Degree of entanglement between excitons in two quantum dots in a cavity

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The dynamical behavior of entanglement for excitons in two coupled quantum dots, that are placed in a cavity is investigated for two different initial conditions. The effects of exciton tunneling and of dissipation of the cavity field and the excitons on entanglement of formation are discussed. It is shown that depending on initial conditions the direct coupling between the dots will enhance or reduce the entanglement.

Introduction

Entanglement constitutes the single most characteristic property that makes quantum mechanics distinct from any classical theory. It has been found that entanglement forms a fundamental resource for quantum-information processing. For application purposes, it became essential to quantify entanglement. Different quantities such as entanglement of formation[1], relative entropy[2] and negativity (or logarithmic negativity)[3] are introduced to measure quantum entanglement. Among them, the entanglement of formation[1] is a good measure for the degree of entanglement for two-qubit states. On the other hand, there has been growing interest in the quantum information properties of semiconductor quantum dots in the quest to implement quantum-dots scalable quantum computers[4]. This interest is stimulated to some extent by recent experimental advances in the coherent observation and manipulation of quantum dots, including the demonstration of the quantum entanglement of excitons in a single dot[5] or in a quantum dot molecule[6], and observations of Rabi oscillations of excitons in single dot[7]. Here we discuss the dynamic behavior of the entanglement for excitons in two coupled quantum dots placed in a cavity.

Model

The model under consideration consists of two quantum dots that are placed in a single-mode cavity. We assume that there are only a few electrons excited from the valence-band to the conduction-band in each quantum dot. Then the exciton number in the ground state for each quantum dot is low so that the mean distance between two excitons is much larger than the extension of an exciton. In this case, the exciton can be described by boson operators[8][9]. In the rotating-wave approximation, the Hamiltonian for the system can be written as

$$H = H_0 + H_{int} = \omega a^\dagger a + \omega \sum_{j=1}^{2} b_j^\dagger b_j + \sum_{j=1}^{2} g_j (b_j^\dagger a + a^\dagger b_j) + \lambda (b_1^\dagger b_2 + b_1 b_2^\dagger), \quad (1)$$

where $a^\dagger$ and a are the operators of the cavity field with frequency ω, $b_j^\dagger$ and b_j represent the creation and annihilation operators of the excitons in the $j-th$ quantum dot. The parameters g_j are the coupling constants between the j-th quantum dot and the cavity field. In order to simplify the calculation, the two dots are assumed to be identical so that g_j can be chosen as $g_1 = g_2 = g$. The term containing λ describes the direct interaction between two dots such as exciton tunneling. Taking dissipations of photons and excitons into account, the density operator $\rho(t)$ of the photon-exciton interaction system in the interaction picture obeys the equation

$$\frac{d}{dt}\rho = J\rho + L_a\rho + L_b\rho \tag{2}$$

with

$$\begin{aligned} L_a\rho &= \kappa_a(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \\ L_b\rho &= \kappa\sum_{j=1}^{2}(2b_j\rho b_j^\dagger - b_j^\dagger b_j\rho - \rho b_j^\dagger b_j). \end{aligned} \tag{3}$$

where the superoperator J is defined as $J\rho = -i[H_{int}, \rho]$, and the quantities κ_a and κ represent the loss rates of photons and excitons, respectively, due to the influence of environment with zero temperature. The solution of equation (2) can be formally written as

$$\rho(t) = \exp[t(J + L_a + L_b)]\rho(0). \tag{4}$$

Here $\rho(0)$ is the initial condition of the photon-exciton interaction system. By tracing out the cavity field variables for the density operator $\rho(t)$, one obtains the reduced density operator $\rho_{12}(t) = tr_f(\rho(t))$ for the excitons in two quantum dots.

Dynamic behavior of the degree of entanglement

In order to study the dynamical behavior of entanglement of excitons in two quantum dots, we use the entanglement of formation proposed by Wootters[1] to quantify the degree of entanglement for two subsystems of $2\otimes 2$ bipartite mixed or pure states. Entanglement of formation is a measurable quantity, at least for a pair of qubits, which is the case we are dealing with here. The underlying quantity is called concurrence[1]. For pure states, concurrence is strongly connected with two-particle visiblity which is a property that cannot exist separately in the parts of a bipartite system. The expression relating the concurrence to the density operator ρ_{12} of a mixed state is given as

$$C(\rho_{12}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{5}$$

where λ_i's are the square roots of the eigenvalues of the operator $\rho_{12}\tilde{\rho}_{12}$ in descending order. Here $\tilde{\rho}_{12}$ results from applying the spin-flip operation to ρ_{12}^* as $\tilde{\rho}_{12} = (\sigma_y^{(1)} \otimes \sigma_y^{(2)})\rho_{12}^*(\sigma_y^{(1)} \otimes \sigma_y^{(2)})$, in which $\sigma_y^{(j)}$ (j=1,2) are the Pauli spin operator in the standard basis and ρ_{12}^* is the complex conjugate of ρ_{12}. The entanglement of formation $E(\rho_{12})$ of the density operator ρ_{12} is connected with the concurrence $C(\rho_{12})$ via the formula

$$E(\rho_{12}) = -x\log_2 x - (1-x)\log_2(1-x) \tag{6}$$

with $x = \frac{1+\sqrt{1-C(\rho_{12})^2}}{2}$.

In our first example, we assume that the photon-exciton interaction system is initially in the state $\rho(0) = |0,1,0\rangle\langle 0,1,0|$, which means that the cavity field is initially prepared in the vacuum state $|0\rangle$, one exciton is excited in quantum dot 1 and no exciton exsits in quantum dot 2. In this case, the concurrence for the reduced density operator $\rho_{12}(t)$ for the excitons in the two quantum dots is explictly given as

$$C(\rho_{12}) = \frac{\exp(-2\kappa t)}{4}(\sqrt{f_1} + \sqrt{f_2} - |\sqrt{f_1} - \sqrt{f_2}|), \quad (7)$$

where $f_1 = (\frac{2g^2}{\beta^2}\sin^2\beta t)^2 + 4(\cos\beta t\sin\frac{3\lambda t}{2} + \frac{\lambda}{2\beta}\sin\beta t\cos\frac{3\lambda t}{2})^2$, $f_2 = (1 + \cos^2\beta t + \frac{\lambda^2}{4\beta^2}\sin^2\beta t)^2 - 4(\cos\beta t\cos\frac{3\lambda t}{2} - \frac{\lambda}{2\beta}\sin\beta t\sin\frac{3\lambda t}{2})^2$, and $\beta = \sqrt{2g^2 + \lambda^2/4}$. Substituting eq.(7) into eq.(6) one obtains the entanglement of formation E of the density operator ρ_{12}. Fig.1 shows the time evolution of the entanglement of formation for the excitons in two quantum dots for different values of λ and κ, and $\hbar g = 0.5 meV (i.e., g \approx 7.6 \times 10^{11} Hz)$[9]. If there is no direct coupling between two quantum dots and the dissipation of cavity field and exciton can be neglected, i.e., $\lambda = 0$ and $\kappa = 0$, the excitons in two quantum dots can be partially entangled periodically due to the interaction between the two dots, mediated by the cavity field. If there exists direct coupling between the two dots such as exciton tunneling, the degree of entanglement of the excitons in two different dots can be enhanced. The stronger the direct coupling, the larger the degree of the entanglement that can be reached. However, the effect of the cavity dissipation and exciton dissipation reduces the entanglement of excitons. In the long time limit, the two quantum dots will be damped into their vacuum state due to the dissipation effect and entanglement decays to zero.

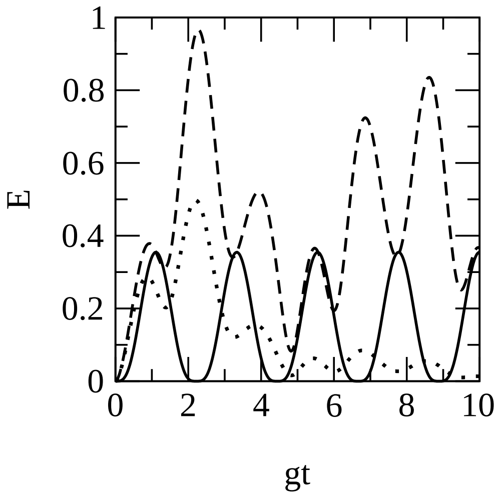

Fig. 1. Time evolution of the entanglement of formation of two quantum dots in a cavity when initially only one exciton is present in one of the dots and no photons in the cavity. Solid line ($\kappa = 0$, $\lambda = 0$), dashed line($\kappa = 0$, $\lambda/g = 0.4$), and dotted line($\kappa/g = 0.1$, $\lambda/g = 0.4$).

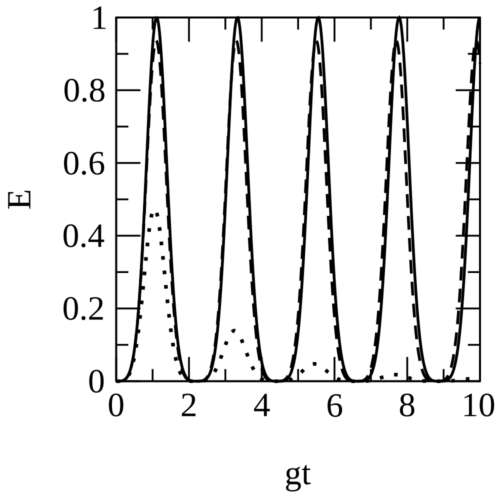

Fig. 2. Same as fig.1, but with initial condition that no excitons are present and the cavity is in the odd coherent state with $|\alpha|= 1.0$ and $\phi = \pi$.

In our second example, we assume as initial condition that the cavity field is prepared in a superposition of two distinct coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ in the normalization form of $|\Psi_a(0) >= (|\alpha\rangle + e^{i\phi}|-\alpha\rangle)/\sqrt{N}$ with $N = 2(1 + e^{-2|\alpha|^2}\cos\phi)$, and that the two modes of excitons are in the vacuum state $|0, 0\rangle$. The concurrence for the reduced density operator

$\rho_{12}(t)$ for the excitons then becomes

$$C(\rho_{12}) = \frac{1 - \exp(-\frac{4g^2}{\beta^2}|\alpha|^2 e^{-2\kappa t}\sin^2\beta t)}{1 + \cos\phi e^{-2|\alpha^2}} \exp[-2|\alpha|^2(1 - \frac{2g^2}{\beta^2}e^{-2\kappa t}\sin^2\beta t)]. \quad (8)$$

Fig.2 displays the time evolution of the entanglement of formation for $\rho_{12}(t)$ when the cavity field is initially in the odd coherent state (i.e., $\phi = \pi$). Different from fig.1, the excitons in the two different quantum dots can evolve into a maximally entangled state if there is no direct coupling between two dots and no dissipation of the cavity field and excitons. The direct coupling between two dots now *reduces* the degree of entanglement of the excitons in two dots. The dissipation of the cavity field and excitons also plays a role in the reduction of the degree of entanglement.

Conclusions

The dynamical behavior of entanglement for excitons in two coupled quantum dots placed in a cavity is discussed. It is found that if there is initially only one exciton in one of the two dots and the cavity field is in vacuum state, the excitons in two dots can be entangled periodically due to the interaction between two dots mediated by the cavity field. If the two dots are close so that there exists direct coupling between them, this direct coupling may enhance the entanglement. Cavity dissipation and exciton dissipation reduce the entanglement. Also when driven by the cavity field which is initially in the superposition of two distinct coherent states, the excitons in two quantum dots can become entangled. In that case, the direct coupling between two dots *reduces* the degree of entanglement between the excitons.

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