On the connection between phase singularities and the radiation pattern of a slit in a metal plate

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We report a new fundamental relation between the minima of the far-zone radiation pattern of a narrow slit in a metal plate and the location of phase singularities in the intermediate field. If a system parameter such as the wavelength is changed, a previously unappreciated singular optics phenomenon occurs, namely the transition of a near-zone phase singularity into a singularity of the radiation pattern. Our results have significance for the design of novel nano-scale light sources and antennas.

Surprising effects in the radiation pattern of sub-wavelength light sources have recently been observed in several studies, for example the "beaming effect" due to surface plasmons on grating structures around sub-wavelength apertures [1, 2]. In this paper we report the prediction of a new phenomenon, namely the relationship between phase singularities in the near-field and intermediate field of the aperture, and the shape of the far-zone radiation pattern [3]. Furthermore, a new singular optics process is described: the transition of a near-zone singularity into a singularity of the radiation pattern.

A rigorous scattering approach, which takes into account the finite conductivity and the finite thickness of the plate, is used to calculate the field in the vicinity of an infinitely long slit in a metal plate. This method, commonly referred to as the Green's tensor method, is described in detail in [4], and has been applied previously by us to study similar configurations [5, 6, 7].

The intensity radiation pattern is defined by the expression

$$I(\theta) \equiv \frac{\lim\limits_{\rho\to\infty} \langle \mathbf{S}^{(\mathrm{sca})}(\rho,\theta)\rangle \cdot \rho}{\int_{\mathrm{slit}} \langle S_z^{(0)} \rangle \, \mathrm{d}x}, \tag{1}$$

with $\rho = (x, 0, z)$ and $\cos\theta = z/\rho$. Furthermore, $\mathbf{S}^{(\mathrm{sca})}$ is the time-averaged Poynting vector associated with the scattered field (i.e, the field minus the incident field) and $S_z^{(0)}$ is the component of the time-averaged Poynting vector associated with the illuminating field (i.e., the field emitted by the laser source) that is perpendicular to the plate. The radiation pattern may be calculated using the angular spectrum representation of the field [3]. In this manner one obtains for the two polarizations the formulae

$$I_{TE}(\theta) = \frac{2\pi k}{w} \cos^2\theta |\tilde{E}_y(k\sin\theta)|^2, \tag{2}$$

$$I_{TM}(\theta) = \frac{2\pi k}{w} \frac{\mu_0}{\varepsilon_0} \cos^2\theta |\tilde{H}_y(k\sin\theta)|^2, \tag{3}$$

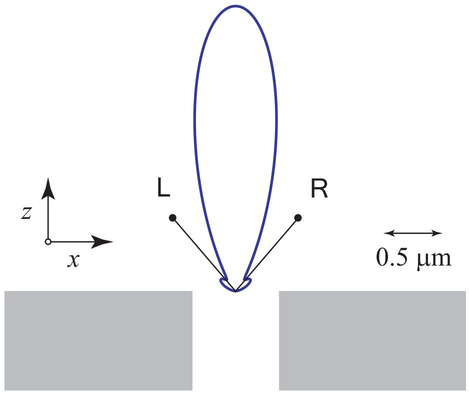

Figure 1: The intensity radiation pattern of a 750 nm wide slit in a 860 nm thick silver plate ($n = 0.05 + i2.87$). Also the positions of the phase singularities of $\hat{E}_y$ behind the slit are shown. The incident light has a wavelength $\lambda = 510$ nm and is TE-polarized.

with μ_0 the vacuum permeability, ε_0 the vacuum permittivity and w the slit width.
If the field ($\hat{E}_y$ for TE-polarization and $\hat{H}_y$ for TM-polarization) behind the slit is plotted for cases such that the radiation pattern has minima, one typically finds a phase singularity *in the direction of the radiation minimum*, see Fig. 1. A phase singularity (also called a wave dislocation) is a point where the amplitude of the field is zero and hence the phase of the field is undefined [8]. The gradient of the phase circulates around the singularity and such a phenomenon is often referred to as an optical vortex. Because of the continuity of the field, the phase change around a phase singularity is necessarily an integer number (usually ± 1) times 2π. The *topological charge* s of a phase singularity is defined as

$$s = \frac{1}{2\pi} \oint_C \mathrm{d}\phi = \pm 1, \pm 2, \ldots \tag{4}$$

with C a closed curve around the phase singularity and ϕ the phase of the field. The topological charge is a continuous function of the system parameters, as long as the contour C is chosen such that no phase singularity crosses the contour. Because s is also an integer, the topological charge is conserved. In Fig. 1 two phase singularities are present, one with charge $+1$ (left-handed) and one with charge -1 (right-handed). Each of these phase singularities gives rise to a vortex (circulation) in the field of power flow. The connection between these two kinds of phase singularities is explained in detail in [6]. The phase singularities in Figs. 1 are found by numerically calculating the topological charge (see Eq. (4)). In this way, we can determine if there is an actual phase singularity present, or just a minimum in the intensity.
In Figs. 1 and 2 the phase singularities near the slits as well as the radiation pattern are plotted for two values of the wavelength of the incident field. It is seen that for $\lambda = 510$ nm (Fig. 1) the phase singularities are closer to the plate and the minima are less pronounced than for $\lambda = 500$ nm (Fig. 2). This behavior was found to be typical for this kind of configuration. If the wavelength is decreased even more, a curious phenomenon is observed: the phase singularities move away towards the far-zone, and at a certain value of the wavelength it is no longer possible to track them numerically. The minima in the intensity radiation pattern correspondingly become deeper with decreasing wavelength until

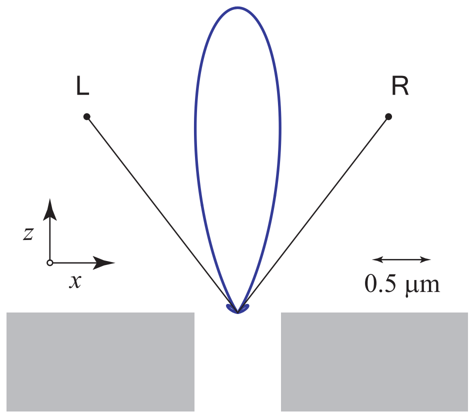

Figure 2: The intensity radiation pattern of a 750 nm wide slit in a 860 nm thick silver plate ($n = 0.05 + \mathrm{i}2.87$). Also the position of the phase singularities of $\hat{E}_y$ behind the slit are shown. The incident light has a wavelength of 500 nm and is TE-polarized.

at a certain critical wavelength, the minima evidently become true zeros. For wavelengths smaller than this critical wavelength, the value of the intensity at the minima rises as a function of decreasing wavelength. To investigate this behavior more closely, we quantify the phase behavior at infinity by first introducing for $\hat{E}_y$ (or for $\hat{H}_y$ for TM-polarized fields) the *reduced field* $\hat{E}_y^{\mathrm{red}}$ defined by the expression

$$\hat{E}_y(\theta, \rho) = \frac{e^{\mathrm{i}k\rho}}{\sqrt{\rho}} \hat{E}_y^{\mathrm{red}}(\theta, \rho). \tag{5}$$

Next we take the limit for the phase $\phi^{\mathrm{red}}(\rho, \theta)$ of $\hat{E}_y^{\mathrm{red}}$,

$$\phi^{\mathrm{inf}}(\theta) \equiv \lim_{\rho \to \infty} \phi^{\mathrm{red}}(\rho, \theta) = \mathrm{Arg}\left[\tilde{E}_y(k \sin\theta)\right] - \pi/4. \tag{6}$$

In ref. [3] it is shown that the phase singularity are actually *disappearing* at infinity.
This disappearance of a phase singularity at infinity can be observed in the behavior of the phase at infinity ϕ^{inf}. This effect is illustrated in Fig. 3: if the phase singularity is present at a large, but finite distance from the slit, the situation on the left-hand side of the figure applies: near the angle where the phase singularity is present, the phase at infinity changes rapidly, but continuously, by π (it increases by π when the angle of observation θ is increased if the topological charge is -1, whereas it decreases by π if the topological charge is $+1$). If the wavelength is decreased, the phase singularity can be exactly at infinity, i.e., at the boundary $\rho' = \pi/2$, as is shown in the middle of Fig. 3. In this case there is an exact zero in the radiation pattern, together with a π phase jump at this point. If the wavelength is decreased even further, then the right-hand side of Fig. 3 applies: the phase singularity does not exist anymore. However, a "residual effect" can still be observed in the phase behavior at infinity: the phase rapidly changes by π. Note that if initially there was a π *increase* when the angle θ is increased, now there is a π *decrease*.
In summary, we have shown that there is a connection between minima in the far-zone radiation pattern and phase singularities in the intermediate field. On changing a system parameter, these singularities can move to infinity and become singularities of the radiation pattern.

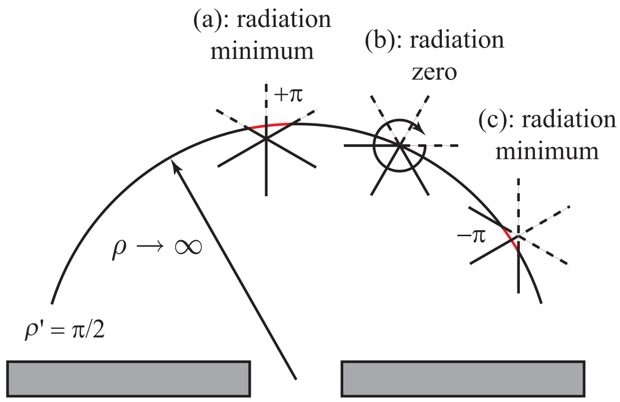

Figure 3: Illustration of the disappearance of a phase singularity at infinity when the wavelength of the field is gradually decreased. A sketch of the equiphase lines of $\phi^{\mathrm{red}}(\rho',\theta)$ is drawn, which on the semi-circle $\rho' = \pi/2$ takes on the value $\phi^{\mathrm{inf}}(\theta)$. Three cases are depicted: in (a) the phase singularity is still present, corresponding to a radiation minimum; in (b) the phase singularity is exactly at infinity, corresponding to a radiation zero; in (c) the phase singularity is no longer present, corresponding to a radiation minimum. In this example the singularity is taken to have a topological charge $s = -1$.

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