Stimulated Brillouin scattering in single-mode fibers above threshold: spectral and statistical properties

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We performed numerical simulations in order to obtain statistical and spectral characteristics of stimulated Brillouin scattering (SBS) initiated from Gaussian noise in singlemode optical fibers. Recently published experimental spectra of SBS power are completely explained by on one dimensional SBS model. We give a clear physical insight into the problem and, for the first time, reveal how the probability function of Stokes power and the SBS spectra evolve as key parameters of the model vary, leading to a modification of Stokes field statistics.

Introduction

Stimulated Brillouin scattering (SBS) in optical fibers, a nonlinear process initiated by spontaneous scattering of the pump wave on occasional hypersound waves caused by thermal noise, demonstrates complicated dynamic behavior [1 - 3] that can be employed in applications. Two dimensionless parameters $N = P_0/P_{th} = T_0/10T_1$ and $\gamma = T_1/T_2$ that determine the Stokes-wave dynamics are defined as ratio between key temporal parameters of the SBS process: the time that light takes to travel through the fiber $T_0 = nL/c$, the time associated with the effective SBS amplification length $T_1 = 2nS/cgP_0$ and the hypersound decay time T_2, where g is the SBS gain factor, P_0 is the pump wave power, L is the fiber length, S is effective mode area, c is the velocity of light in vacuum, and n is the refractive index. A Stokes wave generated in optical fibers near threshold ($N \approx 1$) exhibits 100% depth stochastic power fluctuations with a correlation time in the nanosecond range ($\sim T_2\sqrt{2T_0/T_1}$), and the complex amplitude of the Stokes wave is a Gaussian stochastic process with zero mean [1]. At higher pump powers, pump depletion leads to specific feedback caused by energy exchange between the counter-propagating pump and the Stokes waves. When $\gamma >> 1$ this feedback only slightly influences the statistical properties of the Stokes fluctuations [1,3]. At moderate pump depletion regime ($\gamma \prec 10$) SBS statistics becomes non-Gaussian [4]. Here we present the results of numerical simulation of SBS in single optical fiber aimed to study how the probability function of Stokes power and SBS spectra are affected by pump power level above threshold. In particular, we show that the broadening and hole burning of Stokes power spectrum observed recently in the experiment [5] are connected with modification of the SBS statistics.

1-D SBS model

Numerical simulations were based on the equations for complex amplitudes of the pump wave $E_L(z,t)$, the Stokes wave $E_S(z,t)$ and the hypersound wave $\rho(z,t)$ [1-3]:

$$\begin{cases} \dfrac{n}{c}\dfrac{\partial E_L}{\partial t}+\dfrac{\partial E_L}{\partial z}=-\dfrac{g}{2S}\rho E_S \\ \dfrac{n}{c}\dfrac{\partial E_S}{\partial t}-\dfrac{\partial E_S}{\partial z}=\dfrac{g}{2S}\rho^* E_L \\ T_2\dfrac{\partial \rho}{\partial t}+\rho=E_L E_S^*+f\left(z,t\right) \end{cases} \tag{1}$$

where z is coordinate along the fiber. The complex amplitudes $E_L\left(z,t\right)$, $E_S\left(z,t\right)$ are normalized in such way that $E_L E_L^*=P_L$ and $E_S E_S^*=P_S$, where P_L and P_S are pump and Stokes powers, respectively. The Langevin noise source $f\left(z,t\right)$ is spatially and temporally δ-correlated Gaussian random process with zero mean: $\left\langle f\left(z',t'\right) f^*\left(z'',t''\right)\right\rangle = Q\,\delta\left(z'-z''\right)\delta\left(t'-t''\right)$. In our calculations, noise intensity $Q=320\sqrt{5\pi}\chi_a S^2 \exp\{-20\}\big/\left(g^2 L_a T_2\right)$ was chosen so that the pump-to-Stokes power conversion efficiency was equal to $\chi_a=1\%$ at the pump power level $P_a=20S\big/\left(gL_a\right)$ for a fiber length of $L_a=200\,m$. The boundary conditions correspond to the injection of a monochromatic cw pump wave at $z=0$, i.e. $E_L\left(0,t\right)=\sqrt{P_0}$, $E_S\left(L,t\right)=0$. Other parameters in the calculations are related to SBS in typical telecommunication fiber at pump wavelength of $\lambda_L\approx 1\,\mu m$: $g=2.5\cdot 10^{-9}\,cm/W$, $T_2=10\,ns$, $S=25\mu m^2$. For any given fiber length L the threshold power level is referred as $P_{th}=20S/gL$ [2]. Numerical integration of Eq.(1) based on 4-order Runge-Kutta algorithm with temporal step $h_t\approx\min\{T_0,T_1,T_2\}/10$ have been performed for a wide range of parameters: $N=1\sim 10$ and $\gamma=0.01\sim 10$. For a given pair $\{N,\gamma\}$ we calculated twenty different stationary realizations of Stokes field time series $E_S\left(0,t\right)$ each with a duration of $20\mu s$. Then these data were processed to build the Stokes power probability function, the spectrum of the Stokes field $S\left(\nu\right)$ and the spectrum of the Stokes power $S_P\left(\nu\right)$, where $S\left(\nu\right)=\mathfrak{F}\left\{B\left(\tau\right)\right\}$, $S_P\left(\nu\right)=\mathfrak{F}\left\{C\left(\tau\right)\right\}^{1\!/\!2}$, $\mathfrak{F}\{\;\}$ means a Fourier transform, $B\left(\tau\right)$ and $C\left(\tau\right)$ are autocorrelation functions of Stokes field and Stokes power, respectively.

Probability function of Stokes power

The Stokes power probability function $W\left(P_S/\left\langle P_S\right\rangle\right)$ is shown in Fig.1 with N and γ as parameters. At any value of γ near threshold, the probability function (e.g., $N=1.25$, curves 1) is approximately coincident with the exponential power distribution function $W\left(P_S/\left\langle P_S\right\rangle\right)=\exp\left\{-P_S/\left\langle P_S\right\rangle\right\}$, which is equivalent to Gaussian statistical distribution with zero mean for the Stokes field. Above threshold, the statistics of the Stokes field remains Gaussian only for $\gamma>>1$ (e.g., $\gamma=10$, Fig.1(a)). However, when $\gamma\prec 1$, the probability function is significantly modified above threshold (Figs.1(b-d)). Specifically, it exhibits a maximum near $P_S/\left\langle P_S\right\rangle\approx 1$. The width of the probability function is reduced while N grows and saturates to some minimal value at $N=10$. At lower values of γ, probability function is more densely concentrated near $P_S/\langle P_S\rangle \approx 1$.

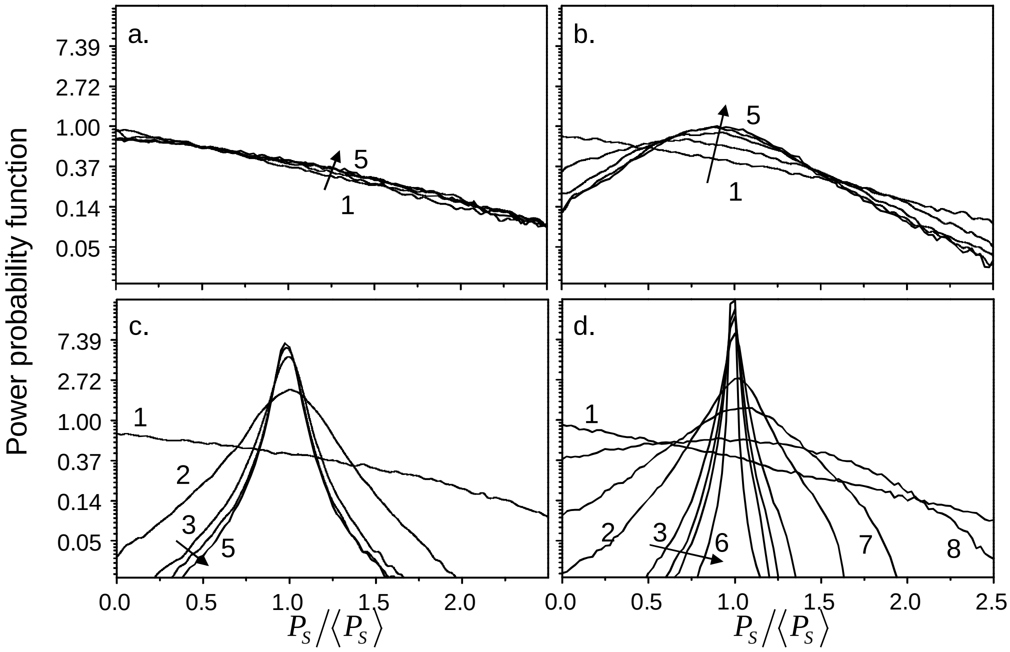

Figure 1. Probability functions of the Stokes power for $\gamma=10$ (a), 1 (b), 0.1 (c) and 0.01 (d); curves 1 to 8: $N=1.25$, 2.5, 5, 7.5, 10, 100, 1.9 and 1.5, respectively.

Spectrum of SBS field

We verified numerically that the drastic modification of SBS statistics shown in Fig.1 does not affect the Stokes field correlation function $B(\tau)$ and the Stokes field spectrum $S(\nu)$. For any pair $\{N,\gamma\}$, both $B(\tau)$ and $S(\nu)$ were found to be Gaussian with correlation time $\tau_c \approx 85\,ns$ and full spectrum width $\Delta\nu_B \approx 7.5\,MHz$, respectively (widths at $1/e$ maximum). The variations of τ_c and $\Delta\nu_B$ were less than ±10% over all $\{N,\gamma\}$. This result is in quite good agreement with experiments [1, 3] and theoretical estimation $\Delta\nu_B \approx \left(\sqrt{20}\pi T_2\right)^{-1}$.

Spectrum of SBS power

Nevertheless, the modification of the Stokes field statistics does affect the spectrum of Stokes power $S_P(\nu)$ as it is shown in Fig.2. At any value of γ near threshold ($N=1.25$) or at $\gamma=10$ above it, while SBS statistics remains nearly Gaussian, the shape of the spectrum $S_P(\nu)$ is Gaussian with about ~15 MHz width at $1/e$ maximum, which is exactly twice the width of the Brillouin line $\Delta\nu_B$. Obvious relation between $S_P(\nu)$ and $S(\nu)$ in this case is a direct consequence of Gaussian statistics of the SBS field, that provides a simple relation between field- and power- correlation functions, i.e. $B(\tau)=\sqrt{C(\tau)}$ [3]. Modification of the SBS statistics caused by either decreasing γ at given $N > 1.25$ or by increasing N at given $\gamma < 10$ leads to the broadening of the spectrum $S_P(\nu)$ with modification of its wings. Besides a dip appears and grows in the center of the spectrum. At moderate values of N, the width of the dip slightly depends on N and is determined by γ. It reaches ~$13MHz$ at $\gamma = 0.01$. Similar specific features of the Stokes power spectrum $S_P(\nu)$ have been observed recently in experiment [5].

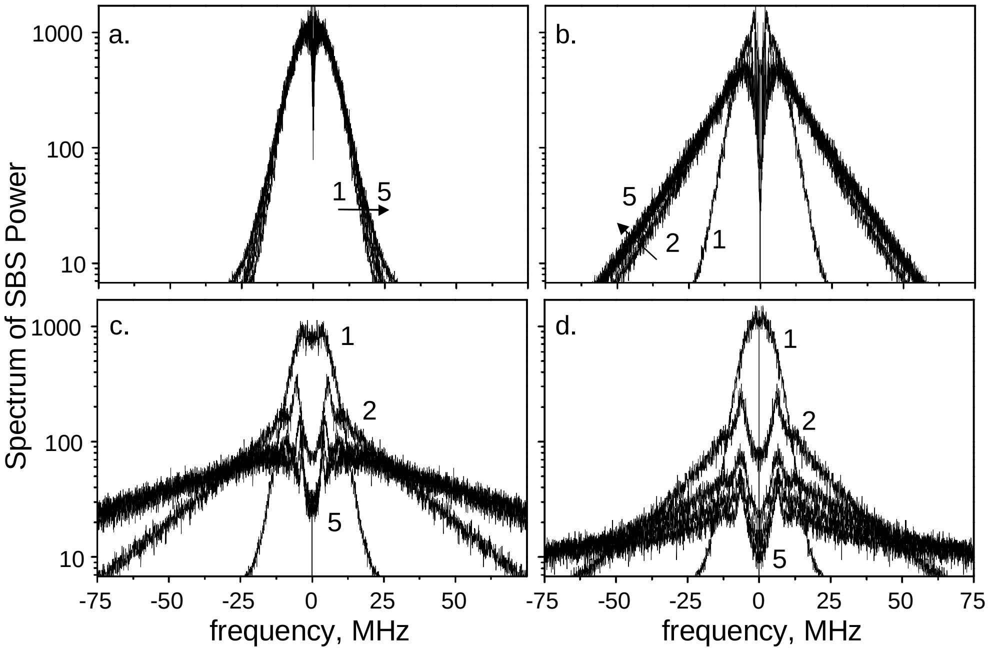

Figure 2. Spectra of the Stokes power (normalized to average power) for $\gamma = 10$ (a), 1(b), 0.1(c) and 0.01(d); curves 1 to 5: $N = 1.25$, 2.5, 5, 7.5 and 10, respectively.

Conclusion and acknowledgments

In conclusion, we numerically investigated SBS statistics in single-mode fiber above threshold. Main spectral and statistical functions were calculated for wide range of parameters. We found that anomalies in spectrum of Stokes power is connected with modification of SBS statistics.

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References

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