Symmetry-breaking and high-frequency periodic oscillations in mutually coupled semiconductor lasers

F. Rogister*[1] and J. Garcia-Ojalvo[2]

[1]Service d'Electromagnétisme et de Télécommunications, Faculté Polytechnique de Mons
31 Boulevard Dolez, 7000 Mons, Belgium

[2]Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Barcelona, Spain

We investigate the dynamical behavior of two mutually coupled laser diodes. We report a symmetry-breaking in periodic solutions at low coupling rate. For higher coupling rate, both lasers can exhibit high frequency periodic oscillations (>10 GHz).

1. Introduction

In this paper, we investigate the dynamical behavior of two laser diodes that are mutually coupled by their optical fields (Fig. 1). The system is assumed to be perfectly symmetric, i.e. internal and operating parameters are identical for both lasers. This system can be modelized by the following equations

$$\frac{dE_{1,2}}{ds} = (1+i\alpha)N_{1,2}E_{1,2} + \eta E_{2,1}(s-\theta)\exp(-i\Omega\theta)\,, \tag{1}$$

$$T\frac{dN_{1,2}}{ds} = P - N_{1,2} - (1+2N_{1,2})\,|E_{1,2}|^2\,. \tag{2}$$

where $E_{1,2}$ and $N_{1,2}$ are the normalized slowly varying complex electric fields and the normalized excess carrier numbers in lasers 1 and 2, respectively. The dimensionless time s is measured in units of the photon lifetime τ_p. η is the normalized coupling rate and θ is the ratio of the flying time of the light between the lasers to the photon lifetime. α is the linewidth enhancement factor and Ω is the product of the angular frequency of a solitary laser and the photon lifetime. P is the dimensionless pumping current above solitary laser threshold and T the ratio of the carrier lifetime to the photon lifetime. We use typical values for the linewidth enhancement factor and the ratio of the carrier lifetime to the photon lifetime, namely α = 4 and T = 1710. The other parameters are P = 1.155 and θ = 20. η is the only variable parameter.

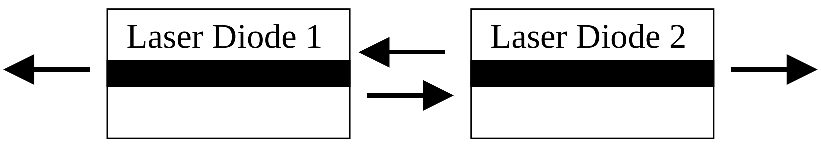

Figure 1

In the following, we show that both lasers undergo a cascade of bifurcations as the coupling rate increases. We demonstrate that a symmetry-breaking in periodic solutions occurs at low coupling rate. For higher coupling rate, both lasers can exhibit high frequency periodic oscillations with identical amplitudes. The frequency of these periodic oscillations is proportional to the inverse of the flying time between the lasers.

* Electronic address: rogister@telecom.fpms.ac.be

2. Bifurcation diagrams and symmetry-breaking

Bifurcation diagrams for laser 1 [Fig. 2 (a) and (c)] and laser 2 [Fig. 2 (b) and (d)] have been calculated by solving numerically Eqs. (1) and (2). They show the extrema of the intensity emitted by laser 1($|E_1|^2$) and laser 2 ($|E_2|^2$) as functions of η. Figs. 2 (c) and (d) are zoom views of Figs. 2 (a) and (b), respectively. At very low coupling, the output of both lasers is stationary. At $\eta = 2.7 \times 10^{-4}$, the system looses its stability through a Hopf bifurcation, and both lasers start to oscillate periodically with a frequency $f = 5.2$ GHz that is very close to the relaxation oscillation frequency of the lasers in absence of coupling. From $\eta = 2.7 \times 10^{-4}$ to 6.1×10^{-4}, both lasers oscillate with the same amplitude and frequency but in antiphase. The amplitude of the periodic oscillations increases steadily until 6.1×10^{-4}. At this point, a new bifurcation

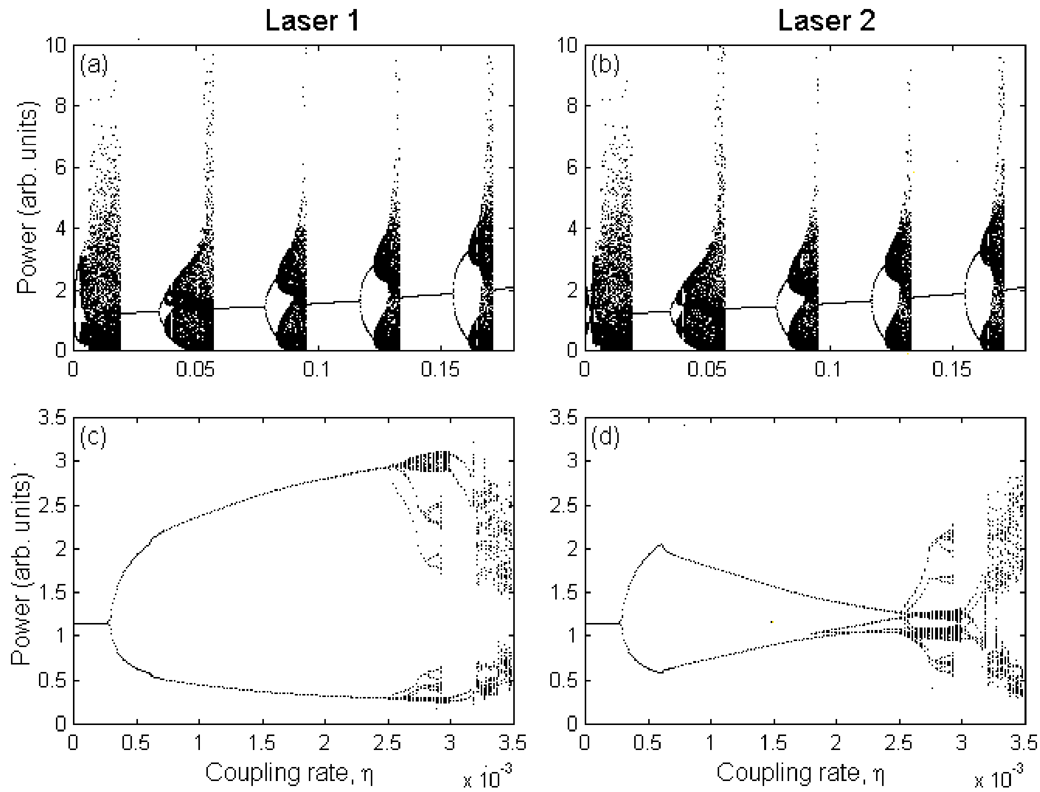

Figure 2

occurs. It is characterized by the appearance of a symmetry breaking in the periodic solutions: although both lasers continue to oscillate at the same frequency, the modulation amplitude is different and this difference increases with the coupling rate. At $\eta = 2.5 \times 10^{-3}$, a bifurcation to quasiperiodic oscillations with two incommensurable frequencies occurs. It must be noted that the quasiperiodic solutions coexist with periodic solutions. Chaos is observed from 3.18×10^{-3} to 1.9×10^{-2}. From $\eta = 1.9 \times 10^{-2}$, the system exhibits a cascade of bifurcations with regions where the intensities emitted by the lasers are stationary interspersed with regions of more complex behaviors as periodic, quasiperiodic and chaotic oscillations. Symmetry breaking in periodic solutions is no longer observed although there are some evidences of symmetry breaking when quasiperiodic solutions are frequency locked. As we will see in the next section, the periodic solutions in these regions are characterized by a high frequency (>10 GHz).

3. High-frequency periodic oscillations

When the system is between the first Hopf bifurcation and the bifurcation leading to symmetry breaking, the laser outputs oscillate periodically with a frequency $f \cong 5.2$ GHz which is close to the relaxation oscillation frequency. The optical spectra of both lasers are identical and exhibit several lines symmetrically distributed around the optical carrier frequency [see Fig. 3(a) which displays the spectrum of laser 1 computed for $\eta = 5 \times 10^{-4}$]. In the regions within which the laser oscillate periodically with a high frequency, the corresponding optical spectra of both lasers are also identical. They are however asymmetric and characterized by the existence of two main peaks, the harmonics being barely visible [see Fig. 3(b) which displays the spectrum of laser 1 computed for $\eta = 0.16$]. This kind of spectra suggests that the high-frequency periodic oscillations result from a beating between two pairs of single frequency rotating wave solutions of the form $E_{1a,b}(s) = A_{a,b}\exp[i(\Delta_{a,b}\text{-}\Omega)s]$ for laser 1 and $E_{2a,b}(s) = B_{a,b}\exp[i(\Delta_{a,b}\text{-}\Omega)s]$ for laser 2, respectively, and that they can be approximated by analytical solutions of the form

$$E_1(s) = A_a \exp[i(\Delta_a - \Omega)s] + A_b \exp[i(\Delta_b - \Omega)s], \tag{3}$$

$$E_2(s) = B_a \exp[i(\Delta_a - \Omega)s] + B_b \exp[i(\Delta_b - \Omega)s]. \tag{4}$$

The corresponding intensities are

$$|E_1(s)|^2 = |A_a|^2 + |A_b|^2 + 2|A_a||A_b|\cos(\omega s + \phi_1) \text{ and} \tag{5}$$

$$|E_2(s)|^2 = |B_a|^2 + |B_b|^2 + 2|B_a||B_b|\cos(\omega s + \phi_2) \tag{6}$$

where ϕ_1 and ϕ_2 are constant phases and ω is the beating normalized angular frequency defined as $\omega = \Delta_a - \Delta_b$. The corresponding frequency is $f = \omega / 2\pi\tau_p$ [Hz]. It is possible to show that solutions of the form of Eqs. (3) and (4) are exact solutions of Eqs. (1) and (2) when the ratio of the carrier lifetime to the photon lifetime is large (i.e. for $T \to \infty$) and for particular values of the coupling rate. These values of η and the corresponding normalized angular frequencies read respectively

$$\eta_k = [-\Delta_1\theta + (2k+1)\pi/2]/[\theta\sin(\Delta_1\theta)] \text{ and } \omega_k = [-2\Delta_1\theta - (2k+1)\pi]/\theta \tag{7}$$

where $k = 0,1,2,3\ldots$. The phases $\Delta_1\theta$ are obtained by solving the following transcendental equation

$$\alpha\cot(\Delta_1\theta)\left[\Delta_1\theta + (2k+1)\frac{\pi}{2}\right] = -\Omega\theta - (2k+1)\frac{\pi}{2}. \tag{8}$$

Figure 4 shows the dependence of the oscillation frequency on the coupling rate as obtained numerically from Eqs. (1) and (2) at the successive Hopf bifurcation points and analytically from Eqs. (7) and (8). The figure shows a good agreement between approximate and exact solutions, the periodic solutions emerging from the first Hopf bifurcation excepted. This is due to the fact that, at low coupling rate, the periodic oscillations result from the undamping of the relaxation oscillations, not from a beating mechanism.

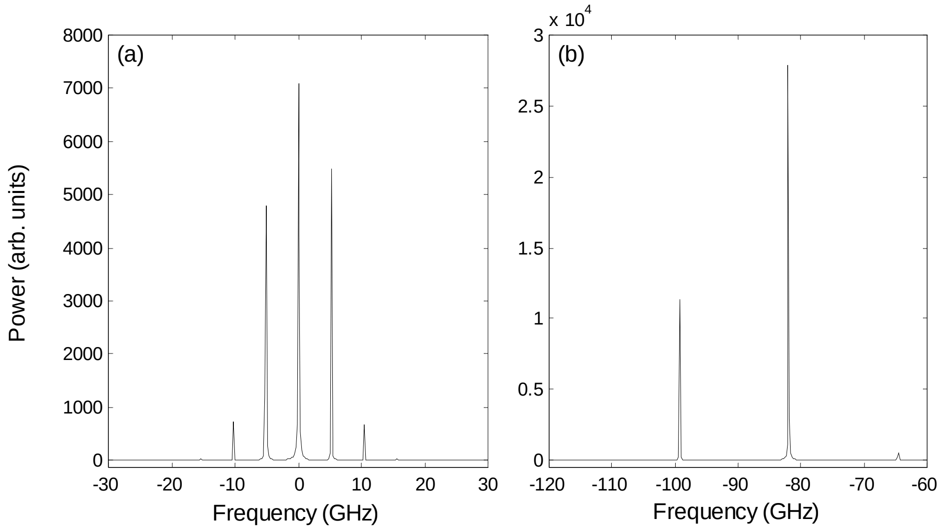

Figure 3

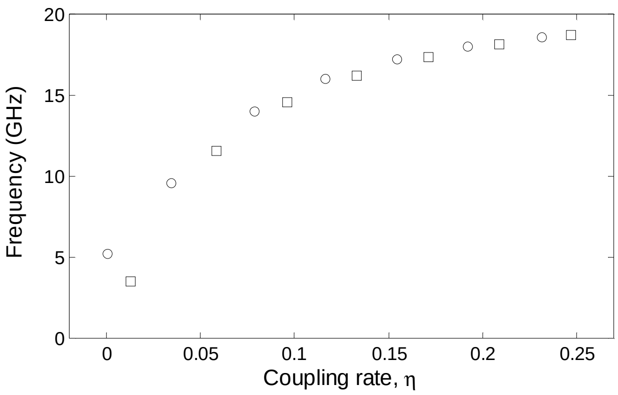

Figure 4

4. Conclusion

We have studied the dynamics of two mutually coupled semiconductor lasers. We have reported a symmetry breaking in periodic solutions at low coupling rate. High-frequency periodic oscillations can be observed at higher coupling rate. The frequency of these periodic oscillations is proportional to the inverse of the flying time between the lasers. The system under study is therefore of practical interest since it constitutes an all-optical source of microwave oscillations.

F. Rogister is a Scientific Research Worker of the Fonds National de la Recherche Scientifique (FNRS). This research was supported by the InterUniversity Attraction Pole (IAP V/18) program of the Belgian government and by the Training and Mobility of Researchers Programme, Access to Installations, under contract ERBFMGECT950062 Access to supercomputing facilities for European researchers established between The European Community and CESCA-CEPBA.