Polarization Switching in Vertical-Cavity Surface-Emitting Lasers: A bridge between the Spin Flip Model and the Gain Saturation Model

G. Van der Sande[1], T. Erneux[2], I. Veretennicoff[1] and J. Danckaert[1]

[1]Vrije Universiteit Brussel, Dept. Of Applied Physics and Photonics (TW-TONA),
Pleinlaan 2, 1050 Brussels, Belgium
tel.: ++32 2 629 36 58, e-mail: guy.van.der.sande@vub.ac.be
[2]Université Libre de Bruxelles, Optique Nonlinéaire Théorique,
Campus Plaine, Code Postal 231, 1050 Bruxelles, Belgium

In the literature different models have been presented to study polarization switching (PS) in vertical-cavity surface-emitting lasers (VCSELs). In this work we have studied two of them: the so-called Spin Flip Model due to San Miguel, Feng and Moloney (SFM), attributing PS to specific amplitude-phase coupling, and the more phenomenological Gain Saturation Model (GSM), attributing PS to linear and non-linear gain effects. By careful use of a multiple-time scale technique we have thrown a bridge between the SFM and GSM models in the parameter domain of high spin-flip rate and large birefringence. This link has been used to discuss the similarities and differences between both models.

Introduction

Different models have been proposed in the literature to explain polarization switching (PS) in VCSELs. A first model is of thermal nature and attributes PS to a spectral shift of the gain maximum with respect to the cavity resonances for the two frequency-split polarization modes [1]. Other authors adopted this idea and explained PS due to thermal lensing [2], or by incorporating the temperature and frequency dependence of both losses and gain [3]. The simplest mathematical description of gain-induced PS is based on standard multimode intensity rate equations. Both gain saturation, as well as a linear gain difference between the two polarization modes are necessary ingredients to describe PS in VCSELs [4].
A distinctively different model to describe the PS was introduced by San Miguel *et al.* [5] and was extended to include frequency and gain anisotropies by Martin-Regalado *et al.* [6]. This model, also called the Spin Flip Model (SFM), describes the active semiconductor quantum-well in terms of a spin-split two level system, where the two spin subsystems are coupled through spin flip processes described by a relaxation rate γ_s. It is desirable to look at possible simplifications of the SFM in a physically relevant parameter domain. In [7] van Exter *et al.* propose a set of intensity rate equations, derived from the SFM, by exploiting the small ratio of carrier to the spin relaxation rate and assuming the birefringence large. The technique, also called rotational averaging, boils down to a projection on the linear axes of the Poincaré sphere. We would like to suggest a reduction technique using a multiple-time scale analysis in the same parameter domain. This will give us the opportunity of approximating the SFM dynamics in a mathematically sound manner and enable us to study how and when such a reduction may fail.

Formulation and Adiabatical Elimination

The electric field is written as:

$$\mathbf{E}(z,t) = [E_+(t)\mathbf{e}_+ + E_-(t)\mathbf{e}_-]exp(i(\omega_0 t - k_0 z)) + c.c. \tag{1}$$

and is formulated mathematically in terms of four rate equations for the slowly varying amplitudes $E_\pm$ of the left and right circularly polarized basis states [i.e. $E_\pm = (1/\sqrt{2})(E_x \pm iE_y)$] and two carrier numbers N and n. N is defined as the population difference between conduction and valence band, while n is the difference between the two spin subpopulations, which couple separately to the circularly polarized fields. Including both phase, γ_p, and amplitude anisotropy, γ_a, the SFM rate equations are written as [6]

$$\begin{aligned}
\dot{E}_+ &= \kappa(1+i\alpha)(N+n-1)E_+ - i\gamma_p E_- - \gamma_a E_-, &(2)\\
\dot{E}_- &= \kappa(1+i\alpha)(N-n-1)E_+ - i\gamma_p E_+ - \gamma_a E_+, &(3)\\
\dot{N} &= -\gamma\left[N - \mu + (N+n)|E_+|^2 + (N-n)|E_-|^2\right], &(4)\\
\dot{n} &= -\gamma_s n - \gamma\left[(N+n)|E_+|^2 - (N-n)|E_-|^2\right]. &(5)
\end{aligned}$$

In eqs. (2)-(5) α is Henry's linewidth enhancement factor and μ is the normalized injection current. κ is the field decay rate, γ is the decay rate of the total population inversion, and the spin flip rate, γ_s, accounts for the mixing of the populations with opposite spin. Typical orders of magnitude can be found in the caption of Figure 1.
Introducing the amplitude/phase decomposition in the x/y-basis, $E_{x,y} = R_{x,y}e^{i\phi_{x,y}}$ and $\phi = \phi_x - \phi_y$, we are able to rewrite eqs. (2)-(5) as a set of 5 scalar equations:

$$\begin{aligned}
R_x' &= \frac{\kappa}{\gamma}(N-1)R_x - \frac{\gamma_a}{\gamma}R_x - \frac{\kappa}{\gamma}\alpha n R_y\cos\phi + \frac{\kappa}{\gamma}nR_y\sin\phi, &(6)\\
R_y' &= \frac{\kappa}{\gamma}(N-1)R_y + \frac{\gamma_a}{\gamma}R_y + \frac{\kappa}{\gamma}\alpha n R_x\cos\phi + \frac{\kappa}{\gamma}nR_x\sin\phi, &(7)\\
\phi' &= -2\frac{\gamma_p}{\gamma} + \frac{\kappa}{\gamma}\alpha n\sin\phi\left(\frac{R_y}{R_x} - \frac{R_x}{R_y}\right) + \frac{\kappa}{\gamma}n\cos\phi\left(\frac{R_y}{R_x} + \frac{R_x}{R_y}\right), &(8)\\
N' &= -N + \mu - N(R_x^2 + R_y^2) - 2nR_xR_y\sin\phi, &(9)\\
n' &= -\frac{\gamma_s}{\gamma}n - 2NR_xR_y\sin\phi - n(R_x^2 + R_y^2), &(10)
\end{aligned}$$

where prime denotes derivation to $s = \gamma t$.
Assuming $\gamma_s >> \gamma(R_x^2 + R_y^2)$ and N to be very close to 1, we can adiabatically eliminate n from eq. (6)-(10) resulting in

$$\begin{aligned}
P_x' &= 2\frac{\kappa}{\gamma}(N-1)P_x - \frac{2\gamma_a}{\gamma}P_x + 4\frac{\kappa}{\gamma_s}\alpha P_xP_y\sin\phi\cos\phi - 4\frac{\kappa}{\gamma_s}P_xP_y\sin^2\phi, &(11)\\
P_y' &= 2\frac{\kappa}{\gamma}(N-1)P_y + \frac{2\gamma_a}{\gamma}P_y - 4\frac{\kappa}{\gamma_s}\alpha P_xP_y\sin\phi\cos\phi - 4\frac{\kappa}{\gamma_s}P_xP_y\sin^2\phi, &(12)\\
\phi' &= -2\frac{\gamma_p}{\gamma} - \alpha\frac{\kappa}{\gamma_s}2\sin^2\phi(P_y - P_x) - 2\frac{\kappa}{\gamma_s}\sin\phi\cos\phi(P_x + P_y) &(13)\\
N' &= -N + \mu - N(P_x + P_y) + 4\frac{\gamma}{\gamma_s}P_xP_y\sin^2\phi, &(14)
\end{aligned}$$

where the intensities, $P_{x,y} = R_{x,y}^2$. We have checked that this approximation is valid as long as γ_s remains larger than γ_p A further reduction of these 4 equations is possible [8], exploiting $\kappa >> \gamma$.

Multiple-Time Scale Analysis

The next step in this work is the elimination of the phase dynamics associated with γ_p assuming $\gamma_s >> \gamma_p >> \gamma$. We could have done this by averaging the equations, but we have opted to use a more sound mathematical technique: a multiple-time scale analysis [9] of eq. (11)-(14). We seek solutions which are functions of two times treated as independent variables: a fast time scale, $T = (\gamma_p/\gamma)s$, and a slow time scale, $\tau = s$. The goal is to eliminate the fast time scale. The formal procedure consists of assuming a perturbation expansion of the form

$$P_x = P_x^0(T,\tau) + \frac{\gamma}{\gamma_p}P_x^1(T,\tau) + \ldots \tag{15}$$

and similar equations for the other variables, where $P_x^0(T,\tau)$ and $P_x^1(T,\tau)$ are $O(1)$. Writing the zero order terms results in

$$\frac{\partial P_x^0}{\partial T} = \frac{\partial P_y^0}{\partial T} = \frac{\partial N^0}{\partial T} = 0\,,\; \frac{\partial \phi^0}{\partial T} = -2. \tag{16}$$

This means that the zero order intensities and population difference are functions of τ alone and that the zero order phase difference pulsates in $2T$. Writing the first order terms results in e.g. (more details can be found in [10])

$$\frac{\partial P_x^1}{\partial T} = -\frac{\partial P_x^0}{\partial \tau} + \frac{2\kappa}{\gamma}(N^0-1)P_x^0 - \frac{2\gamma_a}{\gamma}P_x^0 + 4\alpha\frac{\kappa}{\gamma_s}P_x^0P_y^0\sin\phi^0\cos\phi^0 - \frac{4\kappa}{\gamma_s}P_x^0P_y^0\sin^2\phi^0. \tag{17}$$

(17) is of the form $\frac{\partial X}{\partial T} = f(T) + f_0$, with $f(T)$ a bound periodic function and f_0 a constant in T. On integration f_0 will lead to a secular term, unless the average in T of the right-hand side of (17) equals zero. Applying this constraint to (17) and similar equations results in

$$\frac{\partial P_x^0}{\partial \tau} = \frac{2\kappa}{\gamma}(N^0-1)P_x^0 - \frac{2\gamma_a}{\gamma}P_x^0 - \frac{2\kappa}{\gamma_s}P_x^0P_y^0, \tag{18}$$

$$\frac{\partial P_y^0}{\partial \tau} = \frac{2\kappa}{\gamma}(N^0-1)P_y^0 + \frac{2\gamma_a}{\gamma}P_y^0 - \frac{2\kappa}{\gamma_s}P_x^0P_y^0, \tag{19}$$

$$\frac{\partial N^0}{\partial \tau} = -N^0 + \mu - N^0(P_x^0 + P_y^0) + \frac{2\gamma}{\gamma_s}P_x^0P_y^0. \tag{20}$$

(18)-(20) is equal to the set of intensity rate equations found in [7]. This set is equivalent to a GSM where self-saturation effects are neglected and cross-saturation effects are due to spin dynamics. It should be emphasized that (18)-(20) do not exhibit PS, unless the linear dichroism is assumed to vary with a parameter, e.g. $\gamma_a = \gamma_a(\mu)$. We have verified our approximations numerically and found a good match with the SFM behavior in the parameter domain (see Figure 1). Using the same method we are able to predict the first-order corrections to eq. (18)-(20) e.g.

$$\frac{\partial P_x^1}{\partial T} = \frac{2\alpha\kappa}{\gamma_s}P_x^0P_y^0\sin 2\phi^0 + \frac{2\kappa}{\gamma_s}P_x^0P_y^0\cos 2\phi^0. \tag{21}$$

So the first order terms will add a bound periodic term to the solution pulsating in the fast time scale and only when both modes are present (e.g. during PS).

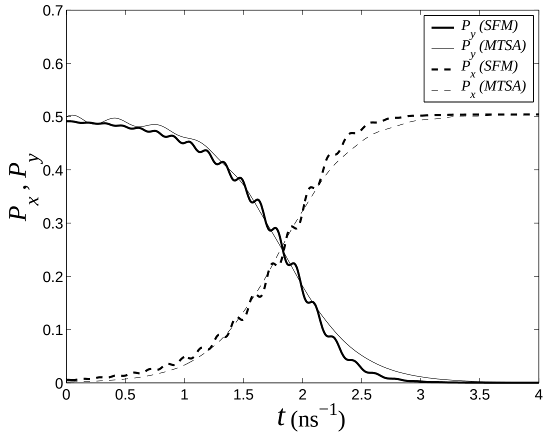

Figure 1: Intensities of the linearly polarized modes during PS. The intensity SFM behavior has small periodic fluctuations in the timescale of γ_p, which are eliminated by the multiple-time scale analysis. PS is induced by switching γ_a from $0.5ns^{-1}$ to $-0.5ns^{-1}$. ($\kappa = 300ns^{-1}$, $\gamma_s = 200ns^{-1}$, $\gamma = 1ns^{-1}$, $\gamma_p = 10ns^{-1}$)

Conclusion

We have reduced the SFM in the physical parameter domain of high spin flip rate and large birefringence to a set of intensity rate equations including cross-saturation effects due to the spin dynamics. With the help of multiple-time scale analysis we are also able to study first-order corrections to the approximation.

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