

# Bloch modes and self-localized waveguides in nonlinear photonic crystals

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*We present a rigorous modeling technique using eigenmode expansion and a spatial grid to simulate infinite periodic structures with the Kerr nonlinearity. Using a unit cell with Bloch boundary conditions, our iterative algorithm efficiently calculates self-consistent two-dimensional Bloch modes. We show how it can be used to study the band structure of nonlinear photonic crystals and to gain rapid insight in the operation of devices. Furthermore, by iterating from a linear defect index, we can simulate nonlinear transversely localized guided modes, soliton-like waves with frequency in the bandgap that induce their own waveguide through a photonic crystal without linear defects.*

## Introduction

Photonic crystals with nonlinear materials are the subject of active research. The Kerr effect for example makes it possible to dynamically tune the strong localization and dispersion effects present in photonic crystals. Moreover, it is not only possible to adjust the linear properties, there are also new effects to be studied such as nonlinear localized modes or gap solitons.

Along with theoretical and experimental progress, there is a need for efficient numerical tools to explore nonlinear devices and phenomena. We recently expanded the mode-expansion technique for two-dimensional Kerr nonlinear structures of finite size. Here we show how this algorithm can be adapted to model the self-consistent Bloch waves of infinite periodic structures. In this way we can observe the shifting of band structures with increasing amplitude. Quickly knowing which frequencies and input intensities can couple to a guided Bloch mode is important for designing devices.

With our method we demonstrate the existence of self-localized waveguides in a two-dimensional photonic crystal without linear defects. These are Bloch modes with frequency in the bandgap, that are confined in the transversal direction because of the bandgap, but propagate longitudinally because they induce their own guide.

We first describe our modeling technique and then show applications of band structure calculations. Self-localized waveguides are discussed in the last section.

## Modeling technique

In the linear mode expansion method one chooses a main propagation direction and divides the structure in sections invariant along this direction. The field in every section can be described as a superposition of eigenmodes derived from the transversal index profile. Using mode-matching one can infer transmission and reflection matrices for the mode amplitudes on the interface between two sections. Concatenating these matrices one obtains the fields throughout a structure composed of various sections [1].

Our extension for nonlinear materials proceeds as follows [2]. A Kerr nonlinearity means

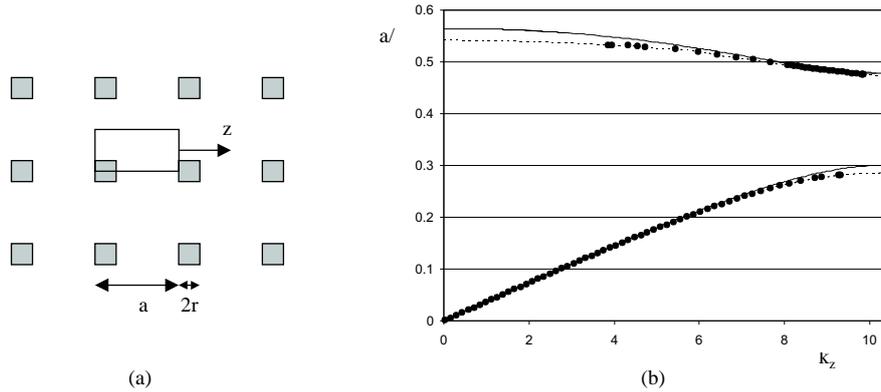


Figure 1: Nonlinear photonic crystal. (a) Geometry. The box shows the used unit cell, exploiting symmetry. (b) Band structure. Solid lines show linear bands, dots and dashed lines present calculation results and extrapolation curves.

the refractive index is dependent on the local field intensity  $I$  as  $n(x, y) = n_0 + n_2 I(x, y)$ , with  $n_0$  the linear index and  $n_2$  the Kerr coefficient. To model this, we divide the nonlinear materials in small rectangles. Each rectangle is assigned its index during each iteration. We start by using the linear index, an estimate or a previously calculated index distribution. By performing a linear eigenmode calculation we get the intensities at the center of each rectangle. This intensity distribution is used to update the index distribution using the Kerr equation. If the new index distribution is equal to the previous index distribution, within a certain tolerance, we converged to a solution of the full nonlinear problem.

To simulate infinite periodic structures we define a unit cell and use this iterative algorithm in combination with Bloch boundary conditions. If we assume that the field, and thus the mode amplitudes, reproduce themselves after a period, including multiplication with a complex factor, the scattering matrix delivers an eigenvalue problem. Solving this eigenvalue problem gives the Bloch modes of the structure at each step. In order to converge to a self-consistent solution we keep the flux of the Poynting vector through a boundary constant during iteration. In practice this simply means scaling the intensity from the linear calculation. By inspecting the guided Bloch waves we can derive a band structure dependent on the power flux in the corresponding mode.

## Applications

Calculation methods of band diagrams for nonlinear photonic crystals have been described in [3, 4, 5]. Here we show results on a square lattice of square rods along the  $\Gamma X$ -direction with TE-polarisation. The nonlinear rods have index 3.4 and  $n_2 = 2 \times 10^{-5} \text{m}^2/\text{V}^2$  in an air background and the ratio of the side of the rods to the crystal period is  $2r/a = 0.25$ . Figure 1 shows the typical down-shifting of the bands with increasing amplitude. For these calculations we used 10 eigenmodes, which is efficient as the plane-wave expansion method e.g. typically requires hundreds of plane waves.

As the nonlinearity increases, some bands become very flat, which are, even in the linear case, difficult to model completely. Moreover, in the nonlinear case it is then hard to iterate to a solution, because small deviations can push the solution into the non-guided

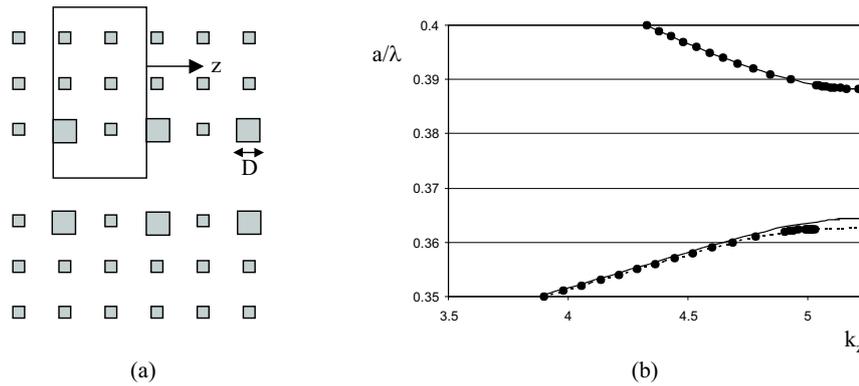


Figure 2: Photonic crystal limiter. (a) Geometry : defect waveguide with larger nonlinear rods. The box shows the used unit cell. (b) Waveguide bands. Solid lines present linear bands, dots and dashed lines present calculation results and extrapolation curve.

region. To overcome this we extrapolate parabolically towards the edge of the Brillouin zone.

Many periodic nonlinear devices depend on a shifting of the allowed modes. Simulating these long structures with tens of periods for applications is necessary, but very time consuming. Instead, studying the infinite structure can rapidly give information on geometry, operational frequencies and intensities.

We illustrate this by studying a waveguide photonic crystal limiter proposed in [6]. Geometry and results are shown in Fig. 2. The same lattice as above is used, but with a defect waveguide and larger nonlinear rods ( $D = 0.4a$ ,  $n_2 = 10^{-5} \text{m}^2/\text{V}^2$ ) alongside every two periods. The band structure shows two waveguide bands within the bandgap. Increasing the intensity shifts the lowest defect band downwards. The upper defect band is very insensitive to the nonlinearity. Field plots indeed show that the fields in this band are concentrated in the smaller linear rods along the waveguide, whereas in the lower band fields are strongest in the large nonlinear rods. Thus, if we operate on top of the lowest waveguide band, low amplitudes will give a high transmission. But as we increase the amplitude our simulations indicate the guided Bloch mode disappears below the working frequency, so we get limiting action.

### Self-localized waveguides

Waves with frequency within the bandgap are exponentially dampened in a linear photonic crystal. With Kerr materials there are however new effects such as localized modes or gap solitons [7, 8]. Here we numerically find waveguide modes, confined in one direction by bandgap effects but propagating in the other direction because the beam itself creates a defect waveguide, see Fig. 3(a).

To model these modes we use the previous technique, but start with a 'seed'. We estimate the required linear index of a center rod to create a defect waveguide. The linear calculation with this index gives an estimate of the flux and a starting index distribution. If we iterate from this distribution, keeping the flux constant, and we converge to a guided Bloch mode, we have reached a stable nonlinear mode. Once we have one mode we can

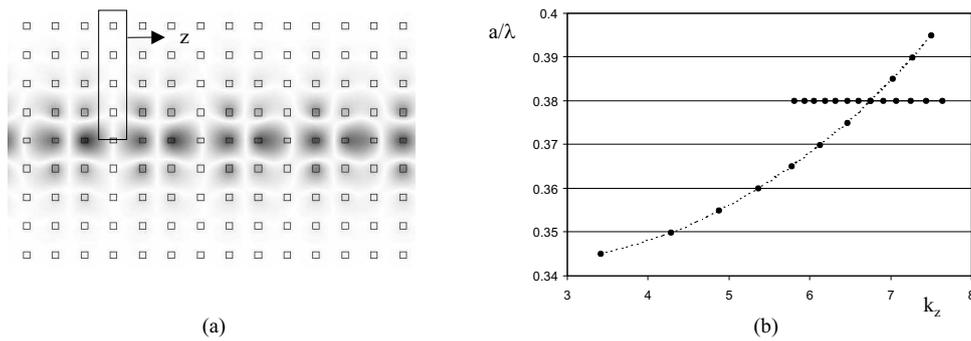


Figure 3: (a) Example of a self-localized waveguide. (b) Properties : the dashed line keeps the flux constant, whereas the solid line has a constant frequency.

slightly change the frequency or the flux to quickly iterate to new modes and map their properties, as in Fig. 3(b). Other propagation directions, indexes and geometry can give rise to similar modes.

## Conclusion

We presented an efficient method to model band structures of two dimensional devices with Kerr nonlinear materials. With this technique we numerically study self-localized solitary modes in photonic crystals.

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