Performing Diffraction Tomography Without Phase Information

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It is demonstrated that the usual theorem of diffraction tomography may, in many cases, be replaced by a method which uses measurements of the intensity of the scattered field on a pair of planes in place of the usual phase measurements of the field. Numerical examples of reconstructions with this new method are given.

Introduction

Tomographic methods such as computed tomography ([1], Sec. 4.11) and diffraction tomography ([1], Sec. 13.2) are important techniques for probing the structure of three-dimensional weakly scattering objects. Computed tomography employs a geometric model for the propagation of wavefields, assuming they are absorbed but not diffracted, while diffraction tomography incorporates both absorption and scattering effects. However, although diffraction tomography is a more general method, it is limited by the requirement that both the phase and the intensity of the scattered field must be measured. At optical wavelengths or higher, phase measurements can present formidable difficulties.

Recently we have introduced [2, 3] a new method of performing diffraction tomography which does not require phase information. Phase measurements are replaced by intensity measurements on a pair of planes along the direction of propagation of the probing field. In this paper we describe this method and present numerical examples of reconstructions using it.

Intensity-only diffraction tomography

The crux of the new tomographic method is that the propagation of a wavefield is determined by both its intensity and its phase. It seems possible, then, that at least in some circumstances the phase of a wavefield could be determined by measuring how the intensity of the field changes as it propagates.

We consider a system as depicted in figure 1. A scalar plane wave $U_i(\mathbf{r}) = e^{ik\mathbf{s}_0\cdot\mathbf{r}}$ is incident upon an object occupying a volume V with a (generally) complex index of refraction $n(\mathbf{r})$, producing a total field $U(\mathbf{r})$ in the region of the object. $U(\mathbf{r})$ may represent an acoustic wave or a polarization component of an optical field; the intensity of the field is given by $I(\mathbf{r}) = |U(\mathbf{r})|^2$. The propagation of the total field is determined by the scalar wave equation

$$[\nabla^2 + k^2]U(\mathbf{r}) = -4\pi F(\mathbf{r})U(\mathbf{r}), \tag{1}$$

where

$$F(\mathbf{r}) = \frac{k^2}{4\pi}[n^2(\mathbf{r}) - 1] \tag{2}$$

is the scattering potential of the object.

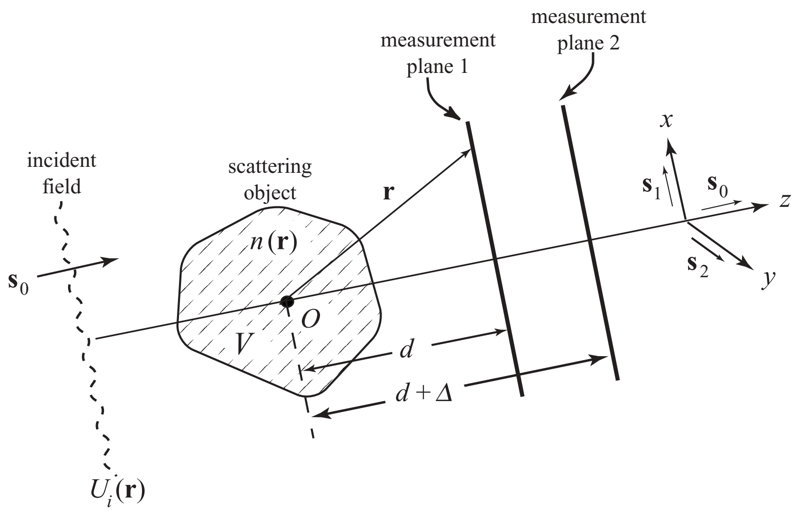

Figure 1: *Illustrating the notation relating to the new tomographic method.*

We define an intensity data function by the relation

$$D_I(x,y;z) \equiv \log[I(x,y;z)], \tag{3}$$

and the two-dimensional Fourier transform of this data function in the xy-plane,

$$D_I(u,v;z) = \frac{1}{(2\pi)^2}\iint D_I(x,y;z)e^{-\mathrm{i}(ux+vy)}\mathrm{d}x\mathrm{d}y. \tag{4}$$

The intensity-only tomography method requires the construction of another data function

$$\hat{D}_\Delta(u,v;d) \equiv \frac{\hat{D}_I(u,v;d) - \hat{D}_I(u,v;d+\Delta)e^{\mathrm{i}(w-k)\Delta}}{\Delta}, \tag{5}$$

where $w = \sqrt{k^2-u^2-v^2}$, and it is assumed that $u^2+v^2 \leq k^2$. This data function uses a combination of intensity data from two measurement planes, one plane at $z = d$ and one at $z = d+\Delta$. If the scattering is sufficiently weak so that the scattered field may be approximated by the first Rytov approximation ([1], Sec. 13.5), it can be shown [2, 3] that the data function takes on the simple form

$$\hat{D}_\Delta(u,v;d) = \frac{(2\pi)^2\mathrm{i}}{w\Delta}\tilde{F}[u\mathbf{s}_1+v\mathbf{s}_2+(w-k)\mathbf{s}_0]\exp[\mathrm{i}(w-k)d]\{1-\exp[2\mathrm{i}(w-k)\Delta]\}, \tag{6}$$

where

$$\tilde{F}(\mathbf{K}) = \frac{1}{(2\pi)^3}\int_V F(\mathbf{r}')e^{-\mathrm{i}\mathbf{K}\cdot\mathbf{r}'}\mathrm{d}^3r' \tag{7}$$

is the *three-dimensional* spatial Fourier transform of the scattering potential. Equation (6) demonstrates that the data function $\hat{D}_\Delta$ is directly related to Fourier components of the scattering potential, and is the basis of the new tomographic method. By combining data from a variety of u,v values and a variety of directions of incidence $\mathbf{s}_0$, it is possible to obtain all Fourier components of $\tilde{F}(\mathbf{K})$ such that $|\mathbf{K}| \leq \sqrt{2}k$. In fact, Eq. (6) differs from

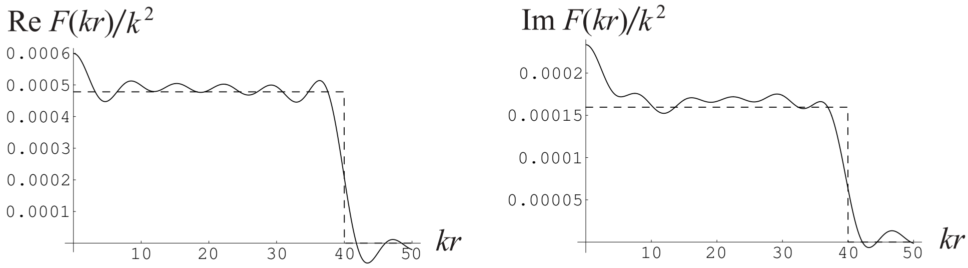

Figure 2: *The reconstruction of a homogeneous sphere by the new tomographic method. The dashed curve indicates the actual scattering potential.*

the traditional theorem of diffraction tomography only in the presence of the term in the curly brackets. This additional term vanishes when the equation

$$2[k - \sqrt{k^2 - u^2 - v^2}]\Delta = 2n\pi \tag{8}$$

is satisfied. It can be deduced from this expression that the low frequency components of $\tilde{F}(\mathbf{K})$ must be extrapolated from neighboring values, and that the measurement planes must be placed sufficiently close to each other to avoid other zeros in the data function.

This method is similar to a method suggested by Teague [4] for the determination of the phase of a paraxial field from intensity measurements, and suffers from some of the same limitations, such as nonuniqueness when the field possesses optical vortices [5]. However, our tomographic method is not in principle limited to paraxial fields, and does not require the solution of an additional differential equation to determine the phase, as does Teague's.

Examples

To illustrate the new method, we first consider scattering from a sphere of scaled radius $ka = 40$ and complex index of refraction $n = 1.003 + .001\text{i}$. The scattered field was determined using a partial wave expansion, and the measurement planes were taken to be at distances $kd = 60$ and $k\Delta = 2$, measured from the center of the sphere. Spatial frequencies of $\tilde{F}$ away from the origin were determined using the data function $\hat{D}_\Delta$; low spatial frequency components were determined by fitting this data to a quadratic function centered on the origin.

The assumed scattering potential and the reconstruction are shown in figure 2. It can be seen that there is good agreement, and that the reconstruction has properly reconstructed both the real and imaginary parts of the scattering potential. The differences between the assumed potential and the reconstruction can be attributed to a Gibbs phenomenon from neglecting higher spatial frequency components of $\tilde{F}$, and from multiple scattering effects.

We next consider scattering from a two-layered spherical object of inner radius $ka = 20$ and outer radius $kb = 40$, with the refractive index of the inner sphere taken to be $n_a = 1.001 + 0.001\text{i}$ and that of the outer shell taken to be $n_b = 1.0005 + 0.001\text{i}$. The measurement planes were again taken to be at distances $kd = 60$ and $k\Delta = 2$. The assumed

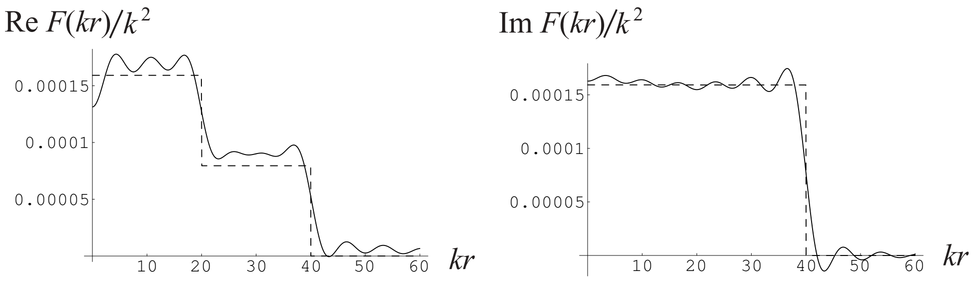

Figure 3: *The reconstruction of a multilayered sphere by the new tomographic method. The dashed curve indicates the actual scattering potential.*

potential and the reconstruction are shown in figure 3. Again, it is seen that there is good agreement between the actual and reconstructed potentials; furthermore, the reconstruction has properly determined the structural differences between the two layers.

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