Ultrafast optical oscillator locked by nonlinear optical feedback

Gregory Kozyreff[1], Pascal Kockaert[2], Thomas Erneux[1],
Cyril Cambournac[2] and Marc Haelterman[1]

[1]Optique non linéaire théorique, Université libre de Bruxelles, CP231
Campus Plaine, B-1050 Bruxelles (Belgium)
[2]Service d'optique et acoustique, Université libre de Bruxelles, CP194/5
50, Av. F. D. Roosevelt; B-1050 Bruxelles (Belgium)

We consider an externally driven double-pass ring cavity with nonlinear incoherent optical feedback. To a certain extent, this cavity is the transposition, in the temporal domain, of the very simple spatial-pattern-generating system made of a Kerr slice and a feedback mirror. An analytical study shows that this system acts like a passive modulator of continuous laser beams, like the self-induced modulational instability laser, with the main difference that the modulation frequency is stable against power fluctuations of the pump beam. Numerics confirm our analytical predictions, and show that we can readily obtain modulation frequencies far above the limits of electronics.

Introduction

We study a temporal analog of the very simple passive optical system [1] made of a Kerr slice and a feedback mirror. It was shown that in the spatial Kerr-slice system, the interaction of nonlinearity, diffraction and diffusion results in a very rich pattern generation dynamics [2], including hexagon formation [3, 4], optical turbulence [5] and fractals [6] generation. In this dynamics, the diffusion inside the Kerr material plays an important role in limiting the minimal size of the patterns. Many experiments have verified the theoretical predictions [7, 8].

In our temporal analog, the interplay between dispersion and nonlinearity leads to temporal modulational instability (MI). Usually, the resulting oscillation frequency scales like the square root of the optical power, but here, this frequency is robust against power fluctuations. This useful property is inherent to the nature of the feedback and contrasts strongly with most systems experiencing modulational instability, as is the case in the MI-laser [9].

In the temporal transposition of the Kerr slice device, the diffusion finds no equivalent. The bandwidth of the instability dynamics is therefore limited by another process: the unavoidable dispersion in the nonlinear medium.

Setup

We study the dynamics of the optical "cavity" depicted on Fig. 1. In contrast with classical cavities, light makes only two turns into the whole setup. This results from the 90-degree polarisation rotation occurring on each roundtrip. The feedback mechanism takes place into the nonlinear medium, where the incoming and the outgoing beams are in orthogonal circular polarisation states. The isotropic Kerr nonlinearity ensures that an amplitude modulation of the outgoing beam is transferred to the incoming beam as a phase modulation, without exchange of energy between the two orthogonally polarised

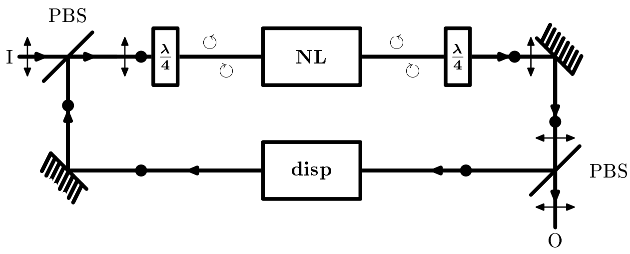

FIG. 1– *The setup includes a Kerr-type nonlinear medium (NL) and a dispersive one (disp). The quarter-wave plates ($\lambda/4$) and the polarising beam splitters (PBS), ensure that light makes two turns into the "cavity", with orthogonal polarisation states during the first pass and the second one.*

beams [10, p. 223]. In the dispersive medium, the phase modulation is transformed into an amplitude modulation, which effectively ensures the incoherent feedback.

Analysis

It is possible to capture the major features of the system dynamics with a very simple model. Our description relies on three main assumptions: (i) the optical phase can be neglected, as the feedback process is incoherent; (ii) the transfer function of the dispersive section is given by $\mathrm{e}^{\mathrm{i}\omega^2\beta_2/2}$; and (iii) in the nonlinear section, the evolution of the circularly polarised envelopes (u, v) of the electric field is given by the simplified coupled nonlinear Schrödinger equations [10, p. 206–208]

$$\frac{\partial u}{\partial z} = \mathrm{i}\,\gamma\left(|u|^2 + \sigma\,|v|^2\right) u, \tag{1}$$

$$\frac{\partial v}{\partial z} = \mathrm{i}\,\gamma\left(|v|^2 + \sigma\,|u|^2\right) v. \tag{2}$$

In these expressions, β_2 is the dispersion coefficient, $\omega = 2\pi\nu$ is the optical pulsation, γ is the nonlinear coefficient, and σ the cross-phase modulation coefficient.
Equations (1) and (2) admit solutions in the form of

$$u(z) = u_0\,\mathrm{e}^{\mathrm{i}\,\gamma\left(|u_0|^2+\sigma|v_0|^2\right)z}, \tag{3}$$

$$v(z) = v_0\,\mathrm{e}^{\mathrm{i}\,\gamma\left(|v_0|^2+\sigma|u_0|^2\right)z}. \tag{4}$$

Using the Fourier transform ($\mathcal{F}$), it is possible to combine the transfer function of the dispersive section with these solutions. In what follows, we assume that the incoming beam is a continuous wave ($P_\mathrm{i}(t) \equiv P_\mathrm{i}$), corresponding to the u polarisation component, while the outgoing beam is related to v. Therefore, the expression of the output power after one turn is given by

$$P_\mathrm{o}(t) = P_\mathrm{i}\,\left|\mathcal{F}^{-1}\left[\mathcal{F}\left[\mathrm{e}^{\mathrm{i}\sigma\gamma L_\mathrm{NL}P_\mathrm{o}(t-\tau)}\right]\mathrm{e}^{\mathrm{i}\beta_2^\mathrm{D}L_\mathrm{D}\Omega^2/2}\right]\right|^2, \tag{5}$$

where we introduced the cavity roundtrip time τ, $P_\mathrm{i} = |u|^2$, $P_\mathrm{o} = |v|^2$, and where L_NL and L_D denote respectively the lengths of the nonlinear and the dispersive media.

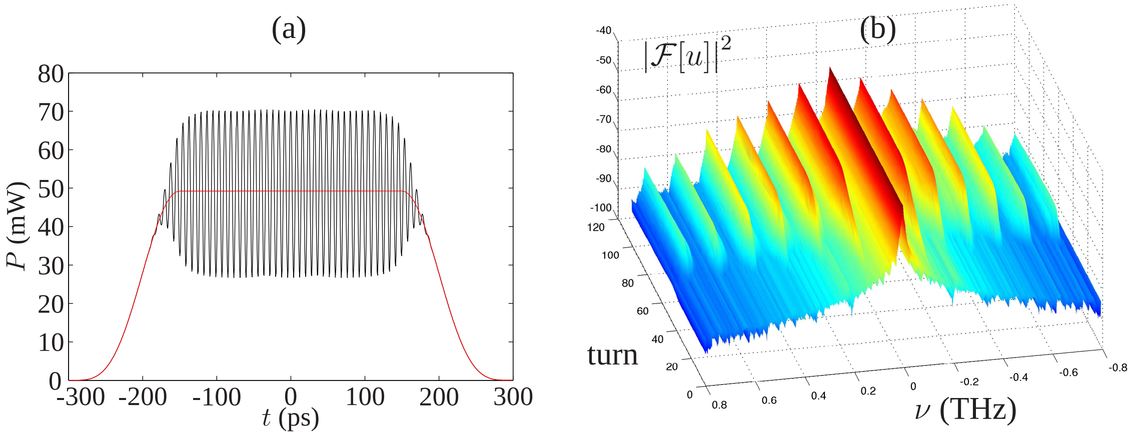

FIG. 2– *(a) Modulation of the beam after one turn in the cavity. The initial state appears in red. (b) Sidebands growing in the spectral domain (logarithmic scale). The parameters are:* $L_{\mathrm{D}} = 160\,\mathrm{m}$, $L_{\mathrm{NL}} = 5\,\mathrm{km}$, $\beta_2^{\mathrm{D}} = 25\,\mathrm{ps}^2/\mathrm{km}$, $\beta_2^{\mathrm{NL}} = 0.2\,\mathrm{ps}^2/\mathrm{km}$, $\gamma = 1/(\mathrm{W}\cdot\mathrm{km})$, $\sigma = 2$, $\alpha = 0.2\,\mathrm{dB/km}$.

From (5), we conclude that $P_{\mathrm{o}}(t) \equiv P_{\mathrm{i}}$ is the stationary solution of the system. We can also use (5) to observe the evolution of a modulation. To the first order in ε, after one turn, the initial modulated state

$$P_{\mathrm{o}}(t) = 1 + \varepsilon \cos(\Omega t)\,, \tag{6}$$

becomes

$$P_{\mathrm{o}}(t) = 1 + g(\Omega, P_{\mathrm{i}}) \cdot \varepsilon \cos(\Omega t)\,. \tag{7}$$

The gain $g(\Omega, P_{\mathrm{i}})$ is given by

$$g(\Omega, P_{\mathrm{i}}) = -2\sigma\gamma P_{\mathrm{i}} L_{\mathrm{NL}} \sin\left(\beta_2^{\mathrm{D}} L_{\mathrm{D}} \Omega^2/2\right). \tag{8}$$

Therefore, the maximal gain $g_{\max}$ is obtained at frequencies $\Omega_{\max}$ that do not depend on the input power:

$$|g_{\max}| = 2\sigma\gamma P_{\mathrm{i}} L_{\mathrm{NL}}, \qquad \Omega_{\max} = \sqrt{\frac{\pi(2N+1)}{|\beta_2^{\mathrm{D}}|\, L_{\mathrm{D}}}}. \tag{9}$$

This last expression shows that for sufficiently high input powers ($g_{\max} > 1$) modulations with arbitrarily large frequencies can grow. This is physically unacceptable and, in practice, the unavoidable dispersion in the nonlinear medium limits the instability bandwidth. This can be verified analytically, as well as numerically.
In the limit of small residual dispersion in the nonlinear section (β_2^{NL}), the gain becomes

$$g(P_{\mathrm{i}}, \Omega, \beta_2^{\mathrm{NL}}) \sim g(P_{\mathrm{i}}, \Omega) \cdot \left(1 - \tfrac{1}{3}\gamma P_{\mathrm{i}} \left|\beta_2^{\mathrm{NL}}\right| L_{\mathrm{NL}}^2 \Omega^2\right). \tag{10}$$

This was confirmed numerically, by solving the coupled cubic nonlinear Schrödinger equations using the split-step Fourier method [10, p. 51]. See Fig. 2(a). The continuous wave input signal was approximated by a long pulse; the output signal was uniformly modulated in its centre, in good agreement with analytical predictions. The growing modes—and their harmonics due to the saturation—are shown on Fig. 2(b).

The set of parameters was chosen to observe a modulation at 100 GHz. Dispersion and nonlinear coefficients are typical of commercially available optical fibres. Losses have also been included in the numerics and were compensated at each roundtrip. However, such a compensation is not necessary to observe growing modes.

Conclusions

Considering the classical transposition from the spatial diffractive systems to the temporal dispersive ones, we designed an equivalent of the well-known Kerr-slice system. The main dynamical properties are similar: the system is modulationally unstable at frequencies that are fixed by the dispersive or diffractive element. Nonetheless, the two systems differ on the physical processes limiting the minimal size of the modulation features, as well as on their dimensionality—limited to (1+1)-D in the temporal case.
Numerical simulations confirmed the theoretical prediction that this device could serve as an optical oscillator, operating at frequencies far above the frequency-limit of electronic devices; and that the generated signal would be robust to power fluctuations.

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