Semi-analytical model for spatial solitons in non-linear photonic crystals

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Numerical simulations have shown the existence of transversely localized guided modes in nonlinear two-dimensional photonic crystals. These soliton-like Bloch waves induce their own waveguide in a photonic crystal without the presence of a linear defect. By applying a Green's function method which is limited to within a strip perpendicular to the propagation direction, we are able to describe these Bloch modes by a nonlinear lattice model that includes long-range interaction and effectively non-local nonlinear response. The advantages of this semi-analytical approach are discussed and a comparison with a rigorous numerical analysis is given in different configurations.

Introduction

Recently, in [1], some of us demonstrated numerically the existence of self-localized waveguides in a two dimensional photonic crystal consisting of a square lattice of Kerr-type rods without linear defects. These are Bloch modes with frequency in the bandgap, that are confined in the transverse direction because of the bandgap and propagate longitudinally as they induce their own guide through the Kerr-effect in the material. This numerical method is based on an eigenmode expansion in combination with an iterative method which updates the local refractive index. Such calculations can be time-consuming. Moreover, numerical approaches do not always provide deep physical insight. It is therefore quite to try and describe the problem in a (semi-)analytical way.
The purpose of this work is to suggest an approach to describe the properties of self-localized waveguides, based on effective discrete equations. By exploiting the clear one dimensional periodicity of the Bloch modes, it is possible to reduce the infinite two dimensional crystal to a strip of material which is finite in the propagation direction and infinite in the orthogonal one. From the Green's function of the strip it is possible to find a set of discrete equations which model the interaction between the field centered at the rods of nonlinear material.
This discrete *Strip-Green* approach is then used to validate the numerical results in [1] with a different number of rows with nonlinear material. Also, a photonic crystal consisting of a two dimensional lattice of diatomic nonlinear system is considered.

A photonic crystal strip

We consider a 2D photonic crystal created by a lattice of parallel, infinitely long dielectric columns (or rods) in air (or any other dielectric). The rods are assumed to be parallel to the z-axis, so that the system is characterized by the dielectric constant $\varepsilon(\mathbf{r}) = \varepsilon(x,y)$. The lattice symmetry is assumed to be square with a lattice constant of a. We assume the electric field to be polarized along the z-axis (of the rods) and propagating in the (x,y)-plane. As is well known, photonic crystals of this type can possess a complete bandgap. The evolution of the slowly-varying envelope of a monochromatic field is governed by the Helmholtz equation:

$$\left[\nabla^2 + \varepsilon_{pc}(\mathbf{r})\left(\frac{\omega}{c}\right)^2\right] E(\mathbf{r}|\omega) = 0. \tag{1}$$

The solitonlike Bloch waves are guided because they introduce a line defect along the x-direction. In this sense, the medium loses its periodicity in the y-direction, while retaining it in the x-direction. We will exploit this fact in the analysis. The strip is denoted by the material between $x = [-\frac{a}{2}, \frac{a}{2}]$. Let us consider the Bloch wave $E(\mathbf{r}|\omega_{k_x}, k_x)$ parameterized in k_x. For each $k_x \in [0, 2\pi/a]$, we can solve in the strip for this one dimensional Bloch wave

$$\left[\nabla^2 + \varepsilon_{pc}(\mathbf{r})\left(\frac{\omega_{k_x}}{c}\right)^2\right] E(\mathbf{r}|\omega_{k_x}, k_x) = 0, \tag{2}$$

with the Bloch condition $E(x+a, y|\omega_{k_x}, k_x) = E(x, y|\omega_{k_x}, k_x)e^{ik_x a}$.
The original dispersion relation and Bloch waves of the entire photonic crystal can be recovered when the Bloch condition in the perpendicular direction is also introduced.
Next, we will study the Green's function for a fixed k_x and ω_{k_x}. The strip Green's function proposed in [2] satisfies:

$$\left[\nabla^2 + \left(\frac{\omega_{k_x}}{c}\right)^2 \varepsilon_{pc}(\mathbf{x})\right] g(\mathbf{r_1}, \mathbf{r_2}|\omega_{k_x}, k_x) = -\sum_{j\in Z} \delta(x_1 + ja - x_2, y_1 - y_2)e^{ijk_x a}, \tag{3}$$

$$g(x_1 + a, y_1, x_2, y_2|\omega_{k_x}, k_x) = g(x_1, y_1, x_2, y_2|\omega_{k_x}, k_x)e^{ik_x a} \tag{4}$$

and ω_{k_x} inside the band gap. The strip Green's function is related to the whole-space Green's function G through

$$g(x_1, y_1, x_2, y_2|\omega_{k_x}, k_x) = \sum_{j\in Z} G(x_1 + ja, y_1, x_2, y_2|\omega)e^{ijk_x a}. \tag{5}$$

By folding the whole space Green's function onto itself, we reduce our study to electric fields which can be considered as one dimensional Bloch modes with the Bloch scalar k_x.

Effective discrete equations

Let us consider the eigenvalue problem in the strip, where the eigenvalue ω_{k_x} lies in the band gap. The eigenvector $E(\mathbf{r}|\omega_{k_x}, k_x)$ satisfies

$$\left[\nabla^2 + \left(\frac{\omega_{k_x}}{c}\right)^2 \varepsilon_{pc}(\mathbf{r})\right] E(\mathbf{r}|\omega_{k_x}, k_x) = -\left(\frac{\omega_{k_x}}{c}\right)^2 \varepsilon_{nl}(\mathbf{r})E(\mathbf{r}|\omega_{k_x}, k_x), \tag{6}$$

where ε_{pc} describes the linear contribution to the dielectric permittivity and ε_{nl} the non-linear part. Using the Green's function it is possible to transform this differential equation into an integral equation

$$E(\mathbf{r}|\omega_{k_x}, k_x) = \left(\frac{\omega_{k_x}}{c}\right)^2 \int_{strip} g(\mathbf{r}, \mathbf{u}|\omega_{k_x}, k_x)\varepsilon_{nl}(\mathbf{u})E(\mathbf{u}|\omega_{k_x}, k_x)d^2\mathbf{u}. \tag{7}$$

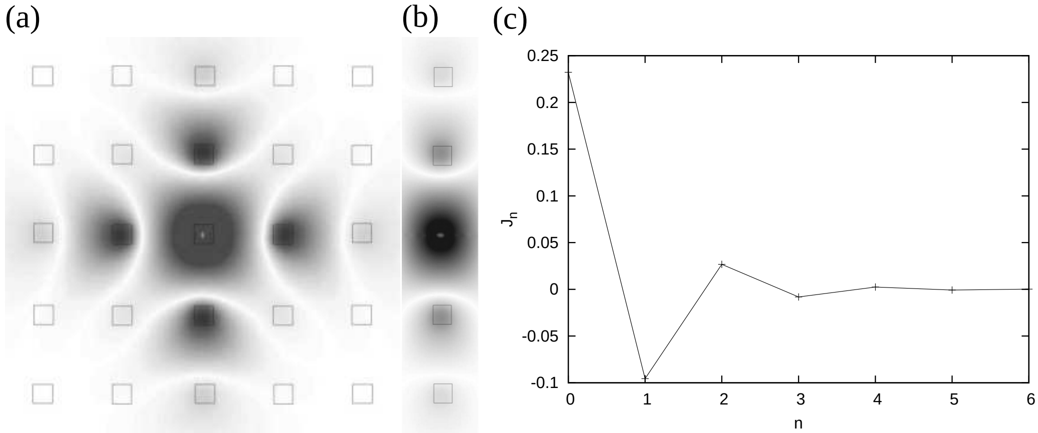

Figure 1: (a) The Green's function at $a/\lambda = 0.38$, (b) The Green's function of the strip at $a/\lambda = 0.38$ and $k_z = 0.7\pi/a$, (c) The corresponding coupling parameters J_n as function of the intersite distance.

The nonlinear permittivity is defined as

$$\varepsilon_{nl}(\mathbf{r}) = -\delta(\mathbf{r})|E(\mathbf{r}|\omega_{k_x}, k_x)|^2, \tag{8}$$

where δ is 1 inside the rods containing nonlinear material and zero outside. The radius of the nonlinear rods in the photonic crystal r_d is assumed to be sufficiently small so that the electric field is almost constant inside the rods. An approximate discrete nonlinear equation for the electric field in rod n can be written as

$$E_n(\omega_{k_x}, k_x) = \sum_m J_{n-m}(\omega_{k_x}|k_x)|E_m(\omega_{k_x}, k_x)|^2 E_m(\omega_{k_x}, k_x), \tag{9}$$

where

$$J_n(\omega_{k_x}, k_x) = \left(\frac{\omega_{k_x}}{c}\right)^2 \int_{r_d} g(\mathbf{r_0}, \mathbf{r_n} + \mathbf{u}|\omega_{k_x}|k_x) d^2\mathbf{u}, \tag{10}$$

with $\mathbf{r_n}$ denoting the center of rod n. This type of discrete nonlinear equation for photonic crystals was earlier introduced by [3]. However, by using the strip Green's function instead of the whole space Green's function, we are able to ensure that the solution is a guided Bloch wave in the x-direction.

Self-induced waveguides: example and summary

We have used the technique discussed in the previous sections to study the self-induced waveguides for different configurations. By solving the set of nonlinear effective discrete equations for a chosen set (ω, k_x) using a Newton-Rhapson algorithm, we find the field amplitudes at the nonlinear rods of the guided solitonlike Bloch wave. This soliton has sufficient energy to induce the waveguide. Basically, this method allows to find the necessary energy ($Q = \sum_m |E_m(\omega_{k_x}, k_x)|^2$) to create a Bloch wave with frequency ω and Bloch number k_x. It is possible to modify the method to find k_x for a given energy and frequency. In Figure 1, we summarize the main results of this work. Figure 1a is an example of the

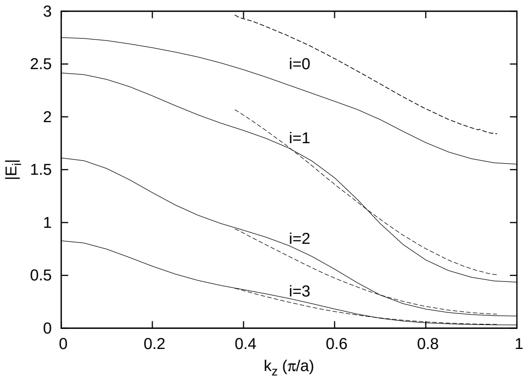

Figure 2: Comparison between the rigorously calculated fields (dashed) of [1] and the predictions from the semi-analytical approach (full). *i* denotes the rod number.

whole space Green's function. By folding this function, we obtain the strip Green's function in Figure 1b. We then calculate the coupling parameters J_n (see Figure 1c). In Figure 2, the resulting field envelopes for a configuration with 7 nonlinear rods are shown (due to symmetry reasons only 4 are shown). These results are compared with more rigorous numerical simulations of self-induced waveguides [1]. The overall qualitative agreement is good. Although, an underestimate of the field on the central rod exists. However, the semianalytical approach is able to explore a larger k_z domain. The quantitative error can be reduced by replacing a central nonlinear rod, by several smaller ones.

Conclusion

We have developed a consistent theory of nonlinear solitonlike Bloch waves, inducing their own waveguide into a photonic crystal with a Kerr-type material in the rods. We have demonstrated that these modes are described by a nonlinear lattice model that includes long-range interaction and effectively nonlocal nonlinear response. This semi-analytical approach was compared with a more rigorous numerical technique. Overall qualitative agreement was found to be good.

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References

[1] B. Maes, P. Bienstman and R. Baets, "Bloch modes and self-localized waveguides in nonlinear photonic crystals", *to be published in JOSA B* (2004).

[2] H. Ammari and F. Santosa, "Guided waves in a photonic bandgap structure with a line defect", *to be published in SIAM J. Appl. Math* (2004).

[3] S.F. Mingaleev, Y.S. Kivshar and R.A. Sammut, "Long-range interaction and nonlinear modes in photonic crystal waveguides", *Phys. Rev. A*, vol. 62, pp. 5777 (2000).