Microcavity Design with Exact Bound Modes

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Cavity modes of dielectric microspheres and vertical cavity surface emitting lasers are never exactly bound, but have a finite width due to leakage at the borders. We propose the designs of dielectric micro structures which sustain exact bound modes of the radiation field when dissipation is neglected. Unlike photonic crystals, the photonic systems that we consider here, rely on periodicity in only one or two dimensions. These structures consist of layers that have an anisotropic dielectric tensor, which could be obtained by making air-holes in the vertical and horizontal directions within isotropic material.

Introduction

Vertical Cavity Surface Emitting Lasers (VCSELs) are examples of very small lasers [1, 2]. These microstructures can have narrow cavity resonances as a result of the localizing effect of the cylindrical wave guide in combination with the Bragg reflection in the stacked disks. The long lifetime of photons created from the recombination of electron-hole pairs in the central layer, makes the stimulated emission efficient. At present, the efficiency of VCSELs is mainly limited by leakage at the borders. There is loss of radiation through spontaneous emission which exits from the side. Also, the light in the cavity mode decays because evanescent waves are not reflected in the vertical direction. The reflection conditions of the interior guided wave and the outer evanescent wave do not match, which gives rise to losses at the boundary and determines the width of the cavity resonances. Exact bound modes do not occur in VCSELs. When the coupling to the lasing mode is enhanced by making the system (and thereby the mode volume) smaller, the evanescent fields become more important and the lifetime of the mode decreases. This kind of incompatibility between small mode volume and high finesse occurs in whispering gallery modes of microspheres [3, 4, 5] too, also because of losses at the boundary.

Bound states for the radiative field can occur in spatially infinite dielectric structures, for example in photonic crystals. When such a crystal has a point defect, it is possible to create a bound state at a frequency inside a three-dimensional photonic bandgap [6, 7]. In absence of dissipation, the state has an infinite lifetime while propagating solutions do not exist at the frequency of the bound mode. Anderson localization in a disordered structure provides an other means to create bound states of light [8, 9, 10]. A two-level atom coupled to a localized field mode would make a perfect realization of the Jaynes-Cummings model [12].

Structures that Support Bound States

In this paper we consider a class of systems where exact bound modes occur, other than photonic crystals and disordered structures. The bound states arize in structures with a simple geometry, but the layers must satisfy specific requirements for the polarization properties. The dielectric tensor of the class of these configurations is given by

$$\varepsilon(\vec{r}) = \Big[1 + \hat{z}\hat{z}U(x) + \hat{z}\hat{z}V(y) + (1 - \hat{z}\hat{z})W(z)\Big]\varepsilon_1. \tag{1}$$

Here the functions $U(x)$, $V(y)$ and $W(z)$ describe structures of layers normal to the x, y and z directions. The background medium, in which the functions U, V and W are zero, is isotropic, given by the dielectric constant ε_1. For air $\varepsilon_1 \approx 1$, but we have in mind a background made of dielectric material and $\varepsilon_1 > 1$. In this paper, we consider real ε_1 and real functions U, V and W, so that the entire system is lossless. The case with lossy layers and an active gain medium is studied elsewhere [13].

The optical properties of dielectric structures, like spontaneous emission, amplification and loss, can be calculated from a complete set of modes of the electromagnetic field. These field modes are the orthogonal solutions of Maxwell's equation for a stationary electric field of frequency ω in a medium with relative dielectric function $\varepsilon(\vec{r})$:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = (\omega/c)^2 \varepsilon(\vec{r})\vec{E}. \tag{2}$$

When the dielectric tensor is of the form given in Eq. (1), two types of solutions can be discerned. Those with the $\vec{H}$ field in the xy plane, hereafter called the s-type modes, and those with the $\vec{E}$ field in the xy plane, hereafter called the p type. This nomenclature is adopted in view of the stack of parallel layers in Fig. 1. The p-type solutions are insensitive to the functions U and V in Eq. (1); they are therefore not localized in the x or y direction. The s-type modes can be expressed as

$$\vec{E}(\vec{r}) = \left(\frac{k_z^2}{k^2}\vec{\nabla} - \hat{z}\frac{d}{dz}\right) f(x)g(y)h(z), \tag{3}$$

in terms of the scalar functions $f(x)$, $g(y)$, $h(z)$. Here, k_z, k are eigenvalues and $\mathcal{R}$ is a normalization constant. This can be proven by direct substitution in Maxwell's equation. As only the second term of (3) survives, when $\vec{\nabla} \times \vec{\nabla} \times$ is applied, one finds immediately

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \left(\vec{\nabla}^2 \hat{z} - \vec{\nabla}\frac{d}{dz}\right)\frac{d}{dz} f(x)g(y)h(z). \tag{4}$$

The substitution of Eqs. (1) and (4) in Eq. (2), give separate scalar equations for the functions $f(x)$, $g(y)$ and $h(z)$:

$$-\frac{d^2}{dx^2}f(x) = k_x^2 f(x) + (k^2 - k_z^2)U(x)f(x), \tag{5}$$

$$-\frac{d^2}{dy^2}g(y) = k_y^2 g(y) + (k^2 - k_z^2)V(y)g(y), \tag{6}$$

$$-\frac{d^2}{dz^2}h(z) = k_z^2 h(z) + k_z^2 W(z)h(x). \tag{7}$$

The three eigenvalues are related by

$$k_x^2 + k_y^2 + k_z^2 = k^2 = (\omega/c)^2\varepsilon_1. \tag{8}$$

In the regions where U, V and W are zero, the functions f, g and h are superpositions of plane waves and the total field is a superposition of eight plane waves with the wave

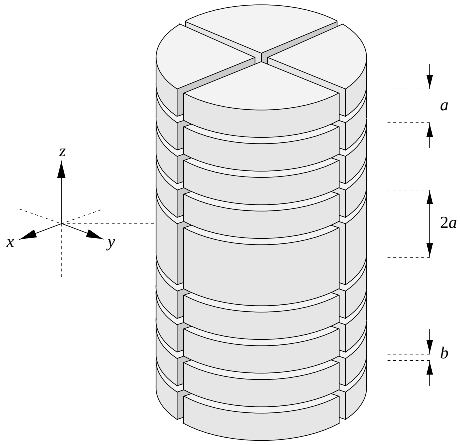

Figure 1: Example of an empty-cavity geometry where bound field modes are realized. For the vertical and horizontal layers, an anisotropic dielectric structure of the form of Eq. (1) is required, as discussed in the text below. The layers are depicted transparent, because their refractive index is lower than the index of the background. These layers, with their specific anisotropy, could be fabricated of air holes. The cylindrical boundary is not essential, because the bound modes decay exponentially with the distance from the origin.

vectors $\vec{k} = \pm\hat{x}k_x \pm \hat{y}k_y \pm \hat{z}k_z$. The polarization vector of each plane wave is proportional to $\vec{k} \times \vec{k} \times \hat{z}$. In the case of a lossless medium, considered here, the components k_x, k_y, k_z must be purely real or purely imaginary. It follows from Eq. (8) that at least one of these wave vector components must be real.

Bound states in structures described with a dielectric tensor of the form of Eq. (1) are found when the equations (5)-(7) allow simultaneously localized solutions for f, g and h. This will be the case for specific choices of the functions U, V and W. It follows from standard wave mechanics that a localized solution in a potential of finite extent is found if the potential is attractive and allows a negative eigenvalue k_x^2, k_y^2, or k_z^2 above the potential minimum. For an extended structure, localized solutions occur when the potential is periodic in two half spaces. In that case, the corresponding eigenvalue k_x^2, k_y^2, or k_z^2 is positive and one needs a discrete solution inside a band gap. Structures with bound states can be designed by combining these localizing effects. Because at least one of the eigenvalues k_x^2, k_y^2, or k_z^2 must be positive, periodicity in at least one dimension is needed.

From these considerations we conclude that the following structures will sustain bound states: 1) Periodic vertical structures in one direction, so that U or V is periodic in two half spaces, combined with a horizontal structure with negative dielectric constant, so that $W(z) < -1$ in a finite region. 2) Periodic horizontal structures combined with low index vertical structures, so that $U(x) < -1/2$ and $V(y) < -1/2$. An example of this case is shown in Fig. 1. The layers must have a principal axis of lower dielectric constant in the vertical direction for the vertical layers, and in the horizontal layers for the horizontal layers. This can be achieved by drilling air holes in isotropic material of high dielectric constant. An even more favorable situation would occur when the air holes would be filled with material with a negative dynamical dielectric constant. 3) A 2d periodic structure with periodicity in one of the two vertical directions and also in the horizontal direction. 4) All three functions U, V, and W are periodic. Then one obtains a solvable model for a 3d photonic crystal.

Conclusions

We identified a class of dielectric structures that are three-dimensional cavities for the optical field. In absence of dissipation these cavities have exact bound states. The structures generally consist of several layers with anisotropic dielectric tensors, placed under right angles with respect to each other. Localization in the three dimensions is obtained as a combination of wave guiding and Bragg reflection. This requires periodic structures in at least one dimension. The simplest realization, shown in Fig. 1, consists of two layers that are placed under right angles and a stack of layers in the third direction, which resembles a VCSEL. The Bragg reflectors localize the waves in the vertical direction inside the crossed wave guide, but also localize the evanescent waves. This results in the exact bound states of our system, in contrast to a standard VCSEL, which does not support bound states.

Because the cavity resonances in our structures are determined only by loss and not by leakage, the line widths will be quite narrow. Due to the ‘evanescent-wave’ nature of most cavity modes, spontaneous emission occurs predominantly in the direction parallel to the dipole moment of the emitter, instead of orthogonal to it. The bound states have a small mode volume (of the order of a few cubic wavelengths), so that the coupling to an emitter placed in the center of the mode can be strong when the cavity dimensions are chosen optimally for the specific transition frequency. Although there is no three-dimensional band gap and spontaneous emission is possible at the same frequencies as the bound states, photons emitted from the central region are likely to end up in the bound state and the noise from the random emission in other modes will then be relatively small. Because the modes are small and lossless, they may prove useful in future semiconductor microlasers with strong coupling and low threshold, and for future cavity QED experiments.

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