

Photonic Crystals Transmission Phenomena

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Light travelling through a slab made of a photonic material is expected to decay exponentially if its frequency lies within a bandgap. Numerical calculations as well as experiments show that sometimes a more complex behaviour occurs: The field inside the crystal may decay bi-exponentially or show saturation i.e. a small part of the incoming field is transmitted without attenuation. These propagation properties can be explained by a generic model which is solved analytically. The model medium consists of a separable dielectric function which is the sum of two one dimensional square well functions or trigonometric functions in two perpendicular directions.

1 Introduction

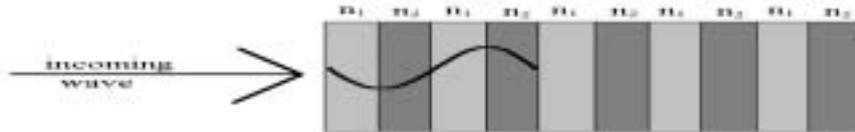


Figure 1: Illustration of a dielectric mirror.

The simplest photonic crystal structure is probably the one dimensional dielectric mirror. This structure is composed out of alternating dielectric layers, with two different dielectric constants, see in figure 1. Its key property is the occurrence of a bandgap, inhibiting the propagation of waves with frequency lying in this gap.

The creation of a complete bandgap is intimately related to the possibility to localize light due to multiple reflection [1]. From a technological point of view, photonic bandgaps are of much interest, as they open new possibilities for, e.g., lossless reflection of light (lossless mirrors), and for the construction of novel types of very efficient and compact waveguides.

For a wave with its wavelength lying in the band-gap one expects that this wave is totally reflected or exponentially decreasing *inside the crystal*. However, instead of this expected exponential decreasing behaviour of the transmitted light, a bi-exponential decreasing behaviour was observed, (see figure 2). This kind of behaviour was experimentally also observed by E. Chow et al [2]. In their experiment and calculation another related phenomena showed up, namely saturation. By saturation we mean that after a certain number of periods in the crystal, the transmission stays unchanged (saturates). There was no explanation given for this saturation, although they observed and mentioned it in their article. D. Labilloy [3] also observed saturation but attributed this to the noise limit, i.e. experimental errors. But they did

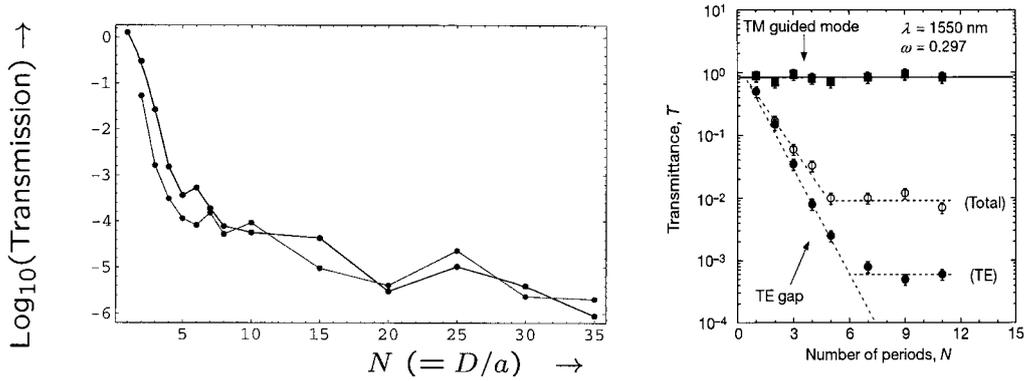


Figure 2: On the left hand side, bi-exponential behaviour is observed. On the right hand side, saturation is observed by E. Chow. Both phenomena start to take place from slab thickness of 5, meaning that the slab is 5 periods thick.

not have theoretical calculations to compare with, whereas E. Chow *did* have this theoretical ¹ backup [4] (the solid and dotted lines in figure 2).

We will investigate this transmission phenomenon of non exponential behaviour. The modes travelling through the crystal are excited by the incoming wave on the edge of the crystal and are expected to show exponential decay. For ordinary exponential decreasing behaviour these modes will all decrease in the same way. Our main observation is that not all the modes decrease in the same way, or better said: *some modes are propagating, i.e. can still penetrate through the crystal although the incoming wave has a wavelength in the band-gap range*. We are aware that this is a first class paradox: a band-gap means by definition that no propagating modes can exist, and we claim that there do always exist some propagating modes. The solution of this paradox is given by the research of G. Allen [5] who made an analysis of the solutions of the stable solutions of a square well potential, see figure 3.

2 Description of the model

We consider the Maxwell equations in two dimensions for a separable dielectric function: $\varepsilon_1(x) + \varepsilon_2(y)$. This leads for TM modes to:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E_z + \frac{\omega^2 \mu}{c^2} \varepsilon(x, y) E_z = 0 \quad (1)$$

and $H_x = \frac{c}{-i\omega\mu} \frac{\partial E_z}{\partial y}$, $H_y = \frac{c}{i\omega\mu} \frac{\partial E_z}{\partial x}$, $E_x = E_y = 0$. The other field components are equal to zero if we only consider TM modes. We next assume that $\varepsilon(x, y)$ are periodic square well dielectrical- or trigonometrical functions, (see figure 4).

Then writing E_z as $E_z = X_f(x)Y_f(y)$, where f denotes the separation constant, Eq. 1 leads to two separated equations for X_f and Y_f :

$$\left(\frac{\partial^2}{\partial x^2} + \mu \left(\frac{\omega}{c} \right)^2 \varepsilon_1(x) + f \right) X_f(x) = 0, \quad \left(\frac{\partial^2}{\partial y^2} + \mu \left(\frac{\omega}{c} \right)^2 \varepsilon_2(y) - f \right) Y_f(y) = 0. \quad (2)$$

¹The theoretical transmittance was calculated using 3D finite difference time-domain simulations

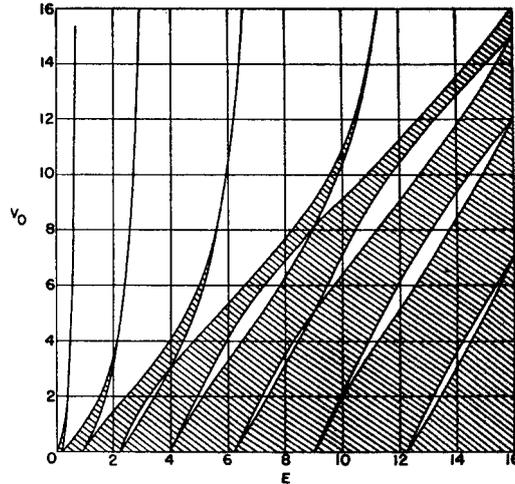


Figure 3: Original plot from the article in 1953 gives transmitting combinations of E and V_0 as shaded areas. Classically the plot would be simple: shaded for the region $E > V_0$ and unshaded for $E < V_0$. But quantummechanics gives another view at reality: in some regions of $E < V_0$ the electrons can still penetrate, which is called 'tunnelling', and in some regions of $E > V_0$ electrons are reflected.

which read for the case of quantum scattering as:

$$\left(\frac{\partial^2}{\partial x^2} + (E + f - V(x)) \right) X_f(x) = 0, (a), \left(\frac{\partial^2}{\partial y^2} + (E + f - V(y)) \right) Y_f(y) = 0, (b). \quad (3)$$

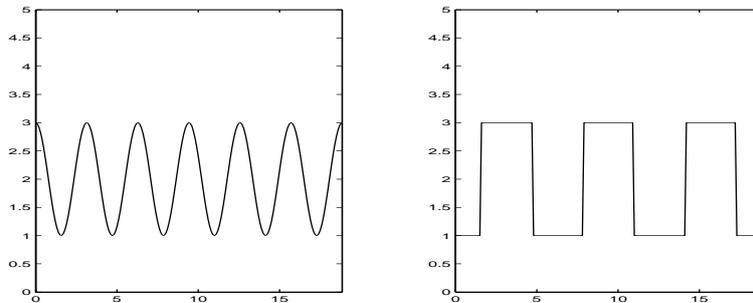


Figure 4: Visualization of the dielectric functions.

3 Field Propagation

The field $E_z(x, y)$ inside a slab made of a model medium with separable dielectric functions depicted in figure 4 is a superposition of the modes $\{X_f(x)Y_f(y)\}$:

$E_z(x, y) = \int_{-\infty}^{+\infty} A(f)X_f(x)Y_f(y)df$ with expansion coefficient $A(f)$:

$$A(f)X_f(0) = \int_{-\infty}^{+\infty} \Psi(x = 0, y')Y_f(y')dy' \quad (4)$$

We will analyze the case of quantum scattering only as for the electromagnetic case similar stability charts like figure 3 are obtained. The integration in Eq.(4) runs over all the *propagating* modes $Y_f(y)$. This means considering figure 3 that starting from a certain point, say ($E = 8, f = 0; V_0 = 8$), only those values of f are allowed which are located within a shaded region and are all lying on the line $V_0 = 8$. These allowed values of f are therefore mainly located to the right of the point ($E = 8, V_0 = 8$) but some values occur also to the left!, see comment below figure 3. We now consider the behaviour of the functions $Y_f(y)$ describing the field propagating in the slab. Because of the minus sign for f occurring in Eq.(3b) we see that the position of the label f of this function in the stability chart 3 is *opposite* to that of the position of f for the function $X_f(x)$. We then infer from the stability chart that:

The modes $Y_f(y)$ are not decaying faster as $|f|$ increases as their counterparts, the evanescent waves, do in free space, but even can show propagating behaviour!

This rather surprising result provides an explanation of the observed bi-exponential transmission behaviour and the saturation as this model shows that one might expect that inside the bandgap a (small) number of propagating- or slowly decaying modes can occur. These modes are likely not observed by a band structure calculating program as the vast majority of the infinite number of modes belonging to a certain value of the parameters are not propagating.

References

- [1] S. John, *Localization of light*, Physics Today **44** (May 1991), no. 5, 32–40.
- [2] Edmond Chow, S.Y. Lin, S.G. Johnson, P.R. Villeneuve, J.D. Joannopoulos, J.R. Wendt, G.A. Vawter, W. Zubrzycki, H. Hou and A. Alleman, *Three-dimensional control of light in a two-dimensional photonic crystal slab*, Nature **407** (2000), 983–986.
- [3] D. Labilloy, H. Benisty, C. Weisbuch, C.J.M. Smith, T.F. Krauss, R. Houdre and U Oesterle, *Finely resolved transmission spectra and band structure of two-dimensional photonic crystals using emission from InAs quantum dots*, Phys. RevB **59** (1999), no. 3, 1649–1652.
- [4] J.S. Foresi, P.R. Villeneuve, J. Ferrera, E.R. Thoen, G Steinmeyer, S. Fan, J.D. Joannopoulos, L.C. Kimerling, Henry I Smith and E.P. Ippen, *Photonic-bandgap microcavities in optical waveguides*, Nature **390** (1997), 143–145.
- [5] G. Allen, *Band structures of one-dimensional crystals with square-well potentials*, Phys. Rev. **91** (1953), no. 3, 531–533.