Origin of uniform pump-to-Stokes relative intensity noise (RIN) transfer in Raman fiber lasers.

A. A. Fotiadi*, S. A. Babin, D. V. Churkin, S. I. Kablukov, and E. V. Podivilov

*Service d'Electromagnétisme et de Télécommunications,
Faculté Polytechnique de Mons, 31 Boulevard Dolez, B-7000, Mons, Belgium
Tel: +32 65 374198; Fax: +32 65 374199; E-mail: Fotiadi@telecom.fpms.ac.be
Also with Ioffe Physico-Technical Institute of RAS, St.Petersburg, Russia

Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, Novosibirsk, 630090 Russia

Recently, the uniform pump-to-Stokes relative intensity noise (RIN) transfer has been discovered for cascaded Raman fiber lasers [1]. It was found that peaks of the radio-frequency (RF) spectrum associated with longitudinal mode beating in the pump laser cavity almost uniformly transfer to the 1st and 2nd Stokes RF-spectra. Here we present a detailed derivation of the analytical expression that explains this effect. We show that interaction between the intensity noise fluctuations related to counter-propagating waves in the Raman cavity is negligible comparing with the interaction of co-propagating fluctuations. This feature results in a simple relation between the fluctuation intensities.

Introduction

Continuous-wave-pumped Raman fiber lasers operating in the spectral range between 1.1 and 1.6 μm are of great interest for many applications in telecommunications, medicine, and other research areas. Although steady-state properties of the Raman lasers have been extensively considered from their early experimental demonstrations in the 1970's, only a limited number of theoretical works has been devoted to the analysis of the dynamical behavior of the Raman lasers. Recently we proposed a simple dynamical model to explain the unexpected effect observed on Raman lasers [1].

The effect is the following. A Raman laser, for simplicity a one-stage one, is pumped with a fiber laser source. The length of the Raman laser cavity L is assumed to be much longer than the length l of the pump laser cavity: $L >> l$. The pump laser is multimode. Therefore, its RF-spectrum $S_{PC}(\nu)$ presents a continuous chain of peaks associated with the longitudinal mode beating in the pump laser cavity. These peaks are observed in the MHz-frequency domain and have a spacing $\Delta\nu_{PC} = c/2ln$. The Raman laser is also multimode. Accordingly its RF-spectrum is expected to present a similar peak chain associated with the mode beating in the Raman laser cavity, with a spacing $\Delta\nu_{RC} = c/2Ln$. However, instead of that the RF-spectrum of the Raman laser, surprisingly, has a peak periodicity corresponding to the mode beating in the pump laser cavity, i.e. a spacing of $\Delta\nu_{PC}$. Inside these peaks the spectrum is modulated with a spacing $\Delta\nu_{RC} << \Delta\nu_{PC}$ determined by the Raman cavity length. Our model completely explains this effect predicting the following dependence between the pump and Raman laser spectra:

$$S_{RL}(\nu) \sim S_{PL}(\nu)\left[a^2 + 1 + 2a\cos\left(2\pi\nu/\Delta\nu_{RC}\right)\right]^{-1},\ a < 1 \text{ is a model parameter} \quad (1)$$

Since our analysis provoked some objections before its recent publication [1], we present here a detailed mathematical derivation of the last dependence (1).

Derivation of Linearized Raman Laser Equations

Our approach is based on the set of Raman propagation equations [2] that are linearized around the steady-state Raman laser solutions [3]. Similar method has already proved to be efficient at describing Brillouin noise spectra [4], [5]. In the cavity of the RFL there are two running Stokes waves: one co-propagating $\left(I^{+}\right)$ and one counter-propagating $\left(I^{-}\right)$ interacting with the pump wave $\left(I^{p}\right)$ that is also a running wave. Their mutual dynamics is described by the propagating Raman equations for their intensities:

$$\begin{aligned} I^{p}\left(z,t+z/v_0\right) &= I^{p}\left(0,t\right)\exp\left(-\alpha_0 z-\int_0^z dz'\left[gI^{-}\left(z',t+z'/v_0\right)+gI^{-}\left(z',t+z'/v_0\right)\right]\right) \\ I^{+}\left(z,t+z/v_1\right) &= I^{+}\left(0,t\right)\exp\left(-\alpha_1 z+\int_0^z dz'\left[gI^{p}\left(z',t+z'/v_1\right)\right]\right) \\ I^{-}\left(z,t+\left(L-z\right)/v_0\right) &= I^{-}\left(L,t\right)\exp\left(-\alpha_1\left(L-z\right)+\int_z^L dz'\left[gI^{p}\left(z',t+\left(L-z'\right)/v_1\right)\right]\right) \end{aligned} \quad (2)$$

Using a stationary approach (i.e. assuming that all intensities are independent on time) we can find the solution $\left\langle I^{i}\left(z,t\right)\right\rangle \equiv I_0^{i}\left(z\right)$ that essentially determines the time-averaged intensity distributions along the cavity. For the further consideration these distributions are given functions. To describe the fluctuations $\delta I^{i}\left(z,t\right)$ Eqs. (2) may be linearized around the steady-state distributions so that $I^{i}\left(z,t\right)=I_0^{i}\left(z\right)+\delta I^{i}\left(z,t\right)$, where $\left|\delta I^{i}\right| << I_0^{i}$, $i=p,+,-$. By definition, $\left\langle \delta I^{i}\left(z,t\right)\right\rangle \equiv \lim_{\Delta T\to\infty}\frac{1}{\Delta T}\int_{\Delta T}\delta I^{i}\left(z,t\right)dt=0$ for any z. The first mean value theorem for integrals [6] obliges the last integral to be a function with a limited maximal amplitude δI_m^{i} and a maximal correlation time τ_c of the fluctuations so as:

$$\left|\int_{\Delta T}\delta I^{i}\left(z,t\right)dt\right| < \delta I_m^{i}\tau_c, \quad (3)$$

We rewrite Eq.(2) with the substitution $I\left(z,t\right)=I_0\left(z\right)+\delta I\left(z,t\right)$, then using the condition $gL\delta I << 1$ expand exponents ($\exp\left(x\right)\approx 1+x$, $x<<1$), subtract the steady-state solutions and omit terms $\sim\left(\delta I\right)^2$. After all simplifications we come to the linearized equations:

$$\begin{aligned} \delta I^{p}\left(z,t+z/v_0\right) &= D\left(z\right)\delta I^{p}\left(0,t\right)-gI_0^{p}\left(0\right)D\left(z\right)\int_0^z\left[\delta I^{+}\left(z',t+z'/v_0\right)+\delta I^{-}\left(z',t+z'/v_0\right)\right]dz' \\ \delta I^{+}\left(z,t+z/v_1\right) &= G^{+}\left(z\right)\delta I^{+}\left(0,t\right)+gI_0^{+}\left(0\right)G^{+}\left(z\right)\int_0^z\delta I^{p}\left(z',t+z'/v_1\right)dz' \\ \delta I^{-}\left(z,t+\left(L-z\right)/v_1\right) &= G^{-}\left(z\right)\delta I^{-}\left(L,t\right)+gI_0^{-}\left(L\right)G^{-}\left(z\right)\int_z^L\delta I^{p}\left(z',t+\left(L-z'\right)/v_1\right)dz' \end{aligned} \quad (4)$$

where $G^{+}\left(z\right)=\exp\left(-\alpha_1 z+g\int_0^z I_0^{p}\left(z'\right)dz'\right)$, $G^{-}\left(z\right)=\exp\left(-\alpha_1(L-z)+g\int_z^L I_0^{p}\left(z'\right)dz'\right)$ and $D\left(z\right)=\exp\left(-\alpha_0 z-g\int_0^z dz'\left[I_0^{+}\left(z'\right)+I_0^{-}\left(z'\right)\right]\right)$. Note that $D\left(z\right)\le 1$ for any z.

Let us now show that at the conditions under consideration $l << L$ the longest correlation length $l_c = v\tau_c$ of the fluctuations associated with the longitude mode beating in the pump laser cavity is small in comparison with both the laser cavity length L and the effective length of Raman amplification $L_{eff} = \left(gI_0^p(0)\right)^{-1}$: $l_c << L < L_{eff}$. Indeed, the fluctuations of the pump laser intensity occur due to beating between the pump laser modes. Thus the longest correlation length l_c is determined by the beat-length between the nearest laser modes $l_c \approx l << L$. On the other hand, the effective length of Raman amplification L_{eff} near the lasing threshold could be estimated from the lasing condition $R_1R_2\exp\left(2L/L_{eff}\right) \approx 1$ to be $L_{eff} \approx NL >> L$, where $N \approx 2\left[-\ln\left(R_1R_2\right)\right]^{-1} \succ 1$, R_1 and R_2 are the reflectivities of the Raman cavity mirrors. So, we can conclude that the relation $L_{eff} > L$ remains true at pump power levels up to N times above the threshold. The last means also that $G^+(z)$ and $G^-(z) < A \prec 2$ are limited functions.

As far as $l_c/L << 1$ and $\eta \equiv L/L_{eff} < 1$, the terms in Eqs. (4) responsible for interaction between counterpropagating fluctuations could be omitted. The resulting set is written as:

$$\begin{aligned}
&\delta I^p\left(z,t+z/v_0\right) = D(z)\delta I^p(0,t) - gI_0^p(0)D(z)\int_0^z \delta I^+\left(z',t+z'/v_0\right)dz' \\
&\delta I^+\left(z,t+z/v_1\right) = G^+(z)\delta I^+(0,t) + gI_0^+(0)G^+(z)\int_0^z \delta I^p\left(z',t+z'/v_1\right)dz' \qquad (5)\\
&\delta I^-\left(z,t+(L-z)/v_1\right) = G^-(z)\delta I^-(L,t)
\end{aligned}$$

In order to prove the last simplification we have to show that the omitted terms are negligible when a solution of Eqs.(5) is substituted to Eqs.(4). In the general case of different group velocities ($v_1 \neq v_0$), the space and running time variables in Eqs.(5) are mixed. The solutions can be represented as a superposition of the boundary fluctuations $\delta I^i(0,t)$ convoluted with the corresponding Green functions (like in [10], [11]). Such convolution obviously implies a non-trivial relation between the pump and Stokes RF-spectra in the Raman laser. In the limiting case of a low velocity dispersion considered here ($v_1 \approx v_0 \equiv v$), the space and running time variables can be separated. Then the solutions of eqs.(5) are simple linear combinations of the boundary fluctuations:

$$\begin{aligned}
&\delta I^p\left(z,t+z/v\right) = \gamma_1(z)\delta I^p(0,t) - \gamma_2(z)\delta I^+(0,t)\\
&\delta I^+\left(z,t+z/v\right) = \gamma_3(z)\delta I^+(0,t) + \gamma_4(z)\delta I^p(0,t), \qquad (6)\\
&\delta I^-\left(z,t+(L-z)/v_1\right) = G^-(z)\delta I^-(L,t)
\end{aligned}$$

where $\gamma_j(z)$ are functions of the steady state solutions $I_0^i(z)$; note that $\left|\gamma_j(z)\right| < A$.

Substitution of Eqs. (6) to the omitted terms gives the following expressions for them:

$$\begin{aligned}
&-D(z)\frac{\eta}{L}\int_0^z G^-(z')\delta I^-\left(L,t-L/v+2z'/v\right)dz' \equiv H^{-p}(z,t)\\
&G^-(z)\frac{\eta}{L}\int_z^L\left[\gamma_1(z')\delta I^p\left(0,t+L/v-2z'/v\right) - \gamma_2(z')\delta I^+\left(0,t+L/v-2z'/v\right)\right]dz' \equiv H^{p-}(z,t)
\end{aligned} \qquad (7)$$

Applying to all integrals in (7) the second mean value theorem for integrals [6] together with the rule (3) we can demonstrate that the omitted terms are indeed negligible:

$$\left|H^{-p}(z,t)\right| < \eta\frac{A}{L}\left|\int_0^z \delta I^{-}\left(L,t-L/v+2z'/v\right)dz'\right| < A\eta\frac{l_c}{2L}\delta I_m^{-}(L) << \delta I_m^{-}(L)$$
$$\left|H^{p-}(z,t)\right| < A\eta\frac{l_c}{2L}\left(\delta I_m^{p}(0)+\delta I_m^{+}(0)\right) << \delta I_m^{p}(0)+\delta I_m^{+}(0) \qquad (8)$$

Results and discussion

We have proved that the solution (6) of Eqs.(5) is also a solution of Eqs.(4). The structure of the solution (6) is rather simple: the functions $\gamma_1(z)$ and $\gamma_3(z)$ describe independent evolution of the fluctuations in the Raman cavity, while $\gamma_2(z)$ and $\gamma_4(z)$ are responsible for interconnect between the fluctuations associated with modulation of the Raman gain. Fourier transform of Eqs. (6) in combination with the boundary conditions leads to the relation between the pump and Raman RF-spectra as observed in the experiment [1]:

$$S_{RL}(\nu) = \gamma_4^2(L)S_{PL}(\nu)\left[a^2+1+2a\cos\left(2\pi\nu/\Delta\nu_{RC}\right)\right]^{-1},\quad a\equiv\gamma_3(L)G^{-}(0)R_1R_2\le 1 \qquad (9)$$

According to (9) the RF-spectrum $S_{RL}(\nu)$ of the Raman laser reproduces the RF-spectrum of the pump laser. The last occurs to be additionally modulated with a period $\Delta\nu_{RC}=c/2Ln$ that is the free spectrum range of the Raman cavity. The parameter a determines the modulation depth. The effect is directly proportional to $\sim\gamma_4^2(L)$.

What is interesting is that the RF-spectra $S_{PR}(\nu)$ of the residual pump radiation emitted by the Raman laser could also exhibit a modulation with the frequency $\Delta\nu_{RC}$, but this effect is much less pronounced than the previous. Fourier transform of Eqs.(6) leads to a noise-like spectrum $S_{PR}(\nu)$ with lower $(-)$ and upper bounds $(+)$ determined as:

$$S_{PR}(\nu) = S_{PL}(\nu)\left[\gamma_1(L)\pm\mu a\left[a^2+1+2a\cos\left(2\pi\nu/\Delta\nu_{RC}\right)\right]^{-1/2}\right]^2, \qquad (10)$$

where $\mu\equiv\gamma_2(L)/\gamma_3(L)$. This result corresponds to the observations [1] at $\mu\approx 0.15$.

Acknowledgements

The research was supported by Interuniversity Attraction Pole program (IAP V 18) of the Belgian Science Policy and the Siberian Branch of Russian Academy of Sciences, Governmental program of support of leading scientific schools in Russia.

References:

[1] S.A. Babin, D. V. Churkin, A.A. Fotiadi, S.I. Kablukov, O.I. Medvedkov, E.V. Podivilov, “Relative intensity noise in cascaded Raman fiber lasers,” IEEE Phot. Techn. Lett., in press, 2005.
[2] J. AuYeung and A. Yariv, “Theory of cw Raman oscillation in optical fibers”, J. Opt. Soc. Am., vol.69, pp.803–807, 1979.
[3] S.A. Babin, D.V. Churkin, and E.V. Podivilov, “Intensity interactions in cascades of a two-stage Raman fiber laser,” Opt. Comm., vol. 226, pp. 329–335, 2003.
[4] A.A. Fotiadi, E.A. Kuzin, M.P. Petrov, and A.A. Ganichev, “Amplitude–frequency characteristic of an optical-fiber stimulated-Brillouin amplifier with pronounced pump depletion,” Sov. Tech. Phys. Lett., vol. 15, pp.434–436, 1989.
[5] E.A.Kuzin, M.P.Petrov, A.A.Fotiadi. “Phase conjugation by SMBS in optical fibers”, in Optical phase conjugation, M.Gower, D.Proch, Eds., Springer-Verlag, 1994, pp.74-96.
[6] http://www.sosmath.com/calculus/integ/integ04/integ04.html