Pure frequency oscillations of semiconductor lasers with filtered optical feedback

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A semiconductor laser subject to delayed filtered optical feedback can show pure frequency oscillations with a period of the order to the delay time, while the power remains practically constant. This is remarkable in light of the strong self-phase modulation in semiconductor lasers that couples frequency and power. It turns out that the dynamics of the filter plays an essential role in this behavior, because it changes the instantaneous amount of feedback in response to the instantaneous laser frequency. By using numerical bifurcation techniques we show how frequency oscillations bifurcate in Hopf bifurcations from the continuous wave solutions known as external filtered modes.

The extreme sensitivity of semiconductor lasers (SLs) to external perturbation has let to a vast amount of studies investigating the onset of instabilities caused by feedback in SLs, and still there are many open questions. A lot of these studies are focusing on the origin of instabilities and chaotic dynamics that degrade the performance of the system [1, 2, 3, 4]. On the other hand, ideas have been developed to utilize chaotic dynamics for applications, such as secure communications [5]. This asks for control of the feedback.
We investige a SL that receives filtered optical feedback; see the sketch in Fig. 1. A fraction of the light emitted by the laser is traveling through a feedback loop, where a Fabry-Perot filter gives control over the feedback light by means of the filter detuning and the filter width; see also Refs. [6, 7]. In particular, we show the important role of the feedback phase on the structure and stability of the system.

This system can be modeled by the set of rate equations for the complex envelope of the laser field E and the filter field F and the laser inversion N given by

$$\frac{dE}{dt} = (1+i\alpha)N(t)E(t)+\kappa F(t,\tau) \tag{1}$$

$$T\frac{dN}{dt} = P-N(t)-(1+2N(t))|E(t)|^2 \tag{2}$$

$$\frac{dF}{dt} = \Lambda E(t-\tau)e^{-iC_p}+(i\Delta-\Lambda)F(t)\,. \tag{3}$$

Here $\alpha = 5$ is the self-phase modulation parameter, κ the feedback rate, $T = 100$ the electron decay rate, $P = 3.5$ the pump rate, C_p the feedback phase, $\Delta = 0.007$ the filter detuning and $\Lambda = -0.007$ the filter width (HWHM). In Eqs. (1)–(3) time t is measured in units of the photon lifetime; see Refs. [7, 8] for more details of the model
The basic solution of Eqs. (1)–(3) are CW states, with a frequency deviation ω_s from the solitary laser frequency at threshold. The amplitudes of the laser field R_s and the filtered feedback field A_s and the inversion N_s of laser are constant in time. The feedback field may have a constant phase shift ϕ. We call these solutions *external filtered modes* (EFMs):

$$E(t) = R_s e^{i\omega_s t}\,, N(t) = N_s\,, F(t) = A_s e^{i\omega_s t+i\phi}\,. \tag{4}$$

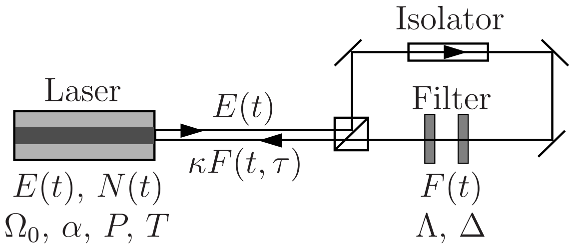

Figure 1: Sketch of the system.

Even though the delay time τ is long the number of EFMs is small, because of the relatively small linewidth of the filter. However, one should note that the distance between EFMs is still approximately given by the roundtrip time in the feedback loop. Since the number of EFMs is small one can trace each EFM individually. We use DDE-BIFTOOL [9, 10] for the numerical continuation of the EFMs.

In Fig. 2 the EFMs are plotted as a function of the feedback rate κ for six different values of the feedback phase C_p. Because of the small number of EFMs the feedback phase C_p has a vital influence on the structure and stability of the EFMs as can be seen from Fig. 2. The branches of EFMs are plotted in black, where thin curves indicate unstable EFMs and thick curves stable EFMs. Apart from the solitary laser modes that exists for all feedback rates κ, EFMs are born in pairs in saddle-node bifurcation $(+)$, one of which may be stable. EFMs destabilize in Hopf bifurcations $(*)$. As C_p is changed the positions of these bifurcations change, and so does the structure and stability of the EFMs. The dark gray (light gray) curves in Fig. 2 are the saddle-node (Hopf) bifurcations in the (κ, N_s)-plane parametrized by the feedback phase C_p. As C_p is changed the saddle-node bifurcations $(+)$ and the Hopf bifurcation $(*)$ move along these curves.

Especially for small values of κ there are several stable EFMs. Up to four EFMs can be stable simultaneously, depending on the feedback rate κ and the feedback phase C_p. Moreover, EFMs can undergo Hopf bifurcations in which periodic orbits are created. In this case the EFM still exists after the bifurcation, only its stability (more generally the number of unstable eigendirections) has changed. (This is in contrast to a saddle-node bifurcation where a pair of EFMs annihilate each other and disappear.) If the Hopf bifurcation is supercritical the created periodic orbit is stable.

There are two Hopf bifurcation denoted by H_{RO} and H_{FO} that lead to stable periodic orbits. H_{RO} leads to stable relaxation oscillations with a frequency $\omega_{RO} \approx \sqrt{\frac{2P}{T}}$. It is well known that external perturbations, such as feedback, may result in an undamping of relaxation oscillations. However, it is a special property of FOF that there is another type of Hopf bifurcation H_{FO} which leads to the so-called frequency oscillations [3]. Figure 3 shows a time series where the characteristic time scale of the frequency oscillations is $\omega_{FO} \approx \frac{2\pi}{\tau}$. The remarkable property of frequency oscillation is that the intensity of the laser is almost constant in time; only the optical frequency of the laser oscillates as shown Fig. 3. This is only possible because of the presence of the filter which changes the amount of feedback according to its lineshape. The structure and stability of the frequency oscillations depends on the feedback phase as well, which is the subject to ongoing research.

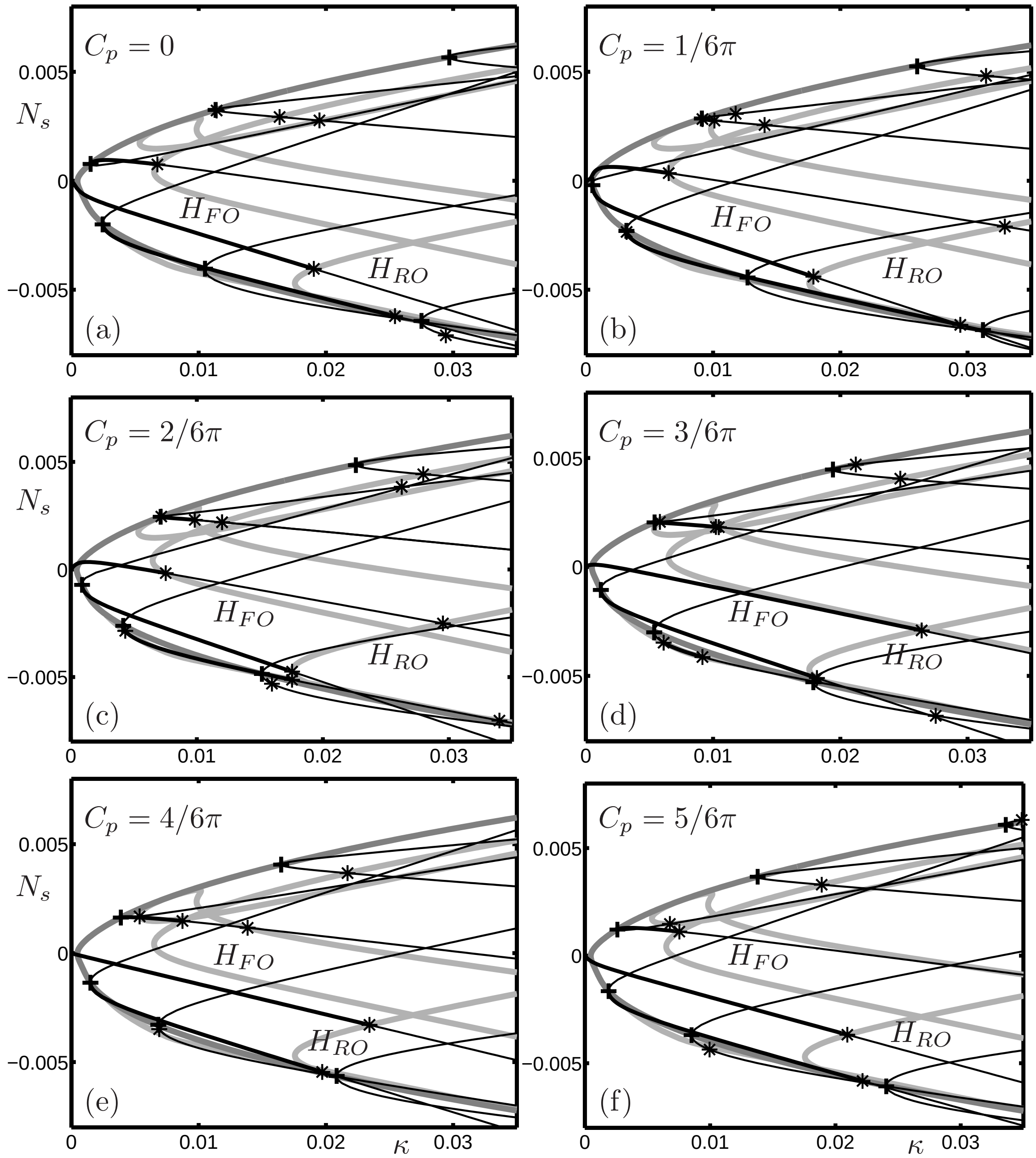

Figure 2: EFMs in the (κ, N_s)-plane for six different values of the feedback phase C_p indicated in the panels. Branches of EFMs are plotted in black, stable parts are thick and unstable parts are thin. Saddle-node bifurcations are indicated by pluses $(+)$ and Hopf bifurcations by stars $(*)$. Saddle-node lines (dark gray) and Hopf lines (light gray) are parametrized by C_p.

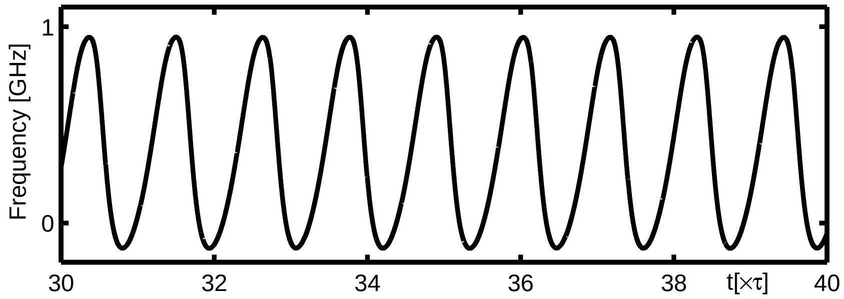

Figure 3: Time series of frequency oscillations; time is in units of the delay time τ.

In conclusion, we discussed the structure and stability of the external filtered modes for a semiconductor laser with coherent filtered feedback in dependence on the feedback phase and the feedback rate. In particular, it turns out that the feedback phase plays a vital role for the dynamics of the system. This is mainly due to the small number of external filtered modes. We showed that, in addition to the well known relaxation oscillations, the system can exhibit frequency oscillations. For frequency oscillations the intensity of the laser is almost constant and only the laser frequency oscillates. Interesting further studies may include the influence of the detuning and the linewidth of the filter.

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