On the robust determination of eigenmodes in 2D stratified wave guiding systems with Berenger-type PML's

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A contour integration method is presented to determine the eigenmodes in a layered structure closed with PML layers at the boundaries of the computational window. Improvements are provided to ensure the accuracy of contour integration and to assure that all the encircled eigenvalues are determined. Numerical examples are presented to test the accuracy of the method .

Introduction

Modal expansion method (MEM) provides a powerful tool to describe the propagation of an input field through a layered 2D structure. In the design of integrated optical devices it is often needed to analyse an open wave guiding system which may vary in the z direction (splitters, gratings etc). MEM also possesses the advantage of giving detailed information on the fraction of power travelling in the guided and radiation mode of the wave guiding system at each longitudinal coordinate.

The key idea of MEM is the discretisation of the continuum of the radiation and evanescent modes by applying an electric wall "sufficiently far" from the core. The performance of the Modal Expansion Method can be improved by introducing artificial absorptive layers, the Perfectly Matched Layers (PMLs) at the boundaries of the computational window. The great advantage of PMLs is, that by means of them we can eliminate the undesired reflections occurring at the edges of the computational window and thus mimic the open system

Finding PML modes can be a hard task, since the effective indices belonging to these modes are complex. The methods used up till now to determine the PML modes use a sort of follow-up technique: having determined the eigenmodes for non PML setting with standard methods the eigenvalues are followed as they move in the complex plane due to gradually increasing the strength of the PMLs. This method may be quite cumbersome in some cases, for example to determine the quasi degenerate system modes of a directional coupler and may eventually lead to losing of eigenvalues.

The problem of losing roots can be circumvented by the method of Contour Integration [1] However, when implementing the existing Contour Integration algorithms we experienced instability and/or numerical inaccuracy for some cases. This paper presents improvements and hints to evaluate the contour integration in order for obtaining the effective indices of the PML modi in a reliable way.

Basic theory

Let us consider a 2D stack of layers with metal boundaries, the first and the last layer being PML's in a finite computational window. The PML modes are the solution of the following transversal Helmholtz equation:

$$\frac{1}{\eta(x)}\partial_x \frac{1}{\eta(x)}\partial_x E_y + (k_0^2 n^2 - \beta^2)E_y \qquad (1)$$

with $\eta(x) = 1 - j\hat{\sigma}$ where $\hat{\sigma}$, being the strength of the PML, differs from 0 only in the PML's with E_y being zero on the computatinal windows.
Making use of the conventional transfer matrix method the following expression can be obtained relating the fields on the first electric wall to those on the last electric wall:

$$\underline{\underline{M}} = \prod_{j=1}^{N} \begin{pmatrix} \cos(\eta\alpha_i d_i) & \frac{j}{\Gamma_i}\sin(\eta\alpha_i d_i) \\ j\Gamma_i \sin(\eta\alpha_i d_i) & \cos(\eta\alpha_i d_i) \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \quad (2)$$

$$\begin{bmatrix} 0 \\ \Gamma_1 A_1 \end{bmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{bmatrix} 0 \\ \Gamma_N A_N \end{bmatrix} \quad (3)$$

with $\alpha_i = \sqrt{\beta^2 - k_0^2 n_i^2}$, $\Gamma_i = -i\alpha_i / (k_0 Z_0)$ and d_i meaning the thickness of the i.-th layer. (3) is equivalent with $m_{12} = 0$. From (3) we get the dispersion relation yielding the effective indices of the PML modes. (2) is evaluated with the multiplying the matrix of each layer with the field vector *(E_y;H_x)*. The dispersion relation can be solved by contour integration in the complex effective index N plane. In the contour integration method we first determine the sum of the n-th powers of the encircled effective indices by evaluating

$$I^{(k)} = \frac{1}{2\pi j} \oint N^k \frac{\frac{d}{dz}[M]_{12}(N)}{[M]_{12}(N)} dN = \sum_{h=1}^{P} N_h^k \quad (4)$$

where P is the number of the eigenvalues encircled by the integration contour. Note that the second equation in (4) is a consequence of Cauchy's Residue Theorem. From the integrals $I^{(k)}$ we construct an equivalent polynomial [2] and solve it with Laguerre's Method [4]. By means of partial integration the integral in Eq.(4) can be transformed to the following form giving us more robust results:

$$\int_{z_1}^{z_2} N^m \frac{f'(N)}{f(N)} dz = \int_{z_1}^{z_2} N^m \frac{d}{dN} \ln(f(N)) dN = \left[(2k\pi + \ln(f)) N^m\right] - m \int_{zl1}^{z_2} N^{m-1} \ln(f) dN \quad (5)$$

$$(k, m) \in \mathbf{Z}$$

where m means the m-th power to be determined and the principal branch of the logarithm function is to be taken. *k* is the number of the branches of the multivalued logarithm function which contribute to the integral. If *m* equals zero then *k* is the number of modes encircled by the contour. Eq. (5) is an important contribution towards robust evaluation. Our other improvements are as follows:

- For large elements of (2) [i.e. large computational window] we rewrite the terms in matrix product (2) to an "assymptotic form" by considering the assymptotic behaviour of the complex valued *sinh(.)* and *cosh(.)* functions. By applying this "assymptotical transfer matrix" together with "logarithmic trick" (5) the danger of computational overflow can be significantly decreased.
- By making use the equivalent polynomial [2] it can be checked whether all the encircled roots are determined
- We can increase the computation speed without loss of accuracy if we make use of a kind of pole-zero compensation method.

Numerical results

The purpose of the numerical tests evaluated by the MEM algorithm is threefold: (a) to check, that there is no reflection on boundary of the PML-adjecent layer (b) to check whether the PML modes with extremely low absorption really coincide with the guided eigenmodes of the open system and (c) to check whether the method is able to find closely located eigenvalues like for example the first symmetric and antisymmetric mode in a symmetrical coupler with large separation. During the numerical tests it has turned out, that criteria (b) and (c) are satisfied by the results obtained for several systems. It could be also observed, that if the PML's are placed "sufficiently far" from the wave guiding system, then the appropriate PML modes coincide with the eigenmodes of the open system. Thus in the forthcoming section we focus how criterium (a) is fulfilled by the results. To do that the propagation of a Gaussian beam having a rather broad angular spectrum in uniform medium is considered.

The parameters of the free space wave propagation problem are as follows: $d_{PML}=1\ \mu$, $d_{material} = 5\ \mu m$, $n_{material}=1.7$, $\hat{\sigma}=0.4$, $k_0=6.28\ \mu m^{-1}$. First let us consider the free space wave propagation:

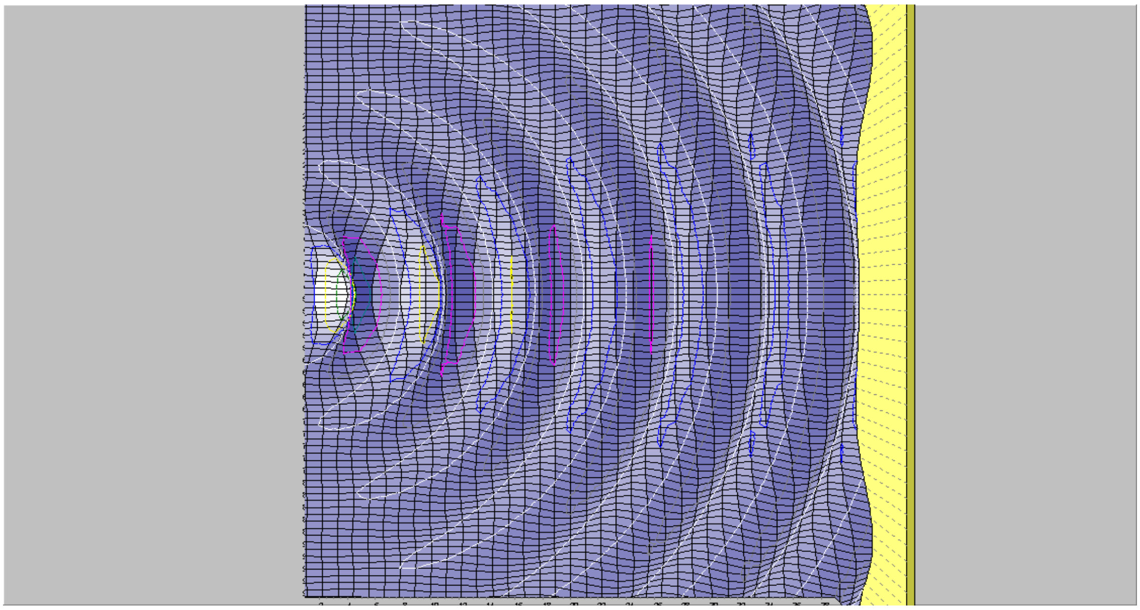
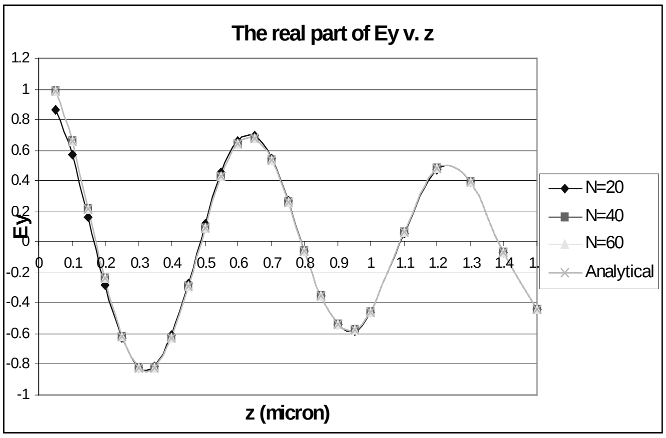


Fig 1. Free space wave propagation of a Gaussian beam. Size of window: $-2.5\mu < x < 2.5\mu$, $0 < z < 5\mu$. For structure & PML parameters see text.

Next let us compare the results obtained with analytical (Green's Function [3]) method to those obtained with MEM with different number of eigenfunctions being involved in the expansion. The results are shown in Fig. 2.



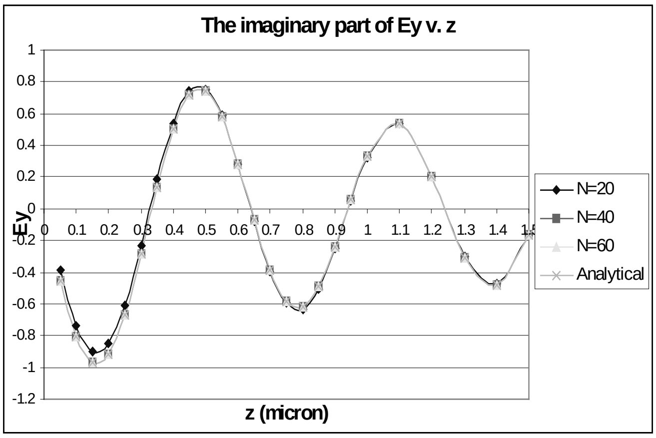


Fig 2. Propagating a Gaussian beam along z at x=0: results obtained by taking different terms of eigenfunctions into account are compared to the exact analytical result..

The results obtained by EEM coincide with the analytical results for other x cross sections, too. From these figures the following conclusions can be drawn. From Fig 1 it can be seen, that the PML's are working properly: there are no back reflections from the computational window. From Fig 2 it can be seen, that it is indeed useless to take many terms into account since the higher order terms vanish within 1 micron in the z direction. The results of the Fourier expansion seem to coincide with the analytical solution quite well.

Conclusion

In this paper a novel computational algorithm was described to determine the PML modes of stratified systems and the efficiency of this determination was tested. It was found, that with this new algorithm the degenerated modes of a coupler as well as highly higher order modes of several PML systems could be found in a robust way. The properties of PML expansion were also investigated. It was found that our results with MEM coincide with analytical results. The no-reflection criterium (at least for the structures which we considered) was satisfied, too on the PML adjacent layer dielectric interface.

References

[1] RE Smith & al: "Mode Determination for Planar Waveguides Using 4-Sheeted Dispersion relation", IEEE QE (28) pp. 1520-1526, 1992

[2] E Anemogiannis & al: "Multilayer Waveguides: Efficient Numerical Analysis of General Structures:, IEEE JLT (10), pp. 1344-1351, 1992

[3] Felsen, Markuwitz: Radiation and scattering of waves, IEEE Press, Oxford, 1994 (ISBN 0-19-859219-1)

[4] Press & al: Numerical Recipes in C, Cambridge University Press, 1992