Analysis of Stokes parameters reflected and transmitted by uniform fibre Bragg gratings written in highly birefringent fibers.

M. Wuilpart, C. Caucheteur, S. Bette, P. Mégret and M. Blondel

Service d'Electromagnétisme et de Télécommunications, Faculté Polytechnique de Mons, 31 Boulevard Dolez, 7000 Mons, Belgium.

We analyze the polarization properties of uniform fiber Bragg gratings written in highly birefringent fibers by studying the evolution with wavelength of the normalized Stokes parameters of the reflected and transmitted signals. In this letter, theoretical expressions for the Stokes parameters of these signals are derived. An experimental work is also reported and compared to the theory.

Introduction

Fiber Bragg gratings (FBG) have become a key technology in the frame of WDM systems where they enable the realization of essential devices such as narrow band optical filters and chromatic dispersion compensators. It has also been shown [1] that FBG can be used to compensate polarization mode dispersion when they are written in polarization maintaining fibers. With the increasing bit rate used in WDM systems, the PMD (Polarization mode dispersion) and, therefore, the polarization properties of fibers and components have more and more impact on the quality of transmission. Consequently, it becomes important to characterize the polarization properties of fiber Bragg gratings.
This paper proposes to analyze the polarization properties of the optical signals reflected and transmitted by a uniform FBG written in a polarization maintaining fiber (hi-bi fiber). Theoretical expressions for the Stokes parameters corresponding to the reflected and transmitted signals are first derived. The evolution of the Stokes parameters with wavelength have then been measured on a real hi-bi FBG and the results are with the theory.

Theory

The birefringence in optical fibers is defined as the difference in refractive index (Δn) between a particular pair of orthogonal polarization modes (called the eigenmodes or modes x and y) and results from the presence of asymmetries in the fiber section. Let us consider a uniform FBG written in a hi-bi optical fiber. When taking into account the birefringence of the FBG, the reflected (transmitted) signal is the combination of the reflected (transmitted) signals corresponding to both the polarization modes x and y. The input signal can be represented by the following bidimensional complex vector (Jones vector) which defines the input state of polarization (SOP):

$$\begin{pmatrix} E_{ix} & E_{iy} \end{pmatrix}^T = \begin{pmatrix} M_x e^{j\theta_x} & M_y e^{j\theta_y} \end{pmatrix}^T \tag{1}$$

where $M_{x(y)}$ and $\theta_{x(y)}$ are the amplitude and the phase angle of the $x(y)$ component of the electric field, respectively. If $\rho_{x(y)}$ and $\tau_{x(y)}$ denote the reflection and transmission coefficients of the uniform Bragg grating corresponding to the mode $x(y)$, the Jones vector corresponding to the reflected (E_r) and the transmitted (E_t) signals can be written as:

$$E_r = \begin{pmatrix} E_{rx} & E_{ry} \end{pmatrix}^T = \begin{pmatrix} \rho_x M_x e^{j\theta_x} & \rho_y M_y e^{j\theta_y} \end{pmatrix}^T \qquad E_t = \begin{pmatrix} E_{tx} & E_{ty} \end{pmatrix}^T = \begin{pmatrix} \tau_x M_x e^{j\theta_x} & \tau_y M_y e^{j\theta_y} \end{pmatrix}^T \tag{2}$$

The coupled mode theory enables to derive the reflection and transmission coefficients [3], which gives:

$$\rho_{x(y)} = \frac{-\kappa \sinh\left(\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2}\, L\right)}{\hat{\sigma}_{x(y)} \sinh\left(\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2}\, L\right) + i\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2} \cosh\left(\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2}\, L\right)} \tag{3}$$

$$\tau_{x(y)} = \frac{i\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2}}{\hat{\sigma}_{x(y)} \sinh\left(\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2}\, L\right) + i\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2} \cosh\left(\sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2}\, L\right)} \tag{4}$$

where

$$\kappa = \frac{\pi v \delta n}{\lambda} \tag{5}$$

$$\hat{\sigma}_{x(y)} = 2\pi n_{eff,x(y)} \left(\frac{1}{\lambda} - \frac{1}{\lambda_{B,x(y)}} \right) + \frac{2\pi}{\lambda}\delta n \qquad n_{eff,x} = n_{eff} + \frac{\Delta n}{2} \qquad n_{eff,y} = n_{eff} - \frac{\Delta n}{2} \tag{6}$$

δn is the index modulation of the FBG, n_{eff}, the fiber effective refractive index, Δn is the fiber birefringence, λ is the wavelength, v is the contrast of the interference pattern and $\lambda_{B,x(y)}$ is the Bragg wavelength defined as $\lambda_{B,x(y)}=2n_{eff,x(y)}\Lambda$ where Λ is the grating period. The Stokes parameters can be easily deduced from the Jones vector [2]. Considering the reflected signal, it gives: $S_0=|E_{rx}|^2+|E_{ry}|^2$, $S_1=|E_{rx}|^2-|E_{ry}|^2$, $S_2=2\mathrm{Re}[E_{rx}{}^*E_{ry}]$ and $S_3=2\mathrm{Im}[E_{rx}{}^*E_{ry}]$. Hence, using relations (2)-(4), it is possible to derive theoretical expressions for the Stokes parameters. When the two eigenmodes (x and y) are the 0 and $\pi/2$ rad linear polarization states and when the input SOP is the $\pi/4$ linear polarization state: $M_x=M_y=1/\sqrt{2}$ and $\theta_x=\theta_y=0$, it gives for the reflected signal:

$$S_0 = \frac{1}{2}\kappa^2 \left[\frac{1}{\hat{\sigma}_x^2 + A_x^2 \mathrm{cotg}^2(A_x L)} + \frac{1}{\hat{\sigma}_y^2 + A_y^2 \mathrm{cotg}^2(A_y L)} \right] \quad S_1 = \frac{1}{2}\kappa^2 \left[\frac{1}{\hat{\sigma}_x^2 + A_x^2 \mathrm{cotg}^2(A_x L)} - \frac{1}{\hat{\sigma}_y^2 + A_y^2 \mathrm{cotg}^2(A_y L)} \right] \tag{7}$$

$$S_2 = \frac{\kappa^2 (A_x A_y \mathrm{cotg}(A_x L)\, \mathrm{cotg}(A_y L) + \hat{\sigma}_x \hat{\sigma}_y)}{(\hat{\sigma}_x^2 + A_x^2 \mathrm{cotg}^2(A_x L))(\hat{\sigma}_y^2 + A_y^2 \mathrm{cotg}^2(A_y L))} \quad S_3 = \frac{\kappa^2 (A_x \hat{\sigma}_y \mathrm{cotg}(A_x L) - A_y \hat{\sigma}_y \mathrm{cotg}(A_y L))}{(\hat{\sigma}_x^2 + A_x^2 \mathrm{cotg}^2(A_x L))(\hat{\sigma}_y^2 + A_y^2 \mathrm{cotg}^2(A_y L))} \tag{8}$$

where

$$A_{x(y)} = \sqrt{\kappa^2 - \hat{\sigma}_{x(y)}^2} / j \tag{9}$$

For the transmitted signal, the Stokes parameters become:

$$S_0 = \frac{1}{2}\left[\frac{1}{\cos^2(A_x L) + \gamma_x^2 \sin^2(A_x L)} + \frac{1}{\cos^2(A_y L) + \gamma_y^2 \sin^2(A_y L)} \right] \quad S_2 = \frac{1}{2}\left[\frac{\cos(A_x L)\cos(A_y L) + (\gamma_x \gamma_y)\sin(A_x L)\sin(A_y L)}{(\cos^2(A_x L) + (\gamma_x)^2 \sin^2(A_x L))(\cos^2(A_y L) + (\gamma_y)^2 \sin^2(A_y L))} \right] \tag{10}$$

$$S_3 = \frac{1}{2}\left[\frac{\gamma_y \sin(A_y L)\cos(A_x L) - \gamma_x \sin(A_x L)\cos(A_y L)}{(\cos^2(A_x L) + (\gamma_x)^2 \sin^2(A_x L))(\cos^2(A_y L) + (\gamma_y)^2 \sin^2(A_y L))} \right] \quad S_1 = \frac{1}{2}\left[\frac{1}{\cos^2(A_x L) + \gamma_x^2 \sin^2(A_x L)} - \frac{1}{\cos^2(A_y L) + \gamma_y^2 \sin^2(A_y L)} \right] \tag{11}$$

where

$$\gamma_{x(y)} = \hat{\sigma}_{x(y)} / A_{x(y)} \tag{12}$$

The normalized Stokes parameters (s_1, s_2 and s_3) can then be computed by using the relationship $s_i=S_i/S_0$ (i=1,2,3). S_0, s_1, s_2 and s_3 are presented in figures 1(a) and 1(b) for the reflected and transmitted signals, respectively. The FBG parameters used for these figures have been calculated from the experimentally measured FBG spectrum described in the next section using a technique derived from [4]. Comments related to figures 1 and 2 are presented in the next III where they are compared with the experimental results.

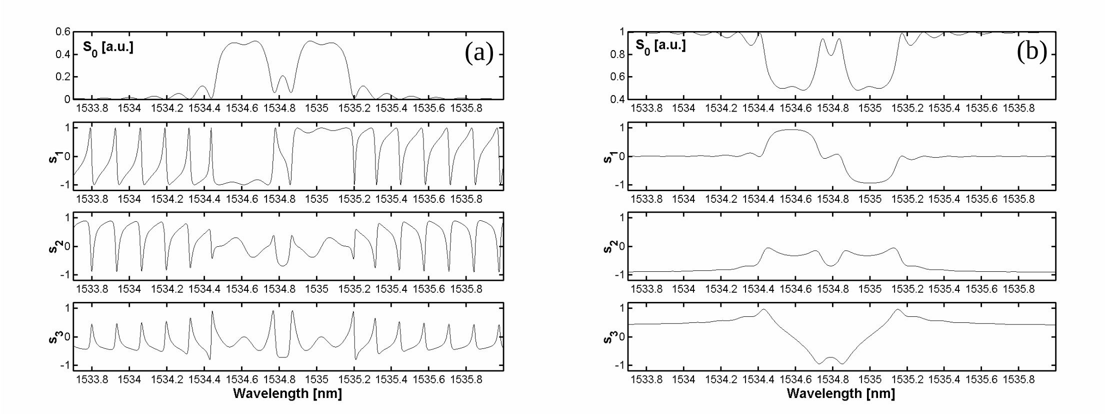

Fig. 1. (a) S_0, s_1, s_2 and s_3 versus wavelength for the reflected signal and with neff=1.4521, L=6 mm, δn=2.10-4, Δn=3.8.10-4, v=1 and Λ=528.4 nm. **(b)** S_0, s1, s2 and s3 versus wavelength for the transmitted signal and with neff=1.4521, L=6 mm, δn=2.10-4, Δn=3.8.10-4, v=1 and Λ=528.4 nm.

Experimental results

The simple measurement set-up is shown on figure 2. A polarization controller (PC) is used to modify the state of polarization of the light emitted by a tunable laser source (TLS) such that the state of polarization at the FBG input is the linear state at π/4 rad between the two eigenmodes. The polarimeter is used to characterize the polarization properties of the transmitted and reflected signal. In our experiment, the TLS has been tuned from 1533.5 to 1536 nm. The O_x and O_y axes of the polarimeter are such that they correspond to the FBG eigenmodes.

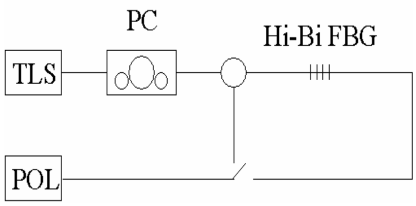

Fig. 2. Measurement set-up. A polarimeter is used to measure both signals reflected and transmitted by the hi-bi FBG.

Figures 3(a) and 3(b) present the results obtained for the reflected and transmitted signals, respectively. For the reflected total power of the signal (S_0), one can clearly observe the two reflection peaks due to the refractive index difference between the two eigenmodes. For s_1, s_2 and s_3, one can notice that the curves have the same general behavior than the ones derived analytically and presented in figure 1. For both theoretical and experimental curves, the normalized Stokes parameters related to the reflected signal quickly vary with the wavelength around the minimums of the FBG power spectrum (corresponding to S_0). One can also observe for both the theoretical and experimental curves that the normalized Stokes parameters vary in a smoother way within the reflection bands corresponding to both the eigenmodes (from 1534.4 to 1534.7 nm and from 1534.8 to 1535.1 nm, approximately). Concerning the transmitted signal, we observe both by theory and measurement that s_1, s_2 and s_3 vary slowly with the wavelength in the transmission band whereas they change more significantly in the rejection bands. Some differences can be observed between the theoretical and experimental curves. This is due to the fact that the FBG parameters used in the analytical expressions are not rigorously identical to the ones of the real FBG. Due to the imperfections of the inscription process, the experimental spectrum of the FBG is not symmetrical, which can explain the asymmetry of the measured polarization properties. When experimentally fixing the input SOP, the PC is optimized in order to obtain the same reflection peak heights for the two eigenmodes. This method is visual and the input SOP is therefore not rigorously at $\pi/4$ rad between the two eigenmodes as supposed in the theory. There is also a residual length of hi-bi fiber at both sides of the FBG which is not taken into account in equations (7) to (12). Despite these differences, the behavior is qualitatively similar between theory and experiment as described previously.

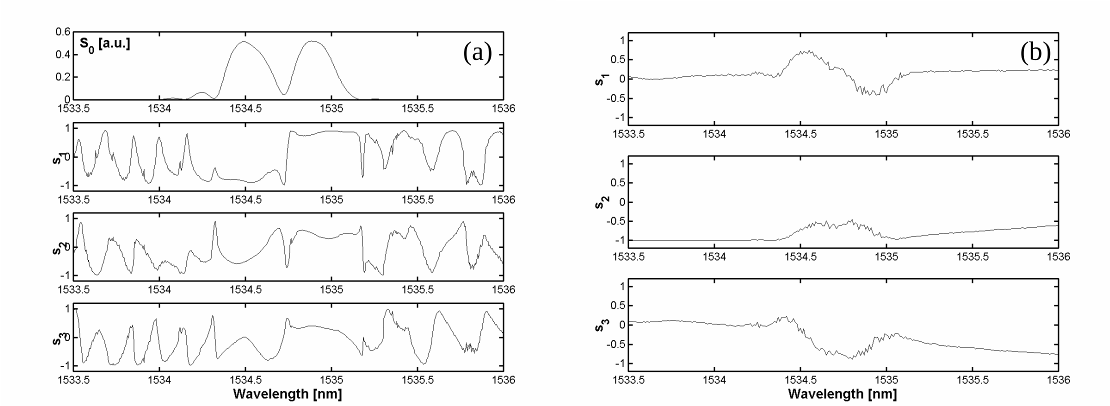

Fig. 3. (a) S_0, s_1, s_2 and s_3 of the reflected signal versus wavelength measured on our hi-bi FBG using the measurement set-up described in figure 2. **(b)** s_1, s_2 and s_3 of the transmitted signal versus wavelength measured on our hi-bi FBG using the measurement set-up described in figure 3.

Conclusion

In this letter, we derived analytical expressions for the evolution of the Stokes parameters versus wavelength of the signals reflected and transmitted by a FBG written in a hi-bi optical fiber. We then compared them to the measured polarization properties of a real FBG. Similar behaviors were obtained for the theoretical and measured Stokes parameters profiles.

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