Improvement in Design of Mach-Zehnder En/Decoder for Implementing New Orthogonal Codes in OCDMA Systems

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We present a design modification of a non-coherent optical CDMA system by giving the shortest path length difference required in each consecutive stage of Mach-Zehnder encoder in order to provide optimum number of codes. The path length differences are much shorter than the one reported in the recent literature, especially for large number of stages, which will facilitate fabrication of the device. The number of orthogonal codes that we can achieve is $2^{\frac{i}{2}+1}$ *for even and* $2^{\frac{i-1}{2}+1}$ *for odd number of stages, where* i *denotes the total number of cascaded stages.*

Introduction

In order to make the optical CDMA system economically attractive and easy to implement we use spectral intensity encoded OCDMA [1],[3]-[5] based on encoding of noncoherent wideband optical sources such as edge emitting LED's or Super luminescent diodes. Such a system has the advantage of being simple, inexpensive and can be realized using optical integrated components. The basic element of the system is a Mach-Zehnder (MZ) encoder.

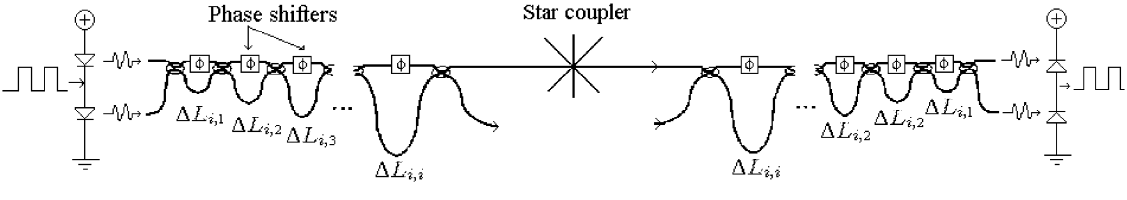

Figure 1: Schematic diagram of the spectrally encoded OCDMA system based on a MZ en/decoder.

The MZ encoder was first represented in [1]. It consists of a cascade of stages and is designed in such a way that the path length difference of each stage doubles that of previous stage. The phase shifters placed in one of the arms of each cascaded Mach-Zehnder filter in the encoder are introduced for optical coding of the non-coherent light source. The power transfer function of a single MZ filter is a squared sinusoid versus frequency. If a $\pi/2$ phase shift is applied to one arm then the original sine transfer function will change to cosine and those functions are orthogonal. Therefore by phase encoding at each stage selected from 0 and $\pi/2$ a very complicated spectral intensity coding scheme can be realized, where "1" and "0" in the code word correspond to phase shifts of 0 and $\pi/2$, respectively. The MZ decoder was made reversed with resepct to the encoder, such that a cascade of MZ filters starts with the one with the longest path length difference.

By making thorough analysis of the system we came to the result that only a limited number of orthogonal codes can be achieved by cascading MZI with FSR's that halve in every successive stage of the MZ encoder, i.e. only a limited number of simultaneous users can access the network. For any number of stages in the MZ encoder the number of orthogonal codes is 4 and they can be described with $x00...0y$, where x and y denote either "0" or "1" in the optical code.

Code and path length difference analysis

In order to support more codes for more simultaneous users a modification in the design must be made.

In this paper a new proposal for the optical CDMA en/decoder is given that offers more orthogonal codes by increasing the path length difference of successive stages in the encoder following the appropriate pattern. Obtained values are much shorter than in [2], especially for large number of stages, which will facilitate fabrication of the device. A complete non-coherent optical CDMA system is presented in Fig. 1. The structure of all the en/decoders that will be used in the system is the same, facilitating the system realization. The path length difference in successive stages of the encoder $\Delta L_{i,j}, j = 1, \ldots, i$ will depend on the number of stages i in the system and is equal to the path length difference of the first stage multiplied by an integer value $K_{i,j}, j = 1, ..., i$, i.e. $\Delta L_{i,j} = \Delta L_{i,1} \cdot K_{i,j}$.

Mathematically we can represent the spectral intensity transfer function of arbitrarly MZ encoder k and decoder l as:

$$T_{k,l}(\omega) = \frac{1}{2} + f_{k,l}(\omega)$$

where $-1/2 < f_{k,l}(\omega) \leq 1/2$, depending on the code, i.e the phase shifts applied on each en/decoder stage. The reference value for the transfer function has chosen to be 1/2 without loss of generality.

If the broadband source has a power spectral density $A(\omega)$ in the frequency range from ω_1 to ω_2, which for simplicity can be assumed to be equal to 1, the balanced detector output of the l-th decoder due to the signal from k-th encoder is:

$$I_{lk} = \int_{\omega_1}^{\omega_2} RT_k(\omega)[T_l(\omega) - \overline{T_l(\omega)}]d\omega \tag{1}$$

where $\overline{T_l(\omega)} = 1 - T_l(\omega)$. If responsivity R is constant in the frequency range of interest the output current is:

$$I_{lk} = \begin{cases} R\int_{\omega_1}^{\omega_2} f_l(\omega) + 2f_k(\omega)f_l(\omega)\ d\omega, & \text{"0" bit} \\ R\int_{\omega_1}^{\omega_2} f_l(\omega) - 2f_k(\omega)f_l(\omega)\ d\omega, & \text{"1" bit} \end{cases} \tag{2}$$

For wide enough encoded spectral range $\omega_2 - \omega_1$ is $\int_{\omega_1}^{\omega_2} f_k(\omega)d\omega = 0$.

Both $f_k(\omega)$ and $f_l(\omega)$ consist of a sum of cosines with various arguments. It is a necessary condition that all the arguments in the functions $f_k(\omega)$ and $f_l(\omega)$ must be different in order to achieve full orthogonality of codes by applying an extra phase shift in each stage of MZ en/decoder. Making the arguments of cosines different can be done by designing the path length differences in each stage of MZ en/decoder to be of a proper size. From the production point of view it is preferable to make these differences as short as possible.

In order to make all the arguments of cosines different to each other using the shortest possible path length difference in each stage of MZ en/decoder we have to follow several steps. In each step the path length difference in any stage of MZ encoder is obtained by multiplying the path length difference in the first stage of MZ encoder $\Delta L_{i,1}$ by an integer $K_{i,j}$. The integer value $K_{i,j}$ is given in Table 1 for up to $i = 8$ stages.

Table 1: Integer values for calculating the path length difference in each stage of MZ en/decoder.

Number of stages (i) ↓	$K_{i,j}$ → j							
1	1	0	0	0	0	0	0	0
2	1	2	0	0	0	0	0	0
3	1	3	5	0	0	0	0	0
4	1	3	7	12	0	0	0	0
5	1	3	7	21	33	0	0	0
6	1	3	7	21	63	96	0	0
7	1	3	7	21	63	189	285	0
8	1	3	7	21	63	189	567	852

The values given in Table 1 are found following the set of rules. These rules can mathematically be described as:

$$\begin{aligned}
\Delta L_{i,i} &= (K_{i,i-1} + K_{i-1,i-1}) \cdot \Delta L_{i,1}, & i &= 2, 3, ... \\
\Delta L_{i,2} &= 3 \cdot \Delta L_{i,1}, & i &= 3, 4, ... \\
\Delta L_{i,3} &= 7 \cdot \Delta L_{i,1}, & i &= 4, 5, ... \\
\Delta L_{i,m} &= (7 \cdot 3^{m-3}) \cdot \Delta L_{i,1}, & &\left\{ \begin{array}{lcl} i &=& 5, 6, ... \\ m &=& 4, ..., i-1 \end{array} \right.
\end{aligned}$$

These rules are very simple to implement and obtained values for path length differences for each consecutive stage of MZ encoder are much smaller than in [2].

In order to check the orthogonality of codes we have to evaluate the validness of the following expression [1]:

$$\int_{\omega_1}^{\omega_2} f_k(\omega) f_l(\omega) d\omega = \left\{ \begin{array}{ll} 0, & k \neq l \\ C, & k = l, C = \text{const.} \neq 0 \end{array} \right. \tag{3}$$

where $\omega_1 - \omega_2$ is the encoded spectral range.

When codes are perfectly orthogonal (3) must be valid. This can be achieved by introducing a proper phase shift at each stage of the MZ en/decoder after properly choosing the path length difference values. When (3) is equal to zero the input spectrum is equally split in a number of subbands at the optical outputs of the receiver giving zero current at the output. For a matched transmitter and receiver pair $f_k(\omega)$ and $f_l(\omega)$ are equal and we get a current out of the balanced detector.

The codes applied to en/decoder are obtained following several steps: First, a table must be made in which first columns and rows contain all possible code combinations applied to coders and decoders, respectively. The codes must be ordered in increasing binary value. Second, we evaluate the expression (1) for every combination of codes and put that value into appropriate place in the table. Third, starting from the first row we eliminate all the columns where a value different from zero appears except the one that starts with a code that can be interpreted as the lowest binary number, thus eliminating codes that are not orthogonal. Fourth, we eliminate all the rows that start with codes which are eliminated in the third step. Finally, we proceed with the next row that is left after elimination and repeat the steps three and four. In the end we will get a table where first row and column give all possible orthogonal codes. This result is given in Table 2 where presented values of (3) are normalized.

Table 2: All orthogonal codes for 4-stage MZ en/decoder.

Codes at the transmitter	Codes at the receiver							
	0000	0011	0100	0111	1000	1001	1110	1111
0000	1	0	0	0	0	0	0	0
0011	0	1	0	0	0	0	0	0
0100	0	0	1	0	0	0	0	0
0111	0	0	0	1	0	0	0	0
1000	0	0	0	0	1	0	0	0
1001	0	0	0	0	0	1	0	0
1110	0	0	0	0	0	0	1	0
1111	0	0	0	0	0	0	0	1

Conclusion

We gave the shortest possible path length difference between successive stages of MZ en/decoders which will provide new orthogonal codes for optical CDMA systems. Furthermore, we gave new codes and showed that increasing the number of stages by 1 will not provide new orthogonal codes if the total number of stages is odd. Therefore, in order to obtain more codes the MZ en/decoder must have an even number of stages and the number of new codes is doubled every time 2 new stages are added. Expressing mathematically, the total number of codes we can get is $2^{\frac{i}{2}+1}$ for even and $2^{\frac{i-1}{2}+1}$ for odd number of stages.

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