Stochastic dynamics of polarization switching in VCSELs

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We present analytical and numerical results on the stochastic properties of the switching time in current-induced polarisation switching in VCSELs. The switching times and their stochastic distributions are compared for different mechanisms causing the switching (thermal and non-thermal). The scaling of the mean switching time and its variance are discussed as a function of the height of the applied current pulse.

Introduction and model

During the last decade, Vertical-Cavity Surface-Emitting Lasers (VCSELs) evolved from laboratory curiosities into successful optical components used in a wide variety of applications. Nevertheless not all their properties are completely understood, e.g. some VCSELs show abrupt polarization switching (PS) between the two orthogonal polarization modes (PMs) when the injected current is changed.[1,2,3,4,5] While un controlled PS is highly undesirable in applications, controlled current-induced PS might be interesting as an alternative switching mechanism. In the present work we investigate the dynamics of current-induced PS, taking stochastic effects into account. Previous experimental studies have indeed demonstrated that stochastic effects cause anomalously large jitter in modulation experiments[6].

Much has been said about the origin of PS in VCSELs. Roughly speaking, the different proposed mechanisms can be divided into two categories: those invoking slow (lattice) thermal mechanisms[3,4,7] and those relying on other, faster mechanisms[2,8,9,10]. Recently, by carefully measuring the polarization modulation response curve, it was shown that PS in gain-guided (proton-implanted) VCSELs was of thermal origin[11], while air-post (strongly index-guided) VCSELs showed PS of faster origin[12].

PS caused by gain switching can be described by stochastic rate eqs, where advantage is taken from the fact that in a VCSEL the two modes are nearly degenerate[13]. We introduce the dimensionless variables E_i and n (the deviation of the carrier density from the clamped value), and the parameters G and $\varepsilon_{s,c}$, the reduced gain difference between the two modes and the rescaled gain saturation parameters, resp. This leads to:

$$\begin{aligned}
\dot{E}_x &= \frac{1+i\alpha}{2}\left(n - \varepsilon_s p_x - \varepsilon_c p_y\right)E_x + \sqrt{\beta}\,\xi_x(t) \\
\dot{E}_y &= \frac{1+i\alpha}{2}\left(n + G - \varepsilon_s p_x - \varepsilon_c p_y\right)E_y + \sqrt{\beta}\,\xi_y(t) \qquad (1) \\
\dot{n} &= \frac{J - p_x - p_y}{T} - n - \left(n - \varepsilon_s p_x - \varepsilon_c p_y\right)p_x - \left(n + G - \varepsilon_s p_y - \varepsilon_c p_x\right)p_y
\end{aligned}$$

Here, α is the usual factor describing phase-amplitude coupling in semiconductor lasers (Henry's alpha-factor), but plays no further role in our context. The factor β describes the strength of the spontaneous emission noise. The Langevin force terms $\xi_i(t)$, describing the spontaneous emission noise, are zero-mean, delta correlated, complex gaussian white noise terms: $\langle \xi_i(t)\xi_j^*(t')\rangle = \delta_{i,j}\delta(t-t')$. Eqs(1) are the typical rate eqs for a two-mode semiconductor laser. In the specific case of a VCSEL, they can also be deduced from the Spin Flip Model (SFM)[7]. This reduction[14] is valid for relatively large birefringence and relatively large spin relaxation rate. The spin flips are then essentially contributing to the cross saturation terms γ_c.

A stability analysis[12] of Eqs(1) reveals that polarization switching is predicted for certain parameters, assuming that the linear gain difference G varies with current and changes sign. A straightforward numerical simulation of such a gain-induced switching event shows that the carrier density essentially remains constant (clamped) during PS. This observation points to a further simplification: from the RHS of the carrier Eq.(1c) is, up to zero order in β, a conservation law can be deduced: $p_x + p_y = J + O(\beta)$. This conservation law physically means that on time scales longer than the inverse of the relaxation oscillations frequency, the two optical modes are anti-correlated. The conservation law can be exploited to further reduce the problem to a single nonlinear dynamical equation[12] for either E_x or E_y:

$$\dot{E}_y = \frac{1+i\alpha}{2}\left[(G-\Delta) + A|E_y|^2 - B|E_y|^4\right]E_y + \sqrt{\beta}\,\xi_y(t) \qquad (2)$$

with $A=3\Delta-G$, $B=2\Delta$, and $\Delta=(\gamma_c-\gamma_s)J$. Δ is in fact a remnant of the gain nonlinearities, and $\Delta>0$ is the sufficient condition to have a (small) region of bistability as is commonly observed in VCSELs.

Eq(3) has the form of a class-A laser equation, and is valid to study the behavior of VCSELs on a time scale larger than the inverse of the relaxation oscillation frequency. For our purposes of calculating the first passage time (or stochastic escape time), i.e. the stochastic time in which p_y crosses a rather large reference value p_0, we only need the linear part of this equation (i.e. we put A=B=0).

Instantaneous (nonthermal) gain switching

In order for the nonlasing y-mode to ignite and depart from the noise level, we suppose that at t=0 a current step is applied and that $G-\Delta$ changes instantaneously from an initial value $G_i-\Delta_i<0$ to a final value $G_f-\Delta_f>0$. The stochastic problem of the switching time (or first passage time) in class-A lasers has already been discussed in the eighties[15], and we can apply these results as we have proven that the problem of PS in VCSELs is equivalent to the case of gain switching in class-A lasers. The passage time statistics can be calculated, yielding

where ψ is the digamma function ($\psi(1)=-0.577$ and $\psi'(1)=1.64$).

$$\langle t^*\rangle = \frac{1}{(G_f-\Delta_f)}\left[\ln\left(\frac{p_0}{\langle h^2\rangle}\right)-\psi(1)\right] \qquad (3)$$

$$\langle(\delta t^*)^2\rangle = \frac{\psi'(1)}{(G_f-\Delta_f)^2}, \quad \text{with} \quad \langle h^2\rangle = \langle|h(\infty)|^2\rangle = \frac{2\beta}{(G_f-\Delta_f)} + \frac{2\beta}{(G_i-\Delta_i)} \qquad (4)$$

Both <t*> and <Δt*> are inversely proportional to the dichroism $G_f–Δ_f$, diverging as $(G_f–Δ_f)$–>0 i.e. as the final value of the current moves closer to the switching point. Also the covariance <h^2>, that plays the role of an effective initial condition, varies with dichroism. Another result that obviously can only be calculated by a stochastic analysis is the expression for the variance <Δt*>, or jitter time. It is an interesting result that <Δt*> does not depend on the noise strength □, and neither on the reference level p_0.
In order to test all the approximations made in the analytical treatment, we compared our theoretical results with numerics, obtained by integrating the full set of Eqs(1) for 10^4 realizations, and calculating the average switching time and the variance. The overall agreement between theory and numerics (see fig. 1) is found to be excellent, giving further confidence in the reduction of the equations, the further linearization and the stochastic analysis.

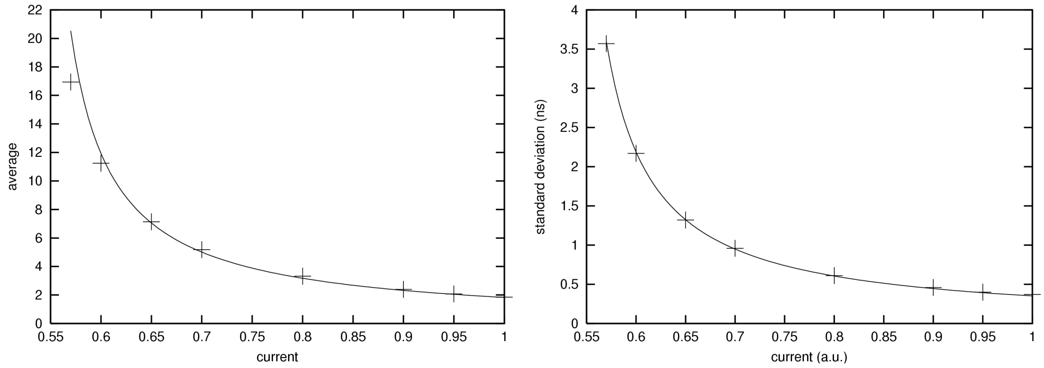

Fig.1: Average switching time <t*> (left figure) and its variance <Δt*> (right figure) as a function of final value of the input current J. Theoretical results (solid line) on the basis of Eqs(4) are compared with ones obtained by numerically solving Eqs(2) (crosses). The linear dichroism is assumed to depend linearly on current J, as follows: $G=-(1–J/J_0)g$, with g=4., J_0=0.471, J_{init}=0.5 and □=1 10^{-4}.

Thermally induced gain switching

Also in the case where the gain varies with temperature, the problem can be solved. After a current step is applied at t=0, the gain relaxes exponentially to its new end value (see Fig.2). The problem can then be split into two parts: first there is a deterministic delay time t_d to reach the switching point, given by: $t_d = \square \ln\left[\left(G_f \square G_i\right)/\left(G_f \square \Delta\right)\right]$
The stochastic escape time from there on can again be calculated using the theory of gain switchin of a class-A laser with ramped control parameter[16], yielding:

$$\left\langle t^{\square}_{thermal}\right\rangle = \square\sqrt{T}\left(1 \square \frac{\square(1)}{2T}\right), \qquad \text{with} \quad \square = \sqrt{\frac{2\square}{G_f \square\square}}$$

$$\left\langle (\square t^{\square}_{thermal})^2\right\rangle = \square^2\frac{\square(1)}{4T}, \qquad T = \ln\left(\frac{p_0}{\left\langle |h_{th}|^2\right\rangle}\right) >> 1 \; ; \text{ and } \left\langle |h_{th}|^2\right\rangle = \sqrt{\square}\square\square$$

One can see that the basic scaling parameter is now □. Through □, both <t*> and <Δt*> now diverge as $(G–Δ)^{-1/2}$ as (G–Δ)–>0. <Δt*> now also depends on the noise level □, unlike the nonthermal case. We have again checked the analytical results with numerical simulations (not shown). Although the correspondence is less good than in the instantaneous case, the theory still adequately predicts the order of magnitude and the basic scaling trends.

Conclusion

In summary, we propose an analytical model to calculate the switching time and its variance of current driven PS in VCSELs. The theory is based on a rate equation approach, where, taking advantage of the clamping of the carriers, we reduce the three rate equations to one single dynamical equation, a linearized class-A laser equation. We can then apply the well-known theory for the statistics of class-A laser switch-on, yielding analytical expressions for the average switching time and its variance for both cases of instantaneous (nonthermal) gain changes with current and thermally induced gain changes. The theoretical results have been succesfully compared with numerical simulations of the original model equations.

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