Modeling the Effects of Spatial Hole Burning in VCSELs using a Rate Equation Framework in C++

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We implement a simplified dynamic multi-mode model for vertical-cavity surface-emitting lasers (VCSELs) developed at KTH Stockholm as a template class inside a C++ framework designed to simulate and analyze rate equation models, deterministically and stochastically. The model uses the modal shapes as basis for the carrier inhomogeneities. The originally spatially resolved rate equation system is reduced to a set of independent rate equations with a minimum of parameters. This system can thus be used to study the effect of spatial hole burning, mode competition and carrier diffusion. We will look at the stationary solutions, small and large signal responses and the influence of the number of modes.

Introduction

In the last decade, VCSELs have evolved from being novelty devices to a point where they are commercially available from several companies. They are increasingly used in short to medium distance high-speed fiber optic data links. Typical current and expected future applications include cable TV distribution, inter- and intra-chip communication, Gigabit and 10 Gigabit Ethernet and Firewire (IEEE1394) extenders, often over plastic optical fiber [1]. The increasing demand for bandwidth has fueled the interest for VCSEL arrays in parallel transmission [2] as well as a quest to improve the bandwidth of the individual devices. To realize this and also to assess the possibility to use more bandwidth effective modulation formats, an in depth understanding of the physical mechanisms affecting the modulation characteristics of VCSELs is necessary.

It has been long known that spatial hole burning (SHB) and mode competition have a profound influence on the characteristics of semiconductor lasers and VCSELs [3][4][5][6]. It is indeed the major effect that induces transitions to higher order modes. Spatial hole burning is referring to the local depletion of the carrier density at points within the laser where the modal intensity is large. This will reduce the overlap between the gain distribution and photon distribution for the lasing mode. This in general leads to an increase of the average carrier density to maintain the threshold gain needed for lasing. It will also often lead to reduced side mode suppression since the gain carrier overlap may increase for competing side modes with different intensity distribution than the main mode [7]. A number of publications have also focused on the effect of SHB on the polarization properties of the emitted light: the modal competition, polarization selection and switching between the two non-degenerate orthogonally polarized modes in slightly birefringent cavities was studied in [8][9].

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While several papers (see refs. above) have addressed the effects of SHB on the static and large signal characteristics, only little [10][11] is written about the effects of SHB on the small signal modulation response. A more detailed study of SHB effects on the VCSEL modulation response is therefore motivated especially since the measured small signal response is commonly used to experimentally characterize the dynamic performance of the laser. In this context it is important to have a good theoretical understanding on how SHB and mode competition affects the small signal response.

Choice of model

Spatially resolved models based on spatial discretisation have few approximations but are computationally heavy. The restricted calculation speed limits the usefulness of these models for modeling the VCSEL's performance when long sequences have to be calculated in order to evaluate statistical and pattern dependent effects. In this paper we use a rate equation model where the SHB induced inhomogeneity of the carrier distribution is approximately described with a weighted sum of the transverse modal distributions [12].

$$N(x,y) \approx N + \sum_j \tilde{N}_j f_j(x,y) \tag{1}$$

Here, N is the average carrier density and $\tilde{N}_j$ are the deviations from uniform carrier density. The modal distributions are assumed known in advance and independent of the carrier distribution. We define the set of functions $f_i(x,y)$ with zero average defined from the modal shapes S_i according to

$$S_i(x,y) = \frac{\Gamma_z P_i}{V}\left(1 + f_i(x,y)\right), \quad \iint_A f_i(x,y)\,dx\,dy = 0 \tag{2}$$

where P_i is the photon number, V the active volume and Γ_z the longitudinal confinement factor.

The approximation (1) is justified for oxide-confined VCSELs commonly used today and leads to a substantial simplification of the equations. Multiplying the spatially resolved carrier equation with each of the modal functions and integrating over the cavity reduce it to a set of spatially independent equations; one equation for the average density and one inhomogeneous carrier equation per photon mode. The main advantage of this approach is the low number of equations to solve. This increases the computational efficiency and enables an intuitive understanding of the impact of different parameters.

Choice of framework

When simulating rate equation models, one is often forced to choose between two alternatives: on the one hand you can go with a general mathematical software environment such as Matlab or Mathematica. These are very powerful and have wide-ranging functionalities. However, this power comes with a speed penalty that can be avoided only if one is very well acquainted with the program. On the other hand, for maximum speed, one can code the implementation directly in a compiled language such as Fortran, C/C++ and others, sacrificing flexibility for speed. However, the speed often implies a certain rigidity which grows worse as time progresses until you are left with a very specialized, hard to maintain program

To avoid these pitfalls and to address some shortcomings (such as the absence of built-in stochastic integration), we have developed a C++ template class framework (MODEL [14]) which offers the power of built-in and proven algorithms such as stationary solution finders, deterministic and stochastic solvers and small signal analysis. The rate equation system (or any system of ordinary differential equations) is specified as a particular specialization of a single class, where after the rest of the framework uses it as a black box. The speed of a compiled language, combined with the power of the framework and the flexibility of the class mechanism creates a very accommodating system: once a particular model has been implemented, it can be easily reused, compared to others and extended without touching the original code.

Implementation

The model used in [12] can be reduced to dimensionless parameters/variables to read

$$
\begin{aligned}
&\rho \tfrac{d}{dT} p_i = (g_i - l_i) p_i + r_i \\
&\tfrac{d}{dT} n = j - n - \sum_j g_j p_j \qquad (3) \\
&\sum_j c_{ij} \tfrac{d}{dT} \tilde{n}_j = e_i j - \sum_j c_{ij} \tilde{n}_j - (n-1) \sum_j c_{ij} p_j + \sum_j D_{ij} \tilde{n}_j + \sum_j \sum_k \tilde{n}_k \left(c_{jk} + C_{ijk} \right) p_j
\end{aligned}
$$

Here, the p_i stand for the photon numbers in each mode, n for the average carrier number and $\tilde{n}_i$ for the carrier inhomogeneities. The injected current profile is embodied in the e_i's. The c_{ij} are due to the overlap between different modes, the D_{ij} correspond to the diffusion between the carrier inhomogeneities, the C_{ijk} are triple overlaps and g_i, l_i and r_i represent the gain, losses and spontaneous emission:

$$
g_i = n - 1 + \sum_j c_{ij} \tilde{n}_j = r_i - 1 \qquad (4)
$$

In MODEL, we have implemented this as a class templated on the number of spatial modes one wants to take into account. If we have N photon modes, the total number of equations is 2N+1 (N photon, 1 average carrier, and N carrier inhomogeneity equations). The class signature becomes `template<counter modes=2> class KTHSHB : public VectorFunction <2*modes+1>{…};`. The last equation of system (3) is implemented internally using a LU decomposition to avoid having to invert c_{ij} explicitly.

Results

Using MODEL we can now generate static and dynamical traces using very few lines of code. An example of the code needed to generate a dynamical trace is given in figure 1. The results of this can be seen in figure 2. In this case, the parameters correspond to a 10 micron radius VCSEL being pumped inhomogeneously in the center. One can clearly see the onset of the first mode, which decays quickly due to the hole being burned. Afterwards, the higher order mode dominates. The effect of SHB is visible in the turn-on of the total power transient as there is a step in the total power due to a pulse from the fundamental mode. The strong dependency of the slope efficiency of the higher order mode on the diffusion coefficient (not shown here) indicates the second mode is mainly getting its carriers through outdiffusion from the center. Finally, in figure 3 one can see the small signal response. The relaxation oscillations are very strongly damped and a low frequency bump appears due to SHB and diffusion effects.

```
N=2; // number of modes
KTHSHB<N> test; // create an instance of the model
TimeFrame t(0.1);  // create a timeframe with a
                   //certain resolution (0.1 ns)

NumVector<2*N+1> start(0.);
ODESystem<2*N+1> dyn(t,test,start,RungeKutta,1e-3);

StepMod up(dyn, test, "current", 0 , 3, 2.); // 0 to 3 @ 2 ns

while(t<30) cout << ++t << "\t" << dyn() <<endl; //30 ns of data
```

Figure 1: Example of code used to do a large signal simulation of the model

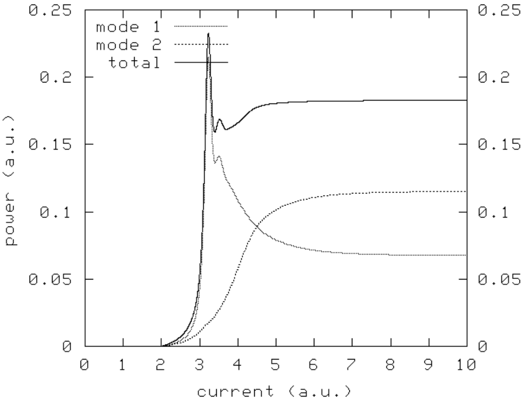

Figure 2: Response to a step modulation. The response of mode 2 is mediated by the carrier diffusion

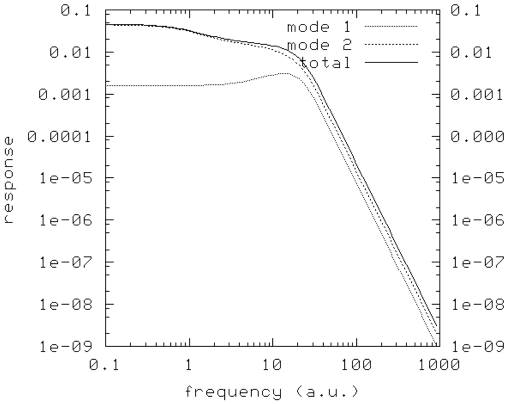

Figure 3: SSA Response at a point corresponding to the high level in figure 2. Note the strong low frequency response due to SHB and diffusion

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