A Rate Equation Model for Self-Pulsing Vertical-Cavity Surface-Emitting Lasers

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Self-pulsations in semiconductor edge-emitting lasers, due to the existence of a saturable absorber surrounding the active region, are well-known. On the other hand, in vertical-cavity surface-emitting lasers (VCSELs), an alternative mechanism can give rise to a similar self gain modulation: unstable Kerr waveguiding. We propose a model based on intensity rate equations that includes non linear gain effects and takes the anisotropy between the polarisation modes into account. This enables us to predict the modal stability and the dynamics originating from the combination of polarisation instability and unstable waveguiding. Our results are compared with experimental results of self-pulsing VCSELs.

Introduction

Self-pulsing in semiconductor lasers containing both an active and an absorbing region within the same structure has been the subject of many studies. A pulsating output intensity is a well-known phenomenon for edge-emitting lasers incorporating a saturable absorber [1]. They have found their use in practical applications for telecommunications [2, 3] and for optical data storage using compact disc [4]. In [5, 6, 7], single-mode self-pulsating edge-emitting lasers are described in terms of three ordinary differential equations for the photon and the two electron densities. In VCSELs, an extra degree of freedom comes into play: the polarization, which could lead to interesting applications. This has been modelled by incorporating the Yamada model and the Spin Flip Model [9] and indeed self-pulsations and even polarization chaos has been predicted for VCSELs where the active region is surrounded by a saturable absorber.

Besides saturable absorption there is another effect, less discussed in the literature, that can also lead to the pulsed operation of the laser: unstable waveguiding. Due to the Kerr effect and the profile of the fundamental mode the refractive index will be enhanced more in the center of the active region than at the sides. As a result the lasing mode will be confined more efficiently, enlarging the effective modal gain and giving rise to self-pulsating behavior. Recently, self-pulsations have been observed in Vertical-Cavity Surface-Emitting Lasers (VCSELs) [8] and unstable waveguiding has been cited as a possible cause.

Fundamental mode confinement

When a mode is better confined in the laser structure, its gain will be higher. If somehow the mode itself could alter the structure in such a way that the confinement is improved, then the mode is able to enhance its own gain, a sufficient ingredient for a self-pulsing

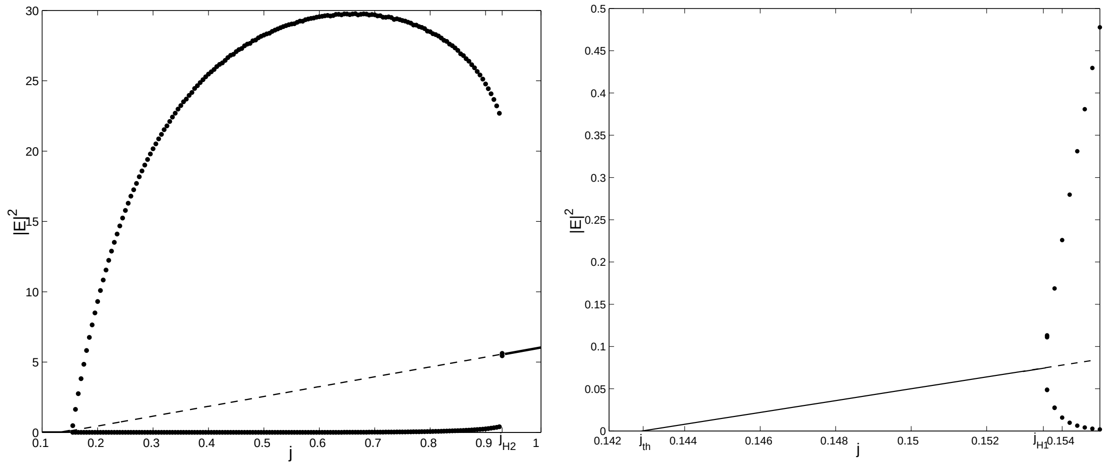

Figure 1: Bifurcation diagram of the intensity of the electric field. Solid lines represent stable fixed points, while dashed lines are unstable fixed points. The filled circles are the minima and the maxima of a stable limit cycle. The right figure is the bifurcation diagram close to threshold. ($\rho = 10^{-3}$, $g = 7.0$, $\alpha = 0.015$, $\beta = 0.5$)

laser. The Kerr effect offers the mode such a possibility. To verify this, we use a variational technique to solve for the fundamental modes in a cylindrical waveguide with a Kerr medium core, and calculate the resulting confinement factor. We assume the electric field to have a gaussian profile and an intensity equal to p. In the weakly guiding approximation, we find that the confinement factor can be approximated by:

$$\Gamma \approx \Gamma_0 \left(1 + \frac{\alpha p}{1 + \beta p}\right). \tag{1}$$

where α and β are parameters, which depend on the difference in refractive index between the core and the cladding and on the strength of the Kerr effect in the core. At laser threshold the confinement factor equals Γ_0.

The gain enhancement model

Here, we present a model without taking into account the polarization degeneracy. To include the effect of unstable waveguiding, we add to the standard set of field rate equations a nonlinear gain enhancement term:

$$\rho \frac{dE}{dt} = \frac{g}{2}(1 + i\beta_c)\left(1 + \frac{\alpha |E|^2}{1 + \beta |E|^2}\right) nE - \frac{E}{2} + \tilde{F_E}, \tag{2}$$

$$\frac{dn}{dt} = j - n - \left(1 + \frac{\alpha |E|^2}{1 + \beta |E|^2}\right) n|E|^2 + \tilde{F_n}, \tag{3}$$

with g the linear gain factor, β_c Henry's line width enhancement factor, j is the renormalized injected current. E and n are the renormalized electric field and carrier density. Time has been rescaled to the carrier life time and ρ is the ratio of photon and carrier life time. We have added $\tilde{F_E}$ and $\tilde{F_n}$ as δ correlated noise sources:

$$< \tilde{F_E}(t) \tilde{F_E}^*(t') > = 2R_{sp}\delta(t - t'), \tag{4}$$

$$< \tilde{F_E}(t) > = 0. \tag{5}$$

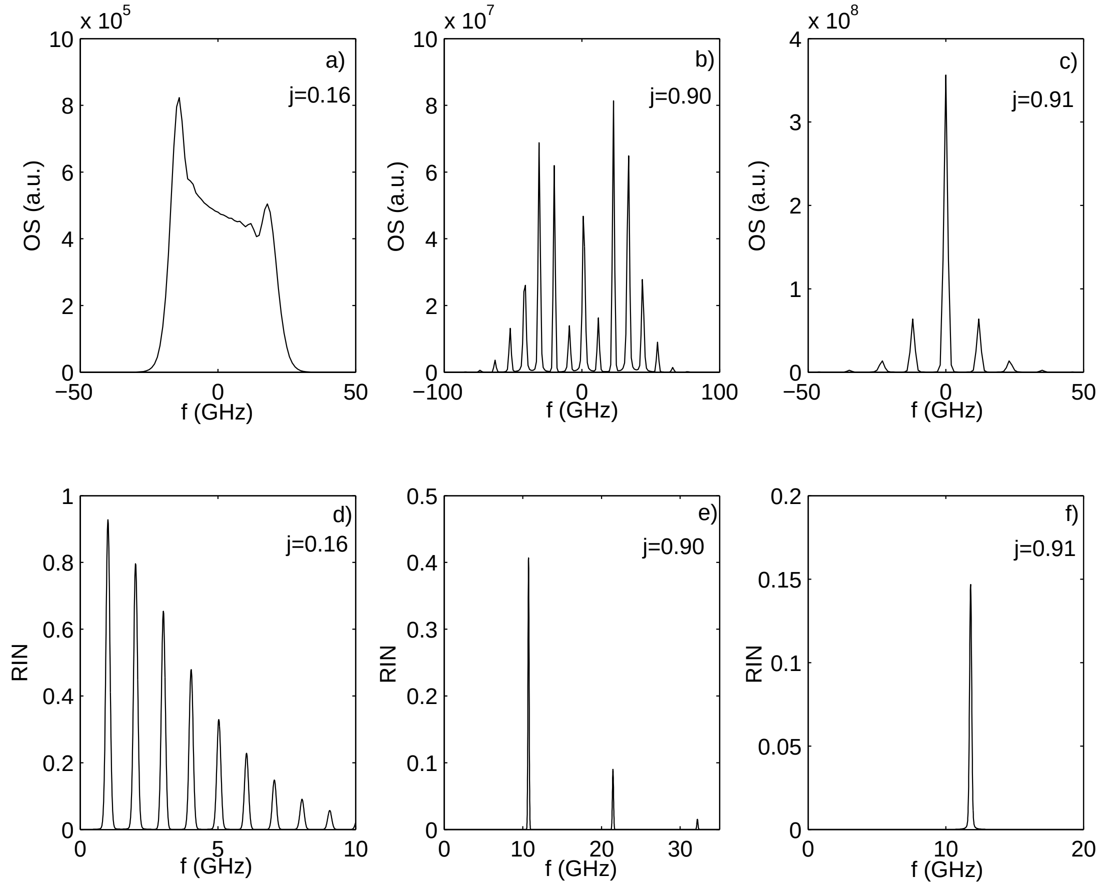

Figure 2: (a)-(c) Optical spectra as calculated from Eqs. (2)-(3) for increasing current. (d)-(e) RIN spectra as calculated from Eqs. (2)-(3) for increasing currents. ($\rho = 10^{-3}$, $g = 7.0$, $\alpha = 0.015$, $\beta = 0.5$, $\beta_c = -5.0$, $R_{sp} = 10^{-4}$)

We will neglect the carrier noise, $\tilde{F}_n$. In Figure 1 we have plotted a typical bifurcation diagram of the field intensity obtained from Eqs. (2)-(3). When the injected current grows the laser achieves a stationary output at threshold, j_{th}. At a first Hopf bifurcation point, j_{H1}, the stationary solution becomes unstable and a stable limit cycle emerges. It is this limit cycle which explains the pulsed behavior. Later, at a higher current, a new Hopf bifurcation point, j_{H2}, is crossed as the saturability takes over.

The RIN and optical spectra at different currents are shown in Figure 2. The optical spectra will be strongly frequency modulated due to the amplitude phase coupling, combined with a strong amplitude modulation. When the current is fixed closely to j_{H2}, we expect the intensity to be nearly harmonic. The resulting optical spectrum is a simple symmetrical FM spectrum (with peaks at multiples of the pulse frequency, Figure 2c). The corresponding RIN spectrum will be just one peak at the pulse frequency (Figure 2f). Lowering the current the pulses become stronger and the spectrum becomes a combination of AM and FM, which destroys the symmetry of the spectrum (Figure 2b). The modulation of the intensity is highly nonlinear which results in the typical RIN spectra in Figures 2d and 2e with harmonics at multiples of the pulse frequency. Lowering the current even further the pulse frequency drops and the frequency peaks start overlapping, effectively constructing the ”bat ears” spectrum (Figure 2a). All the spectra shown in Figure 2 are in excellent agreement with the experimental spectra shown in [8]. We have expanded the model to a two mode intensity rate equation description of a VCSEL and have studied the combination of polarization instability and unstable waveguiding.

Conclusion

We have treated the effect of unstable waveguiding for the fundamental mode in a VCSEL using a variational technique and showed that it boils down to a nonlinear gain enhancement term in a rate equation model. On the basis of this gain enhancement rate equation model, we showed that self-pulsations emerge at the relaxation oscillation frequency from a Hopf bifurcation. Our theoretical results are in excellent agreement with the experimental results reported in [8], confirming that unstable waveguiding due to the Kerr effect could be the cause of the observed self-pulsating behavior of VCSELs.

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References

[1] A. E. Siegman, Lasers, Sausalito, CA: Univ. Science Books, 1986, p. 1024.

[2] P. Rees, P. McEvoy, A. Valle, J. O’Gorman, S. Lynch, P. Landais, L. Pesquera and J. Hegasky, “A theoretical analysis of optical clock extraction using a self-pulsing laser diode”, *IEEE J. Quantum Electron.*, vol. 35, pp. 221-227, 1999.

[3] C. Juang, T. M. Hwang, J. Juang and W. W. Lin, “A synchronization scheme using self-pulsating laser diodes in optical chaotic communication”, *IEEE J. Quantum Electron.*, vol. 36, pp. 300-304, 2000.

[4] M. Yamada and T. Higashi, “Mechanism of the noise reduction method by superposition of high-frequency current for semiconductor injection lasers ”, *IEEE J. Quantum Electron.*, vol. 27, pp. 380-388, 1991.

[5] M. Yamada, “A theoretical analysis of self-sustained pulsation phenomena in narrow-stripe semiconductor lasers”, *IEEE J. Quantum Electron.*, vol. 29, pp. 1330-1336, 1993.

[6] C. R. Mirasso, G. H. M. van Tartwijk, E. Hernández-García, D. Lenstra, S. Lynch, P. Landais, P. Phelan, J. O’Gorman, M. San Miguel and W. Elsäßer, “Self-pulsating semiconductor lasers”, *IEEE J. Quantum Electron.*, vol 35, pp. 764-770, 1999.

[7] T. W. Carr and T. Erneux, “Dimensionless Rate Equations and Simple Conditions for Self-Pulsing in Laser Diodes”, *IEEE J. Quantum Electron.*, vol. 37, pp. 1171-1177, 2001.

[8] M. B. Willemsen, A. S. van de Nes, M. P. van Exter, J. P. Woerdman, M. Brunner and R. H. Hövel, “Self-pulsations in vertical-cavity semiconductor lasers”, *Appl. Phys. Lett.*, vol. 77, pp. 3514-3516, 2000.

[9] A. Scirè, J. Mulet, C. R. Mirasso and M. San Miguel, “Intensity and polarization self-pulsations in VCSELs”, *Optics Letters*, vol. 27, pp. 391-393, 2002.