

Left-Handed Materials in Nonlinear Fabry-Perot Resonators

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We have studied a double layered Fabry-Perot resonator filled with both a traditional nonlinear material and a nonlinear left-handed material. First, we derived a set of equations for the propagation of light through a nonlinear left-handed material. These equations were subsequently used for developing a mean-field model for the nonlinear resonator. We show that the introduction of left-handed materials has a number of interesting implications, of which the most important is the elimination of diffraction. It is also possible to design a resonator in a negative diffraction regime, for which stability is enhanced with respect to the classical resonator.

Introduction

Materials with simultaneously negative electric permittivity and negative magnetic permeability have been first studied theoretically by Veselago [1]. Because the electric field, the magnetic field and the wavevector of a plane monochromatic wave form a left-handed system of reference in these materials, he termed them “left-handed”. Veselago showed that left-handed materials exhibit a number of remarkable properties unlike those of any known materials, such as reversed Doppler effect, reversed Cerenkov radiation, radiation tension instead of radiation pressure and negative refraction. Although no natural materials with left-handedness have been identified, artificial materials – or metamaterials – with negative electromagnetic parameters have recently been designed and experimentally demonstrated in the microwave range [2]. These metamaterials derive their electromagnetic properties from their (sub-wavelength) structure, rather than from their bulk properties.

Linear devices filled with left-handed materials have received a lot of attention. Most noteworthy, it has been predicted that a parallel-sided plate of left-handed material is an imaging system with sub-wavelength resolution properties [3]. This sub-wavelength resolution can be explained by the amplification of evanescent wave components that occurs in the plate. The nonlinear properties of left-handed materials, on the other hand, are still largely unexplored. One of the first contributions was by Zharov *et al.*, who analysed the nonlinear behaviour of the microwave metamaterial constructed by Shelby [4]. Agranovich *et al.* showed that well-known nonlinear effects like second harmonic generation, wave mixing and Raman scattering are more or less identical in left- and right-handed materials [5]. Feise *et al.* observed bistability in the transmission through a number of alternating slabs of two nonlinear materials with positive and negative refractive index [6]. Although the publications mentioned above provide a first step in the study of nonlinear optics in left-handed materials, a lot of work still needs to be done.

The Double Layered Resonator

In this contribution, we will present our study of a new device containing left-handed materials: a double layered Fabry-Perot resonator, of which one layer is filled with a traditional nonlinear material and the other layer with a nonlinear left-handed material (see Figure 1). The Kerr type nonlinear optical materials are assumed to be homogeneous, cubic, centrosymmetric and impedance matched. The sides of the resonator are coated with dielectric mirrors to enhance the finesse of the cavity.

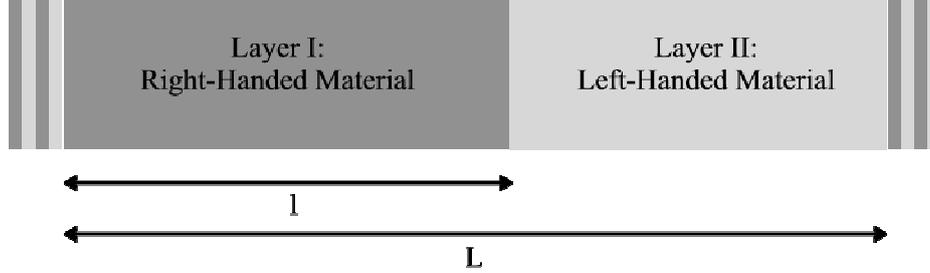


Figure 1: The device under test: a double layered Fabry-Perot resonator.

Propagation of Light in a Nonlinear Left-Handed Material

Light coupled into the resonator will first propagate through the right-handed layer and then through the left-handed layer. A rigorous model of our device should therefore include a description of both interactions. Propagation through a traditional Kerr type nonlinear material can be properly treated with the nonlinear Schrödinger equation. However, propagation through a nonlinear left-handed material has not been considered before. It is not obvious at all that the nonlinear Schrödinger equation is still valid, because it is commonly derived for nonmagnetic materials with $\mu(\omega) = \mu_0$. In [7], we have presented a generalisation for magnetic media. In the first step, we have derived from Maxwell's equations the wave equation governing the evolution of the electric field of light in a nonlinear dispersive magnetic dielectric:

$$-\text{rot rot } \mathbf{E} - \frac{1}{c^2} \int_{-\infty}^{\infty} \mu_r(t-\xi) \frac{\partial^2}{\partial \xi^2} \int_{-\infty}^{\infty} \epsilon_r(\xi-\tau) \mathbf{E}(\vec{r}, \tau) d\tau d\xi = \mu_0 \int_{-\infty}^{\infty} \mu_r(t-\xi) \frac{\partial^2 \mathbf{P}^{(3)}}{\partial \xi^2} d\xi. \quad (1)$$

Subsequently, we assumed the nonlinear term in (1) to be small and applied the slowly varying envelope approximation. A perturbation analysis then yields

$$\frac{\partial A}{\partial \xi} = i \frac{c}{2\omega_0 n} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) A - \frac{ik''}{2} \frac{\partial^2 A}{\partial t^2} + i \frac{3\omega_0}{2c\eta} \chi_0 (|A|^2 + 2|B|^2) A, \quad (2)$$

$$\frac{\partial B}{\partial \xi'} = -i \frac{c}{2\omega_0 n} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) B + \frac{ik''}{2} \frac{\partial^2 B}{\partial t^2} - i \frac{3\omega_0}{2c\eta} \chi_0 (|B|^2 + 2|A|^2) B, \quad (3)$$

where A and B are the envelopes of forward and backward waves, respectively. Please note the difference between Eqs. (2)-(3) and the traditional nonlinear Schrödinger equations for right-handed media. The coefficient of the nonlinear term of the former contains the characteristic impedance η of the medium instead of the index of refraction n written traditionally. Nevertheless, the coefficient of the diffraction term remains inversely proportional to the index of refraction, which implies that the diffraction term becomes *negative* in a left-handed material.

The Mean-Field Model

The propagation model, which consists of the nonlinear Schrödinger equations completed with appropriate boundary conditions, is a rigorous model, because it describes in every point the interaction of light with the materials and the structure of the nonlinear Fabry-Perot resonator. However, it is also too complex for a detailed analytical study. Therefore we have considered an approximation to the propagation model for cavities near to resonance [8]. This means that (1) the cavity has a high finesse – i.e. that it has a high photon lifetime – and (2) that the optical path length of light in the cavity is approximately an integer multiple of the wavelength of the laser light. In this case, we find the following equation relating the input field Σ and the output field A of the double layered resonator:

$$\frac{\partial A}{\partial t} = \Sigma - (1 + i\Delta)A + i\alpha\nabla_{\perp}^2 A + i|A|^2 A, \quad (4)$$

where Δ is the detuning from perfect resonance and α the diffraction coefficient. Eq. (4) is called the *mean field* equation. Since the same equation also describes traditional nonlinear Fabry-Perot resonators [9], it looks like our resonator exhibits the same behaviour as a traditional resonator. Nevertheless, there is an important difference. Let us look to the equation for the diffraction coefficient α :

$$\alpha = \frac{\lambda_0}{2\pi\sigma} \left(\frac{l}{n_R} + \frac{L-l}{n_L} \right). \quad (5)$$

The first term of the right-hand side of Eq. (5) is positive, while the second term is negative (remember that n_L is negative. Both terms will thus cancel each other to some extent. In this way, it is possible to *engineer* the strength of the diffraction by adjusting the lengths l and $L-l$ of the layers. This should be compared with classical resonators, where the diffraction coefficient can only be positive.

Diffraction Compensation and Negative Diffraction

If the ratio of the thicknesses of the layers equals the ratio of the indices of refraction, the diffraction coefficient vanishes. This is the regime of *diffraction compensation*. It is straightforward that this regime is very advantageous for a large number of applications of the nonlinear Fabry-Perot resonator, such as high finesse resonators, optical transistors and optical memory elements. With the diffraction compensated resonator, it is indeed possible to eliminate diffraction losses, which leads naturally to resonators with higher quality factors. Elimination of diffraction losses is even more important for nonlinear Fabry-Perot resonators, where a too small interaction length currently limits their use. A higher quality factor will here increase the photon life time of the cavity, so that the nonlinear phase change is accumulated over more roundtrips.

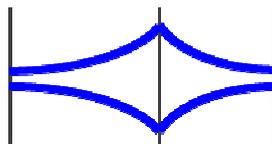


Figure 2: A Gaussian beam in the double layered resonator will first be widened due to diffraction in the right-handed layer, but will then again be focused due to the negative diffraction in the left-handed layer. Overall, it looks as if there is no widening and, hence, as if there is no diffraction.

If the negative term in Eq. (5) is larger than the positive term, the overall diffraction coefficient α becomes negative. The resonator is said to be in the regime of *negative diffraction*. We have studied the dynamics of this new regime and showed that the resonator's input-output characteristic has the same form as for positive diffraction (see Figure 3). The difference here lies in the stability. For the classical resonator with positive diffraction, the upper branch of the curves is unstable, whereas the upper branch is stabilised for a resonator in the negative diffraction regime. This stabilisation is again advantageous for applications, such as switches or optical memories, where the resonator is switched from one to another branch.

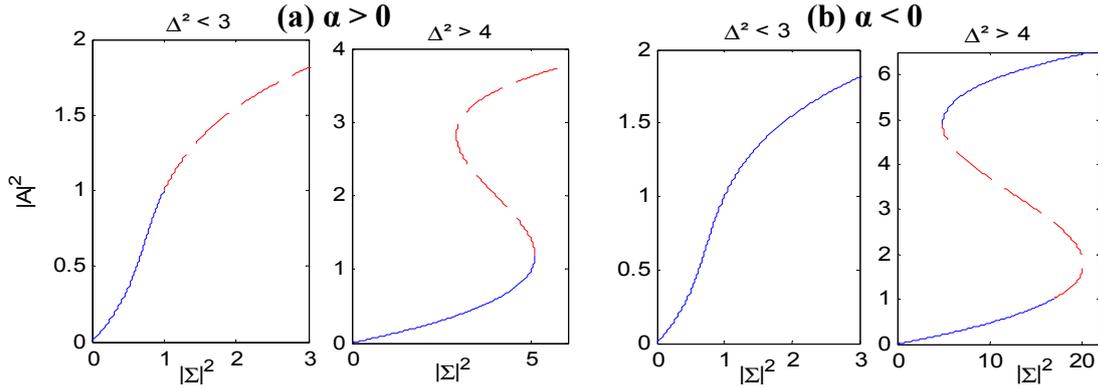


Figure 3: Input-output characteristics of the double layered resonator. (a) Positive diffraction. (b) Negative diffraction. Stable states are denoted by a solid line, unstable states by a dashed line.

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