The Sommerfeld Precursor in Photonic Crystals

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We calculate the Sommerfeld precursor that results after transmission of a generic electromagnetic plane wave pulse with transverse electric polarization, through a one-dimensional rectangular N-layer photonic crystal with two slabs per layer. The shape of this precursor equals the shape of the precursor that would result from transmission through a homogeneous medium. However, amplitude and period of the precursor are now influenced by the spatial average of the plasma frequency squared instead of the plasma frequency squared for the homogeneous case.

Introduction

Due to dispersion and absorption, a pulse separates into distinct parts in configuration space [1]. Its wavefront always propagates at the speed of light in vacuum. Immediately behind it, the Sommerfeld precursor [1, 2] emerges. The amplitude and period of this precursor are very small as compared to the period and the carrier frequencies of the applied pulse. The precursor has been observed for microwaves [3] and for optical pulses [4, 5]. In this paper, the Sommerfeld precursor is calculated after transmission through an N-layer medium.

Model for the N-Layer Medium

Fig. 1 depicts our model for the rectangular 1D N-layer medium. Each of the layers $\lambda = 1, \ldots, N$ contains two homogeneous dielectric slabs of widths a and b such that $a + b = \Lambda$. These slabs have refractive indices n_a and n_b respectively. The surrounding medium has refractive index n_s. The media are infinitely extended in the yz-plane. In medium j $(j = a, b, s)$, the frequency dependence of the refractive index is [6]

$$n_j(\omega) = \left(1 + \frac{2}{3} \sum_{r=1}^{m_j} \frac{f_{rj}\omega_{pj}^2}{\omega_{rj}^2 - \omega^2 - i\gamma_{rj}\omega}\right)^{1/2} \left(1 - \frac{1}{3} \sum_{r=1}^{m_j} \frac{f_{rj}\omega_{pj}^2}{\omega_{rj}^2 - \omega^2 - i\gamma_{rj}\omega}\right)^{-1/2}. \quad (1)$$

Here the atoms of medium j have m_j resonances and f_{rj} and γ_{rj} are the oscillator strength and absorption parameter for the r-th resonance at $\omega = \omega_{rj}$. The plasma frequency of medium j is ω_{pj}.

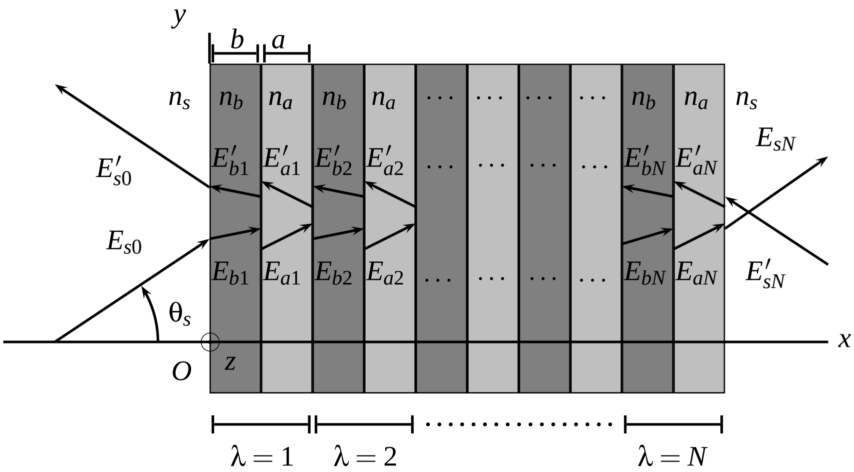

Figure 1: Model for a 1D photonic crystal with electric fields

The Pulse

The TE-polarized fields propagate in the xy-plane along the arrows (Fig. 1). The unit propagation vector of the fields that move to the right in medium j is

$$\hat{\pi}_j = \hat{\mathbf{x}}\cos\theta_j + \hat{\mathbf{y}}\sin\theta_j; \qquad -\pi/2 < \theta_j < \pi/2, \tag{2}$$

specified for $j = s$. The applied field E_{s0} is a 1D-plane wave packet,

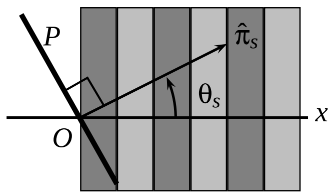

Figure 2: The line P is the wavefront at $t = 0$.

$$E_{s0}(t,x,y) = \int \mathrm{d}\omega\, \tilde{E}_{s0}(\omega)\mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}k_s(\omega)(x\cos\theta_s + y\sin\theta_s)}, \tag{3}$$

where $k_s = \frac{\omega}{c}n_s$ and $\tilde{E}_{s0}$ is the spectral density at (Fig. 2) the line $P : x\cos\theta_s + y\sin\theta_s = 0$. For a generic pulse that is nonzero at P at times $t \in [0,T]$,

$$\tilde{E}_{s0}(\omega) = \frac{1}{2\pi}\sum_m \frac{\omega_m \tilde{E}^P_{s0}(\omega_m)}{\omega^2 - \omega_m^2}\Big((-1)^m \mathrm{e}^{\mathrm{i}\omega T} - 1\Big), \tag{4}$$

where $\omega_m = \frac{m\pi}{T}$ and $\tilde{E}^P_{s0}(\omega_m)$ is the spectral density over a period T of the T-periodically continued signal at P. Let t_N be the plane wave transmission coefficient for the N-layer medium [7]. The transmitted pulse is then

$$E_{sN}(\tau,x,y)=\int d\omega\ t_N(\omega)e^{-i\frac{\omega}{c}n_sN\Lambda\cos\theta_s}e^{i\frac{\omega}{c}(n_s-1)(x\cos\theta_s+y\sin\theta_s)}e^{-i\omega\tau}\tilde{E}_{s0}, \quad (5)$$

where $\tau=t-\frac{x\cos\theta_s+y\sin\theta_s}{c}$ is the time elapse after the wavefront has passed.

Results

We take the vacuum as surrounding medium. The relative slabwidth of slab b is $\beta=\frac{b}{\Lambda}$ and let $\Delta=\frac{\omega_{pb}^2-\omega_{pa}^2}{\omega_{pa}^2}$. Further, denote $\delta=\beta\Delta$. For high frequencies, or small values of τ, the effect of having an inhomogeneous medium is fully captured in this parameter δ. The Sommerfeld precursor field S_{sN} is the small τ approximation to Eq. (5), and equals

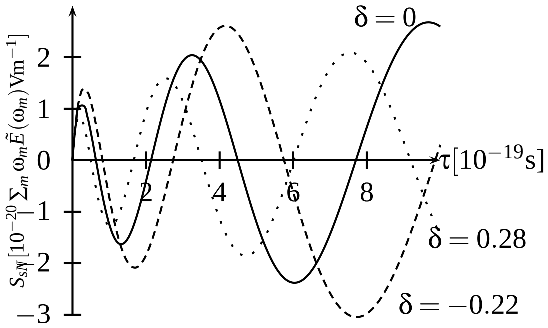

Figure 3: Sommerfeld precursor in a homogeneous Si medium (solid line) and for two different photonic crystals of the same length.

$$S_{sN}(\tau)=\sum_m\omega_m\tilde{E}_{s0}^P(\omega_m)\sqrt{\frac{\tau}{(1+\delta)\xi_a}}J_1\left(2\sqrt{(1+\delta)\xi_a\tau}\right). \quad (6)$$

Here J_1 is the Bessel function of the first kind and order and $\xi=N\frac{a\omega_{pa}^2+b\omega_{pb}^2}{2c\cos\theta_s}$ contains the medium properties. For a homogeneous medium a, $\xi|_{\omega_{pb}=\omega_{pa}}\equiv\xi_a$. In the expression for S_{sN}, the plasma frequency squared for a homogeneous medium is therefore replaced by its spatial average for the N-layer medium. Fig. 3 shows the initially transmitted field for $\theta_s=0$, $N=100$ and $\Lambda=600$nm. Medium a is made of Si ($\omega_{pa}=2.4\cdot10^{16}\mathrm{s}^{-1}$). There are three distinguishable situations: the case $\delta=0$ (solid line) corresponds to either $\beta=0$ or $\omega_{pb}=\omega_{pa}$, representing a homogeneous medium a. The case $\delta>0$ (dotted line: $\omega_{pb}=3.0\cdot10^{16}\mathrm{s}^{-1}$, $\beta=0.5$) is obtained when $\omega_{pb}>\omega_{pa}$ together with $\beta\neq0$. The amplitude and period of the initially transmitted field have decreased with respect to the case $\delta=0$. The case $\delta<0$ (dashed line: $\omega_{pb}=1.8\cdot10^{16}\mathrm{s}^{-1}$, $\beta=0.5$) is obtained when $\omega_{pb}<\omega_{pa}$ and $\beta\neq0$. The amplitude and period increase with respect to the case $\delta=0$.

Rotated Medium

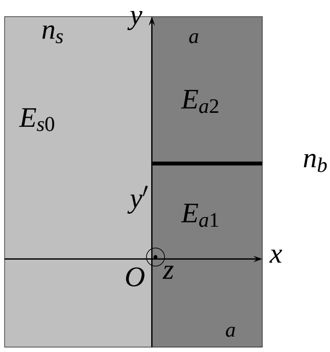

Figure 4: Delta-peak dielectric function with electric fields.

We have further calculated in Born approximation the field distribution far from the entrance plane at $x = 0$ resulting from a generic applied field that has been transmitted through a 90 degrees rotated N-layer half-infinite medium for the case $N = 1$ and for a weakly perturbing slab b of infinitesimal width (Fig. 4)

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