Left-Handed Materials: The Key to Sub-Wavelength Resolution?

P. Tassin, G. Van der Sande and I. Veretennicoff

Department of Applied Physics and Photonics (TW-TONA), Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium

We have studied left-handed materials, which exhibit several unusual properties like negative refraction and focusing of light even with flat surfaces. More specifically, we have investigated the properties of Veselago's lens, for which we have provided a description using both a geometrical and a full wave approach. We have been able to confirm its sub-wavelength resolution properties and to generalize the model given by Veselago [V.G. Veselago, Sov. Phys. Usp., vol. 10, p. 509-514, 1968] and Pendry [J.B. Pendry, Phys. Rev. Lett., vol. 85, p. 3966-3969, 2000] to substances with refractive indices different from -1.

Left-Handed Materials: An Overview

One of the well-known laws of wave optics states that the wave vector, the electric field and the magnetic field of a plane monochromatic wave form a right-handed Cartesian reference system. In left-handed materials, however, this is no longer true. V.G. Veselago [1] theoretically investigated the electromagnetic properties of linear materials with simultaneously negative permittivity and permeability. He concluded that these materials are left-handed and exhibit some unusual properties like negative refraction. Veselago also noted that a parallel-sided slab of left-handed material (Veselago's lens) could focus light, but he did not prove this statement and only considered a special case. Because left-handed materials do not exist in nature, at least to the best of our knowledge, interest in Veselago's idea rapidly faded away. Nevertheless, substances with negative dielectric permittivity are known: metals at optical frequencies, for example. Moreover, artificial materials – the so-called metamaterials – with simultaneously negative permittivity and permeability can now be designed for microwave frequencies. D.R. Smith *et al.* [2] demonstrated experimentally that a periodic array of metal wires and metal coils does indeed have negative electromagnetic parameters at microwave frequencies. Shelby *et al.* [3] constructed a prism-shaped section from such a metamaterial. They have been able to observe the negative refraction, as illustrated in Figure 1.

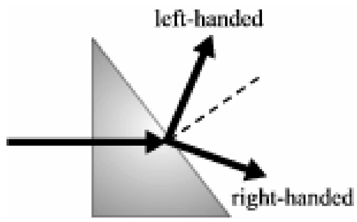

Figure 1: Negative vs. positive refraction

Pendry [6] wrote a remarkable paper in which he analysed the resolution limitation of conventional lenses. He inferred that this limitation was due to the decay of the evanescent waves coming from a two-dimensional object. Further, he reconsidered Veselago's lens. Using a multiple beam approach, Pendry showed that – unlike conventional lenses – Veselago's lens is able to restore the amplitude of the evanescent

waves. Since both types of waves then contribute to the resolution of the image, perfect sub-wavelength reconstruction of the object is possible. Pendry's paper was not only remarkable, but also controversial. Immediately after publication, some authors have raised objection to Pendry's argument. 't Hooft [7] agreed upon the conclusions, but argued that the series of multiple beam reflections in the lens does not converge. Meanwhile, left-handed materials have become an exciting subject for both fundamental and applied optics. A nice overview of recent results can be found in [4].
In this paper, we will address two issues about Veselago's lens. First, we will rigorously solve the problem of sub-wavelength resolution. Secondly, we will show that the techniques of ray tracing can be used to find the imaging properties of the lens.

Veselago's Lens: A Full Wave Approach

Veselago's lens is an infinite, parallel-sided slab of left-handed material with thickness L, index of refraction n_2 and characteristic impedance $\eta_2 = \sqrt{\varepsilon_{r1}/\mu_{r1}}$ (see Figure 2).

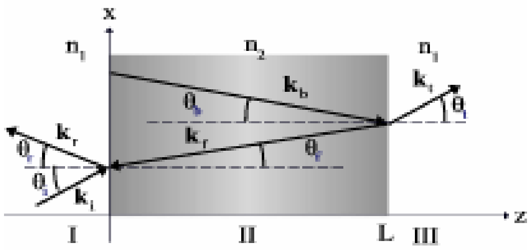

Figure 2: Veselago's lens

If we look to this structure, it is in fact that of a Fabry-Pérot resonator filled with left-handed material. Since this structure is linear, we can work in the frequency domain. The response of the system to a monochromatic light distribution with pulsation ω and wave vector $\mathbf{k} = k_x\mathbf{1_x} + k_y\mathbf{1_y} + k_z\mathbf{1_z}$ is then given by its transfer function, which we have been able to calculate by integrating Maxwell's equations on the geometry the lens:

$$H_l(k_x,k_y) = \frac{e^{\mathrm{iBgtg}\left(\frac{\varsigma^2+\xi^2}{2\varsigma\xi}\mathrm{tg}\,\kappa L\right)}}{\sqrt{1+\frac{\left(\varsigma^2-\xi^2\right)^2}{16\varsigma^2\xi^2}\sin^2\kappa L}}, \tag{1}$$

where ζ, ξ and κ are defined by

$$\varsigma = \eta_1\cos\theta_i,\ \xi = \eta_2\cos\theta_f \text{ and } \kappa = (\omega/c)n_2\cos\theta_f. \tag{2}$$

If the impedances of the two media are matched – i.e. the reflection coefficient equals zero – the transfer function simplifies to

$$H_l(k_x,k_y) = e^{-iL\sqrt{\frac{\omega^2}{c^2}|n_2|^2-k_x^2-k_y^2}}. \tag{3}$$

Of course, the light beam also propagates a distance $d_i + d_o$ through the air. Including this contribution, the transfer function becomes:

$$H(k_x,k_y) = e^{i(d_i+d_o)\sqrt{\frac{\omega^2}{c^2}n_1^2-k_x^2-k_y^2}}\,e^{-iL\sqrt{\frac{\omega^2}{c^2}|n_2|^2-k_x^2-k_y^2}}. \tag{4}$$

Focusing of the object onto the image plane will occur if the phase transformation is the same for all spectral components. Making a paraxial approximation, we find the equation for conjugate planes:

$$d_i = L\left|\frac{n_1}{n_2}\right| - d_o . \quad (5)$$

An object at a distance d_o from the one edge of the lens will be imaged at a distance d_i from the other edge. As a result, Veselago's lens can be used for image formation. Veselago considered only the case where $n_2 = -1$, but we see that this condition is not necessary. Whatever the refractive index of the left-handed substance, an image will be formed at the position given by our generalised lens formula (5), provided the lens is impedance matched. Because of its flat edges, we also expect production of this lens to be much easier than for conventional convex lenses. Furthermore, spherical aberrations will be eliminated. Note also that for the ideal lens, where $n_2 = -n_1$, no paraxial approximation must be made and the total transfer function becomes exactly unity.

Sub-Wavelength Resolution

In the previous section, we have silently assumed that the z-component of the wave vector was real for all spectral components of the object. However, for high spatial frequencies, it will become imaginary:

$$k_z = \sqrt{\frac{\omega^2}{c^2}n_1^2 - k_x^2 - k_y^2} = i\sqrt{k_x^2 + k_y^2 - \frac{\omega^2}{c^2}n_1^2} \text{ if } k_x^2 + k_y^2 > \frac{\omega^2}{c^2}n_1^2 . \quad (6)$$

These spectral components cannot be interpreted as propagating waves. They are exponentially damped waves and are therefore called evanescent waves. It is not impossible that the object contains such spectral components. The Fourier expansion shows that an arbitrary object can contain spectral components with x- and y-wave numbers up to infinity. It is therefore unavoidable to account for these spectral components.

Pendry [6] has investigated the influence of these high-frequency components on the resolution of the imaging process. He has concluded that, when the evanescent waves are not restored at the image plane, the resolution is limited by

$$\delta \approx \frac{2\pi}{k_{\max}} = \frac{2\pi c}{\omega} = \lambda , \quad (7)$$

i.e. details that are smaller than the wavelength cannot be properly imaged.

In the same paper [6], it has been shown that Veselago's lens with $n_1 = -n_2$ and $\eta_1 = \eta_2$ does not focus only the low-frequency components, but also amplifies the evanescent waves to their original amplitude. However, Pendry uses a multiple beam approach to do so. When adding the contribution of all beams, he calculates the sum of a geometrical series. Other authors (e.g. [7]) complained that this series does not converge. We are able to sidestep this geometrical series by extending the discussion of the previous section. Using analogous arguments, we have derived the transfer function for the evanescent components in the specific case for $n_1 = -n_2$:

$$H(k_x, k_y) = e^{k(L-d_o-d_i)} = 1 . \quad (8)$$

This means that the amplification inside the lens just cancels the decay in free space. Since all spectral components are exactly reproduced in the image plane, the resolution limitation (7) does not apply to the ideal Veselago's lens. We can therefore conclude that perfect focusing should be possible with Veselago's lens. Note that we have proven this result by providing an exact solution to Maxwell's equations, and that our derivation does not suffer from the problems raised by 't Hooft [7].

Veselago's Lens: A Ray Tracing Approach

From a paraxial ray-tracing point of view, we can consider Veselago's lens as a composite system containing three subsystems: a refractive boundary from the air to the lens, the propagation of the ray from the first to the second surface and a second refractive boundary. It is not difficult to show that Snell's law remains valid for a left-handed material provided its index of refraction is taken negative. Consequently, the translation and refraction matrices keep the same form as their counterparts in right-handed media [5]:

$$R(n_1,n_2)=\begin{pmatrix}1 & 0\\ 0 & n_1/n_2\end{pmatrix}, \text{ and } T(L)=\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}. \quad (9)$$

Note, however, that the fourth element of R is now negative. The system matrix of the lens can then be obtained by multiplying the matrices of the subsystems:

$$S=R(n_2,n_1)\cdot T(L)\cdot R(n_1,n_2)=\begin{pmatrix}1 & L\dfrac{n_1}{n_2}\\ 0 & 1\end{pmatrix}. \quad (10)$$

This system matrix contains all information about the input-output behaviour of the lens in the paraxial approximation. Using the classical techniques (see, for example, [5]), we have derived an equation for conjugate planes of the lens, i.e. an equation that relates object and image:

$$d_i=L\left|\frac{n_1}{n_2}\right|-d_o. \quad (11)$$

This is exactly the same relationship as the one found with a full wave treatment (5). The second conclusion that we can draw is therefore that optical systems containing left-handed materials can be studied and designed using the techniques of paraxial geometrical optics.

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