

Differential Group Delay Analysis of Uniform Bragg Gratings Written in Hi-Bi Optical Fibers

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Fibre Bragg Gratings (FBG) have become key components in optical communications systems, and, in the same time, lightwave transmissions are more intolerant to the polarization mode dispersion. Consequently, the polarization properties of FBG are of paramount importance. In this frame, we analyze the wavelength dependency of Differential Group Delay in the particular case of a uniform Bragg Grating written in a highly birefringent fibre. Based on the coupled mode theory, a theoretical expression for the differential group delay is derived. We also present experimental results obtained by the Jones Matrix Eigenanalysis Method. A good agreement with theory is reported.

1. Introduction

Nowadays, many different optical communication devices can be realized based on Fibre Bragg Gratings (FBG): add-drop multiplexers, tunable optical filters, laser diodes and fibre lasers, Raman amplifiers, gain-equalizer filters for EDFA, chromatic dispersion compensators, polarisation mode dispersion compensators, ... In the frame of high-speed Wavelength Division Multiplex (WDM) optical communication systems, FBG technology is indeed very attractive due to the possibility to realize tunable and/or multi-channel devices. Moreover, particular functions can be obtained by writing FBG in non-standard fibres as highly birefringent fibres (Hi-Bi fibres). Consequently, FBG components are widely used.

In the same time, due to the increasing bit rate, lightwave transmissions are more intolerant to the polarization mode dispersion. In this frame, the polarisation properties of FBG must be study. The classical parameter used to characterize the polarisation properties in optical communication systems is the Differential Group Delay (DGD).

This paper analyzes the wavelength dependency of DGD in the particular case of uniform FBG written in Hi-Bi fibres (called Hi-Bi FBG).

2. Definitions

Birefringence in optical fibres is defined as the difference in refractive index Δn between a particular pair of orthogonal polarization modes (called the eigenmodes or modes x and y) and results from the presence of circular asymmetries in the fibre section. The refractive index for both the x and y modes is defined as:

$$n_{eff,x} = n_{eff} + \frac{\Delta n}{2} \quad \text{and} \quad n_{eff,y} = n_{eff} - \frac{\Delta n}{2} \quad (1)$$

where n_{eff} is the fibre effective refractive index. In the case of hi-bi fibres, the order of magnitude of the birefringence Δn is $4 \cdot 10^{-4}$ (for a bow-tie configuration).

When a uniform FBG is written into a Hi-Bi fibre, the transmission coefficient is different for the two eigenmodes because of the fibre birefringence, and the transmitted signal is the combination of the transmitted signals corresponding to both the x and y polarization modes.

Bidimensional complex Jones vector is used to define the state of polarization (SOP) of the input signal as:

$$\begin{pmatrix} E_{i,x} \\ E_{i,y} \end{pmatrix} = \begin{pmatrix} M_x e^{j\theta_x} \\ M_y e^{j\theta_y} \end{pmatrix} \quad (2)$$

where $M_{x(y)}$ and $\theta_{x(y)}$ are the amplitude and the phase angles of the $x(y)$ component of the electric field, respectively. If the eigenmodes corresponds to the x and y axes, the transmission properties of the Hi-Bi FBG are described by the diagonal Jones Matrix J (no modes coupling) and the Jones vector corresponding to the transmitted E_t signals can be written

$$\mathbf{E}_t = \begin{pmatrix} E_{t,x} \\ E_{t,y} \end{pmatrix} = J \cdot \begin{pmatrix} E_{i,x} \\ E_{i,y} \end{pmatrix} = \begin{pmatrix} t_x M_x e^{j\theta_x} \\ t_y M_y e^{j\theta_y} \end{pmatrix} \quad \text{with} \quad J = \begin{pmatrix} t_x & 0 \\ 0 & t_y \end{pmatrix} \quad (3)$$

where $t_{x(y)}$ denote respectively the transmission coefficient of the uniform Bragg grating corresponding to the mode $x(y)$.

The coupled mode theory is used to derive the complex transmission t and the power transmission coefficients ($T = |t|^2$) of uniform FBG. Reference [1] gives

$$t = \frac{i\alpha}{\tilde{\sigma} \sinh(\alpha L) + i\alpha \cosh(\alpha L)} \quad \text{and} \quad T = \frac{\alpha^2}{\kappa^2 \cosh^2(\alpha L) - \tilde{\sigma}^2} \quad (4)$$

where

$$\alpha = \sqrt{\kappa^2 - \tilde{\sigma}^2} \quad ; \quad \tilde{\sigma} = 2\pi n_{eff} \left(\frac{1}{\lambda} - \frac{1}{\lambda_B} \right) + \frac{2\pi}{\lambda} \delta n \quad ; \quad \kappa = \frac{\pi v \delta n}{\lambda} \quad (5)$$

δn is the index modulation of the FBG, λ is the wavelength, L is the grating length, λ_B is the Bragg wavelength defined as $\lambda_B = 2n_{eff}\Lambda$ where Λ is the grating period and v is the contrast of the interference pattern. Note that the transmitted spectrum is characterized by a main peak at the wavelength defined as $\lambda_{max} = 2(n_{eff} + \delta n)\Lambda$.

In the case of a FBG written in a Hi-Bi fibre, we can define two transmission coefficients t_x and t_y for the x and y modes respectively and two wavelengths ($\lambda_{max,x}$ and $\lambda_{max,y}$) for the corresponding main peaks:

$$t_{x(y)} = \frac{i\alpha_{x(y)}}{\tilde{\sigma}_{x(y)} \sinh(\alpha_{x(y)} L) + i\alpha_{x(y)} \cosh(\alpha_{x(y)} L)} \quad (6)$$

$$\lambda_{max,x(y)} = 2(n_{eff,x(y)} + \delta n)\Lambda$$

where the $\alpha_{x(y)}$ and $\tilde{\sigma}_{x(y)}$ parameters depend on $n_{eff,x(y)}$. Note that the exact fibre birefringence value can be deduced by measuring the two wavelengths $\lambda_{max,x}$ and $\lambda_{max,y}$.

3. Analytical analysis of Differential Group Delay

Let us consider that the two eigenmodes are the 0° and 90° linear polarization states. If the state of polarization of the input signal is the linear state at 45° between the two eigenmodes (in this case, $M_x = M_y = 1/\sqrt{2}$ and $\theta_x = \theta_y = 0$ in (2)), the input signal is divided equally into the two eigenmodes. The power of the transmitted signal $E_{T,total}$ is the combination of the transmitted signals of both x and y modes (equation (7)) and the total transmission coefficient T_{total} is defined by (8).

$$E_{T,total} = (E_{i,x} \cdot t_x)^2 + (E_{i,y} \cdot t_y)^2 \quad (7)$$

$$T_{total} = \frac{(E_{i,x} \cdot t_x)^2 + (E_{i,y} \cdot t_y)^2}{(E_{i,x})^2 + (E_{i,y})^2} = (E_{i,x} \cdot t_x)^2 + (E_{i,y} \cdot t_y)^2 = \frac{1}{2} T_x + \frac{1}{2} T_y \quad (8)$$

The Differential Group Delay $\Delta\tau$ is defined as the difference in propagation time between the eigenmodes: $\Delta\tau = |\tau_{i,x} - \tau_{i,y}|$. Group delay $\tau_{i,x(y)}$ is the derivative, versus w , of the phase of the complex transmission coefficient $t_{x(y)}$. After development and simplification of negligible terms, we can show that $\tau_{i,x(y)}$ can be expressed as (9). It can be used to obtain the DGD expression defined previously.

$$\tau_{i,x(y)} = \frac{n_{eff,x(y)}}{c} \cdot \frac{\frac{\kappa^2}{\hat{\sigma}_{x(y)}^2} \sinh(\alpha_{x(y)} L) \cosh(\alpha_{x(y)} L) - L}{\frac{\kappa^2}{\hat{\sigma}_{x(y)}^2} \cosh^2(\alpha_{x(y)} L) - 1} \quad (9)$$

4. Experimental results

Evolution with wavelength of the transmission coefficient T_{total} is presented in Figure 1. The FBG used here was written into hydrogenated bow-tie fiber using the phase mask method with an Argon laser followed by a frequency doubler and emitting at 244 nm. The laser beam width was ± 6 mm and the optical power on the phase mask was ± 50 mW. A tunable laser source and a power meter were used to measure the transmission coefficient versus wavelength presented in Figure 1 (dotted-line curve). The state of polarization of the input signal is adjusted using a polarization controller such that the input SOP is at 45° between the two eigenmodes. In this case, the transmission coefficient includes two main rejection bands with the same contribution as seen in Figure 1. Based on this measured spectrum, we use the method described in [2] to adjust FBG parameters in order to fit the theoretical curve defined by (8).

To measure the DGD evolution with wavelength, we use the Jones Matrix Eigenanalysis Method (described in [3]). In our experiment, the laser source is tuned from 1533.5 nm to 1536 nm with a wavelength step of 5 pm. Measurement result is presented in Figure 2 in dotted-line curve. The theoretical DGD curve (black heavy curve in Figure 2) is obtained using equation (9) with the deduced FBG parameters values.

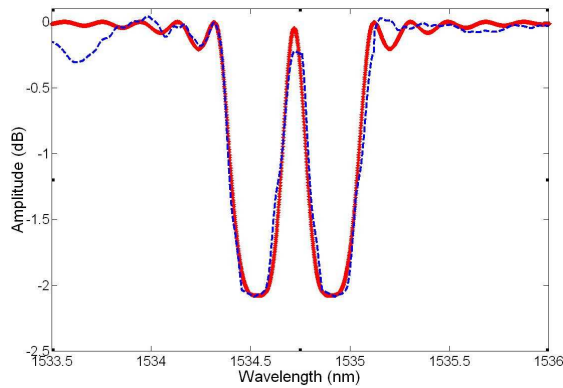


Figure 1 – Transmission coefficient T versus wavelength: measured curve (dotted-line) and theoretical curve (black heavy) with $n_{\text{eff}}=1.4521$, $\Lambda=528.392$ nm, $L=4.23$ mm, $\delta n=1.55 \cdot 10^{-4}$, $\Delta n=3.7 \cdot 10^{-4}$

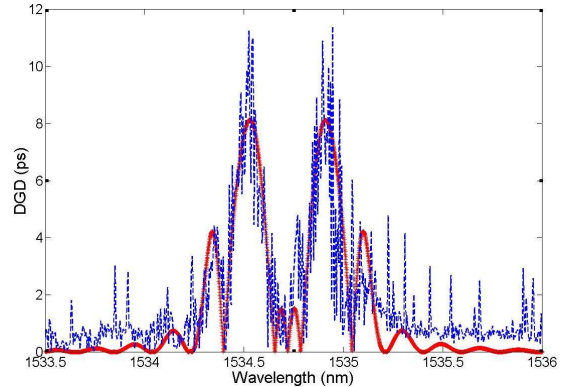


Figure 2 – DGD evolution with wavelength: measured curve (dotted-line) and theoretical curve (black heavy) with $n_{\text{eff}}=1.4521$, $\Lambda=528.392$ nm, $L=4.23$ mm, $\delta n=1.55 \cdot 10^{-4}$, $\Delta n=3.7 \cdot 10^{-4}$

One can clearly observe a good agreement between experiment and theory concerning the wavelength dependency of the DGD. This analysis also shows that the DGD exhibits important values (up to 10 ps) in the useful bandwidth (in the two main bandwidths). Consequently, this information must be taking into account when the FBG is used in a telecommunication system. In this frame, the theoretical formula presented in this paper can be useful to predetermine the wavelength dependency of the DGD in FBG written in a Hi-Bi fibre.

5. Conclusions

In this paper, we analyzed the wavelength dependency of Differential Group Delay in the particular case of a uniform Bragg Grating written into a highly birefringent fibre. Based on the coupled mode theory, an analytical expression for the differential group delay was derived. Measurements performed on a real FBG written in Hi-Bi fibre pointed out that the DGD value can reach important values (up to 10 ps) and, consequently, cannot be neglected in optical telecommunication systems. Moreover, we showed a good agreement between theory and experiment. Consequently, the theoretical formula developed in this paper can be useful to determine the DGD value for FBG written in a Hi-Bi fibre.

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