Beyond the Size Limit on Cavity Solitons with Left-Handed Materials

L. Gelens,[1] P. Tassin,[1] G. Van der Sande,[1,2] I. Veretennicoff,[1] P. Kockaert,[3] M. Tlidi,[2] D. Gomila[4] and J. Danckaert[1]

[1]Department of Applied Physics and Photonics, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussel, Belgium

[2]Optique non linéaire théorique, Université Libre de Bruxelles, CP 231, Campus Plaine, B-1050 Bruxelles, Belgium

[3]Service d'optique et acoustique, Université Libre de Bruxelles, CP 194/5, 50, Av. F. D. Roosevelt, B-1050 Bruxelles, Belgium

[4]Institut Mediterrani d'Estudis Avançats (IMEDEA, CSIC-UIB), Campus Universitat Illes Balears, E-07122 Palma de Mallorca, Spain

We propose a technique to produce cavity solitons with arbitrarily small diameter by placing a nonlinear left-handed material in a Fabry-Perot resonator together with a traditional nonlinear material. This configuration allows engineering the diffraction strength. Considering the typical nonlocal behaviour of the nano-structured left-handed metamaterial, we develop a mean-field model describing the spatiotemporal evolution of this system. We focus our numerical studies on the influence of the nonlocality on the size and linear stability of these localised structures as the diffraction strength is decreased. A minimal width beyond the diffraction limit is discovered and applications are discussed.

Introduction

Transverse localised structures in nonlinear optical cavities – often referred to as cavity solitons (CSs) – have been predicted and demonstrated in a large number of systems [1,2]. Such structures have been suggested for use in optical data storage and information processing. Typically, diffraction constrains their size to be of the order of the square root of the diffraction coefficient.

Some of us have shown recently that the diffraction coefficient of a nonlinear optical resonator can be engineered by inserting a left-handed material in the cavity [3]. The value of the diffraction coefficient can then be reduced to arbitrarily small values by tuning the relative thicknesses of right-handed and left-handed material layers. As, in the Lugiato-Lefever equation, length scales with the diffraction coefficient, this system potentially allows for CSs beyond the size limit imposed by natural diffraction. However, intuitively, one expects that the sub-wavelength structure of left-handed metamaterials will impose a new limit on the width of CSs. Here, we investigate the consequences of this shorter space scale on the properties of CSs.

Linear nonlocality

Technically, the Lugiato-Lefever equation can be derived from a perturbation analysis applied to Maxwell's equations in a nonlinear Kerr medium. The diffraction term arises as a first-order contribution and higher order spatial derivates can be neglected with respect to the diffraction term. However, when the diffraction coefficient is tuned to small values, higher order terms should be taken into account. The nonlocal linear polarisation, for example, results in an additional term in the Lugiato-Lefever equation,

$$\frac{\partial E}{\partial t} = -(1+i\theta)E + E_{in} + i\Gamma|E|^2 E + i\mathcal{D}\nabla_\perp^2 E + \iint \sigma(\mathbf{r}_\perp - \mathbf{r}_\perp^{'})E(\mathbf{r}_\perp^{'})d^2\mathbf{r}_\perp^{'}, \quad (1)$$

which couples the electric field envelopes at different transverse locations. We assume that this term can be expanded in a series of spatial derivatives of E:

$$\iint \sigma(\mathbf{r}_\perp - \mathbf{r}_\perp^{'})E(\mathbf{r}_\perp^{'})d^2\mathbf{r}_\perp^{'} = \sigma_0 E + \sigma_1 \nabla_\perp^2 E + \sigma_2 \nabla_\perp^4 E + \ldots \quad (2)$$

The first two terms in Eq. (2) will contribute to the phase velocity and the diffraction effects, respectively. Keeping only the next higher order effect, we finally arrive at

$$\frac{\partial E}{\partial t} = -(1+i\theta)E + E_{in} + i\Gamma|E|^2 E + i\mathcal{D}^{(1)}\nabla_\perp^2 E + i\mathcal{D}^{(2)}\nabla_\perp^4 E, \quad (3)$$

where $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$ describe the diffraction strength and the strength of the nonlocal effects, respectively.

Linear stability analysis

When examining the homogeneous, stationary solutions of Eq. (3), it is found that the output-field vs. input-field can have a mono- and bistable characteristic. The study here is limited to the monostable region. At higher background intensities a modulational instability gives rise to the appearance of CSs. A linear stability analysis has been performed on these homogeneous, stationary solutions, showing that the modulational instability depends critically on the dimensionless parameter η, defined by the ratio of the nonlocality vs. the diffraction: $\eta = \mathcal{D}^{(2)} / \mathcal{D}^{(1)2}$.

For $\eta > 0$, it is possible to predict the width of the CSs in the limit of small diffraction:

$$\Lambda_+ = 2\pi\sqrt{2\eta\mathcal{D}^{(1)}} = 2\pi\sqrt{\frac{2\mathcal{D}^{(2)}}{\mathcal{D}^{(1)}}}. \quad (4)$$

Eq. (4) predicts that the CS-width increases with decreasing $\mathcal{D}^{(1)}$, while the classical diffraction limit (without the nonlocal term) predicts a decreasing width. This suggests the existence of a minimal CS-width for an optimal value of the diffraction coefficient $\mathcal{D}^{(1)}$. In order to determine this optimum, numerical simulations are needed. It is always possible that the CS with optimal width is unstable. This information shall also be provided by the numerical simulations.

For $\eta < 0$, we find that the CS-width will still decrease with increasing diffraction strength $\mathcal{D}^{(1)}$, but that due to the width will eventually saturate at

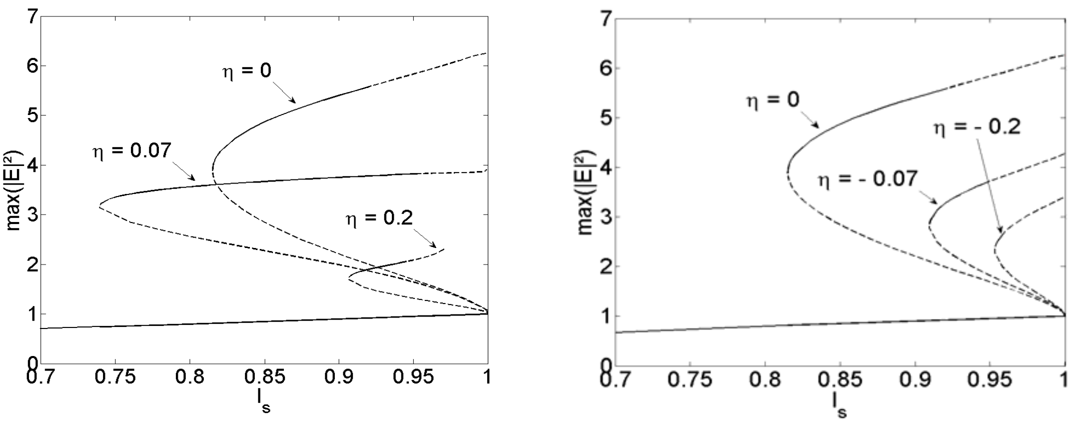
$$\Lambda_{\lim} = 2\pi\sqrt[4]{-\frac{\mathcal{D}^{(2)}}{2-\theta}}. \quad (5)$$

Stability and minimal width of cavity solitons

In order to find stationary, localised solutions of Eq. (3) and their stability, we follow a Newton method [4]. The radial form of Eq. (3) is discretised, from which a set of nonlinear coupled complex equations is found. Spatial derivatives are calculated in the Fourier space by using a Fast Fourier Transform and taking zero boundary conditions. With an appropriate initial condition, the Newton-Rhapson method converges to a solution of this set of equations. This initial condition is found by performing a 2D spatiotemporal simulation of a radial version of Eq. (3). This approach is extremely accurate and automatically generates the Jacobian operator, which provides insight into the stability of the solutions. This method also allows finding unstable solutions and thus creating full bifurcation diagrams.

In Figure (1), bifurcation branches of the CSs are shown as a function of the background intensity of the homogeneous, stationary solutions I_s and the maximum intensity of the solitons (for $\theta = 1.23$). The lowest, solid line represents the stable homogeneous solution. For $\eta > 0$ [Figure 1 (a)], the branches expand to lower intensities for increasing values of η, also enlarging the region of stability (solid line). This trend is reversed at a certain η, narrowing the stability region with larger values of η. For $\eta < 0$ [Figure 1 (b)], the branches always shift to higher I_s and the region of stability narrows with increasing values of $|\eta|$. Note also that in both cases the peak intensity of the CSs diminishes with increasing $|\eta|$. This is a first sign of broadening of the CSs.

(a) (b)



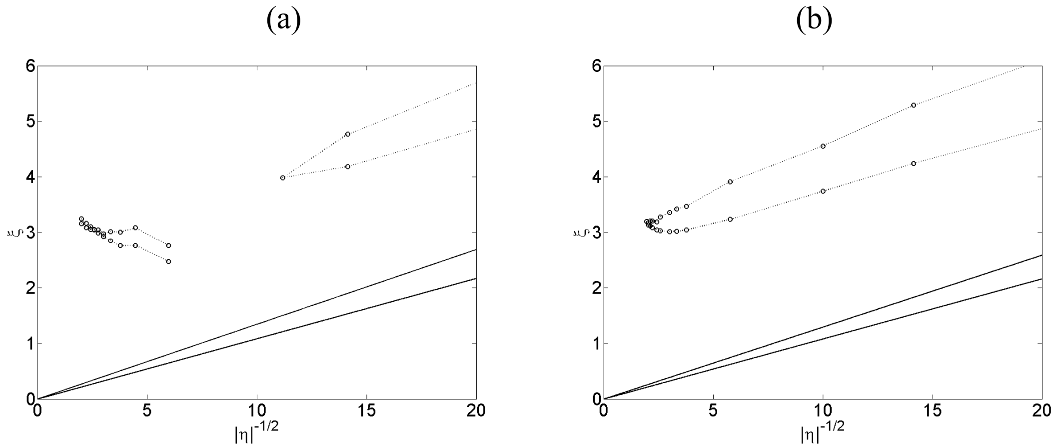
Figuur 1: Bifurcation diagrams with a detuning θ = 1.23 and for different values of η, with η (a) positive and (b) negative. The lowest solid line is the stable homogeneous solution. The branches above are CSs. Solid lines represent stable CSs, dashed lines unstable CSs.

To find the minimal CS-width, a renormalised full width at half maximum (FWHM) is defined:

$$\xi = d_{FWHM} / \sqrt[4]{\left|\mathcal{D}^{(2)}\right|}. \qquad (8)$$

This renormalization allows getting a general result that only depends on the detuning θ. The renormalised FWHM of the stable CSs is plotted in Figure (2). For $\eta = 0$, the solid lines give the width for the lowest and highest values of I_s for which the CS is stable.

We find that smaller $\mathcal{D}^{(1)}$ correspond to narrower CSs. When taking into account linear nonlocality (dashed lines), the linear dependency is lost and the FWHM saturates around $\xi = 3$ for both positive [Figure 2 (a)] and negative [Figure 2 (b)] values of η. For too large values of $|\eta|$ ($|\eta|^{-1/2} < 2$), the CS branch becomes entirely unstable. This Figure provides us with the minimal width and the corresponding optimal value, which is mainly determined by the scaling law [Eq. (4), (5)] and by the loss of stability.



Figuur 2: The renormalised FWHM in function of $|\eta|^{-1/2}$ for (a) positive η and (b) negative η.

Conclusion

The ultimate physical limit on the width of cavity solitons in a diffraction-compensated system is investigated. This limit is due to the destabilisation of the cavity solitons caused by the nonlocal interaction of the electric field with the left-handed metamaterial in the cavity.

Acknowledgements

We acknowledge support from the Belgian Science Policy Office (project IAP 5/18). L.G. and P.T. are Ph.D. fellows of the Research Foundation – Flanders (FWO). G.V. is a Postdoctoral fellow of the Research Foundation – Flanders (FWO). M. T. is a Research Associate of the Fond National de la Recherche Scientifique (FNRS).

References

[1] S. Trillo and W. Torruellas, Spatial Solitons, Berlin: Springer-Verlag, 2001.

[2] S. Barland, J.R. Tredicce, M. Brambilla, L.A. Lugiato, S. Balle, M. Giudici, T. Maggipinto, L. Spinelli, G. Tissoni, T. Knödl, M. Miller, and R. Jäger, Nature,Vol. 419, pp. 699-702, 2002.

[3] P. Kockaert, P. Tassin, G. Van der Sande, I. Veretennicoff, and M. Tlidi, Phys. Rev. A, Vol. 74, 033822, 2006.

[4] W. J. Firth, G. K. Harkness, A. Lord, J. M. McSloy, D. Gomila, and P. Colet, J. Opt. Soc. Am. B, Vol. 19, pp. 747-752, 2002.

[5] D. Gomila, Dynamics of Spatial Structures in Nonlinear Optics, Ph.D. Thesis, IMEDEA-UIB, 2003.