  $p-i-n$  devices. In comparison with the classical Bower's frequency response [6], our expression [7] takes into account any velocity-field curve, allowing to simulate overshooting effects due to the peak of the electron drift velocity in III-V material compounds.

## Noise consideration

Noise arises from the transmitter, photodetector and electrical amplifier. In this paper we neglect the laser noise while the receiver noise is modeled by an additive noise process. We assume in the photodetector a shot noise current essentially white, gaussian, with a power spectral density (PSD):

$$S_{sh}(f) = 2qRP \quad (6)$$

where  $P$  is the average power of the light signal and  $R$  the photodetector responsivity. The equivalent input current noise from the receiver electronics, including photodiode thermal noise, has been computed as in [8]. The general expression for the equivalent input current noise can be more simply expressed in a power series, where, in many practical cases, only three terms are needed.

## Computer simulation results

Physical device models of the 1.55  $\mu\text{m}$  IMDD link have been used to simulate a 10 Gb/sec optical system. Both laser and fiber parameters are described in [9], while photodetector geometrical parameters are tuned to investigate the link performance. Fig. 1 shows the pulse patterns after transmission over 50 km. An input current with a 40 mA amplitude, NRZ format, has been used to drive the laser. The voltage output signal is obtained by filtering the fiber output signal through an InGaAs non-ideal  $p-i-n$  photodiode followed by a low-noise trans-impedance amplifier. The amplifier presents a limited bandwidth and a gain depending on the signal frequency. The eye-diagram Fig. 1(b) is determined for both ideal square-law (b-d) and non-ideal photodetector (a-c) over 10 and 50 km transmission distance. Electrical receiver performance can be improved by a proper selection of  $p-i-n$  detector. The non-ideal photodiode yields a better performance with respect to an ideal detector due to a reduction of the signal overshoot at the receiver output. When increasing the  $p-i-n$  intrinsic region thickness, the receiver bandwidth decreases, determining a sharper signal filtering. Described analysis, based on physical analytical models, allows to simulate optical links with a virtually zero computational cost, offering a tool to investigate the impact of each device with their physical parameters on the whole system.

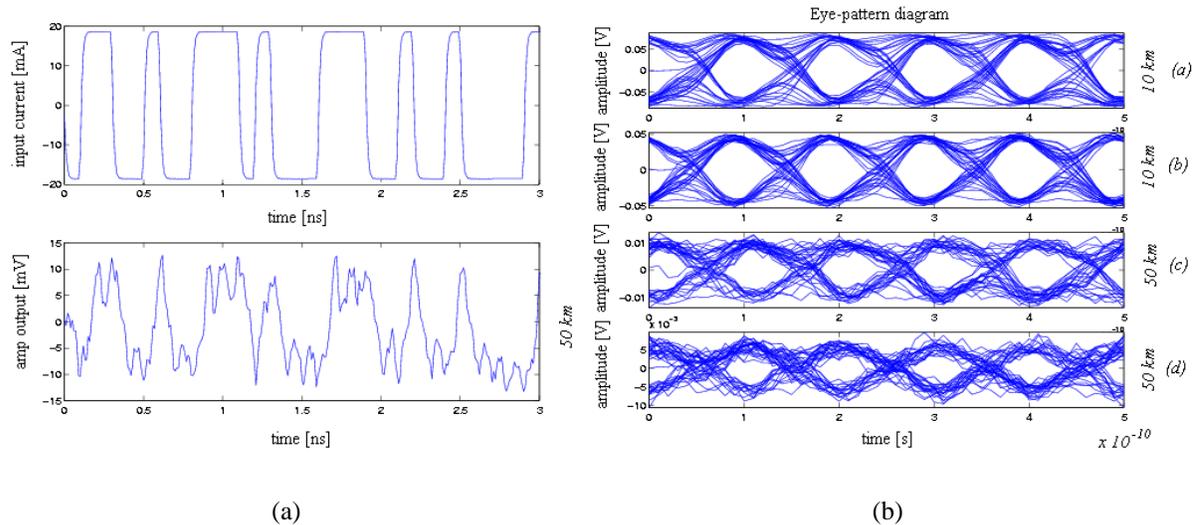


Figure 1: Pulse patterns (a) and eye diagrams (b) for the simulated link

## Acknowledgment

This work is partially funded by project PAI-IV/13 of the Belgian Federal Office for Scientific, Technical and Cultural Affairs.

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