Complete rate equation modelling for the dynamics of multi-mode semiconductor lasers.

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We propose a rate equations model for the description of a multilongitudinal mode semiconductor laser. We show how usually disregarded effects such as parametric interaction, carrier-diffusion and interference between the fields of the different longitudinal modes play an important role in the final operating characteristics of the laser. Parametric interaction induces a non-trivial distribution of spectral power among different cavity modes. Interference results in nonlinear gain terms in the equations, which favour longer wavelengths versus shorter ones. Carrier-diffusion is regulating the magnitude of such effects. In the limit of very strong diffusion, previous rate equations models are recovered.

Introduction

Nonlinear dynamics in semiconductor lasers is nowadays a hot topic in laser research From an experimental point of view, a broad variety of dynamical regimes have been reported, for instance bystablility and mode hopping, sequential mode switching, and transitions to chaos [1]. Compared to other classes of laser, semiconductor devices have a particular tendency to display multimode dynamics. A recent experiment showed a resonant external cavity semiconductor laser operating in chaotic regime with more than 100 active external cavity modes [2].

From the theoretical point of view, one aims to develop a model that reliably describes the features of the dynamics even in the most complicated regimes such as chaos. Rate equations models are very suitable for this task, as they deal with a limited number of dynamical variables that evolve according to a set of ordinary differential equations (ODE). However, developping a reliable rate equations model for devices operating in multimode regime is a challenging task. In the models available in literature, the mode coupling is described as a simple depletion of a common gain, whereas phase effects induced by the interference between field components in different modes are usually neglected. This rules out important effects related to the parametric interaction. Moreover, other mechanisms of mode coupling induced by spatial hole burning or diffusive processes are often disregarded.

In this paper we propose a set of rate equations for the description of multimode dynamics of semiconductor lasers, which takes into account in a consistent way the phase effects induced by the interference of the electrical fields in different modes, the diffusion of carriers and the spatial hole burning. We predict parametric interaction and symmetry breaking between long and short wavelengths.

Model

For sake of concreteness, we discuss here the model in the case of a single-transverse and multi-longitudinal mode Fabry-Pérot semiconductor laser. It will be clear from the

derivation that the principles presented here are more general and work for a generic class B laser.
For a class B laser, the electric field inside the cavity evolves according to the following differential equation:

$$\frac{\partial}{\partial t}E(z,t) = \sum_j \frac{1}{2}\left(-\Gamma_j + 2i\omega_j + G_j(z,t)\right) E_j(t)\psi_j(z)e^{i\omega_j t}, \tag{1}$$

where $\psi_i(z)$ are the spatial profiles of the cold cavity modes, $E_i(t)$ are the modal amplitudes of the electric field, $G_i(z,t)$ is the modal gain, Γ_i the modal losses and ω_i are the eigenfrequencies of the cavity modes.
The gain profile $G_i(z,t)$ depends on the properties of the semiconductor material and can have a complicate dependence on the carrier inversion $N(z,t)$. In what follow we will consider a device operating not too far from its threshold for which a linear dependence on the inversion applies: $G_i(z,t) = g_i + \xi_i(1+i\alpha_i)N(z,t)$, where g_i is the gain of the mode i when the laser is at threshold, ξ_i is the differential gain and α_i is the linewidth enhancement factor [3]. $N(z,t)$ is the inversion level relative to the laser threshold.
Assuming the eigenfunctions $\psi_i(z)$ to be orthonormal, a set of ODEs for the dynamics of E_n can be obtained by projection of Eq. (1) on the cavity modes:

$$\frac{dE_n}{dt} = -\frac{\Gamma_n E_n}{2} + \frac{1}{2}\sum_j \left(\int_0^L dz\left(g_j + \xi_j\left(1+i\alpha_j\right)N(z,t)\right)\psi_n^*(z)\psi_j(z)e^{i\omega_{jn}t}\right)E_j \tag{2}$$

where the frequencies $\omega_{ij} = \omega_i - \omega_j$ indicate the detunings between different modes.
The integral in Eq. (2) regulates the coupling between different modes; one sees that the coupling intensity is regulated by the overlap of the interference term $\psi_n^*\psi_j$ with the carrier profile $N(z,t)$.
In order to express the dynamics of the modal amplitudes E_i, the evolution of the carrier density $N(z,t)$ must be known. The carrier dynamics is assumed to be described by:

$$\begin{aligned}\frac{\partial}{\partial t}N(z,t) &= J(z,t) - \frac{N(z,t)}{T} + D\frac{\partial^2}{\partial z^2}N(z,t)\\ &- \frac{1}{2}\left[\sum_{nj}\left(\left(g_j + \xi_j\left(1+i\alpha_j\right)N\right)\psi_n^*(z)\psi_j(z)e^{i\omega_{jn}t}\right)E_n^*(t)E_j(t) + c.c.\right]\end{aligned} \tag{3}$$

Here $J(z,t)$ describes the pumping of carriers in the laser junction; the second term on the right hand side describes phenomenologically the spontaneous decay of the inversion, with T being a phenomenological carrier lifetime which summarises a range of different decay processes such as spontaneous radiative decay (decay with emission of radiation), non-radiative decay (for example after interaction with phonons) and Auger recombination (collision of different electrons leading to a loss of energy available for stimulated emission). The third term accounts for the spatial diffusion of the carriers in the active layer, with D the diffusion coefficient. The last terms describe the loss of inversion due to the stimulated emission of photons in the laser. To achieve energy balance, this term must equal the number of stimulated emission photons in (2). Note that, when more field components are present in the cavity, their interference terms contribute to the formation of oscillating gratings in the spatial inversion profile. On the other hand, the presence of

diffusion tends to wash away such gratings, driving the carrier distribution towards a flat profile. The actual spatial profile of the inversion results from the competition of these two effects. In general, one has to expect that the diffusion will be very efficient in washing away spatial profiles containing many ripples and less efficient for smoother profiles.
We choose to expand the carrier distribution on an orthonormal set of base functions ϕ_n: $N(z,t) = \sum_j N_j(t)\phi_j(z)$. The diffusive kernel in Eq. (3) introduces a diffusion-induced decay of the different components N_j; in particular, N_j decays fast if the corresponding profile ϕ_j contains many ripples; on the other hand, when ϕ_j is relatively flat, the corresponding N_j decays more slowly. Thus, during the operation of the device, the actual inversion profile will be sufficiently smooth to be reliably described with a finite number of components N_j. From the dynamical point of view, we can state that only a limited number of N_j can actively participate in the laser dynamics.
Performing the projection of Eq. (3) on the base functions ϕ_n, one obtains the following set of coupled ODEs:

$$\frac{dN_n}{dt} = J_n - \frac{N_n}{T} + \int dz D \frac{\partial^2 N}{\partial z^2}\phi_n - \frac{1}{2}\sum_{jk} g_j f_{jkn} e^{i\omega_{kj}t} E_j^* E_k - \frac{1}{2}\sum_{jk} g_j f_{kjn} e^{i\omega_{jk}t} E_k^* E_j$$
$$+\frac{1}{2}\sum_{jkh}\xi_j\left(1+i\alpha_j\right) N_h f_{jknh} e^{i\omega_{kj}t} E_j^* E_k + \frac{1}{2}\sum_{jkh}\xi_j\left(1-i\alpha_j\right) N_h f_{kjnh} e^{i\omega_{jk}t} E_k^* E_j. \quad (4)$$

Where the phase matching coefficients f_{ijk} and f_{ijkh} have been defined as $f_{ijk} = L\int \psi_i^* \psi_j \phi_k dz$, $f_{ijkh} = L\int \psi_i^* \psi_j \phi_k^* \phi_h dz$. With these definitions, the final set of equations for the modal fields amplitudes is:

$$\frac{d}{dt}E_n = -\frac{1}{2}\left(\Gamma_n - g_n\right) E_n + \frac{1}{2}\sum_{jk} f_{njk}\xi_j\left(1+i\alpha_j\right) N_k E_j e^{i\omega_{jn}t}, \quad (5)$$

which, in combination with Eq. (4) is the complete set of multimode laser rate equations.

Numerical investigation of the model

Consider the case of a two-mode system. We assume the diffusion to be fast enough to wash away all components of the carrier density except the two first. The component N_0 describes a common gain available for both modes; the component N_1 regulates the parametric coupling between the modes.
Let the modal parameters be the same for both modes; the equations for the two modes are then completely symmetric and can be distinguished only by their optical frequency. It can be verified from Eqs. (4) and (5) that the following single mode operation scenarios are possible: $|E_{1,2}|^2 = \frac{J_0}{\Gamma}$, $|E_{2,1}|^2 = 0$, $N_0 = 0$ and $N_1 = 0$.
As expected by the symmetry in the device equations, both solutions correspond to the same lasing intensity; however, the stability for these two solutions is different: the solution corresponding to operation in the longer wavelength mode is robust versus fluctuations in the other mode, whereas, the operation in the shorter one is unstable: a fluctuation in the mode with longer wavelength will grow until the short wavelength operation will be completely suppressed. We illustrate this effect in Fig. 1 (left), where we show a 80ns time series obtained by direct integration of (4,5) by predictor-corrector algorithm. The system switches spontaneously from operation at short wavelength to operation at long

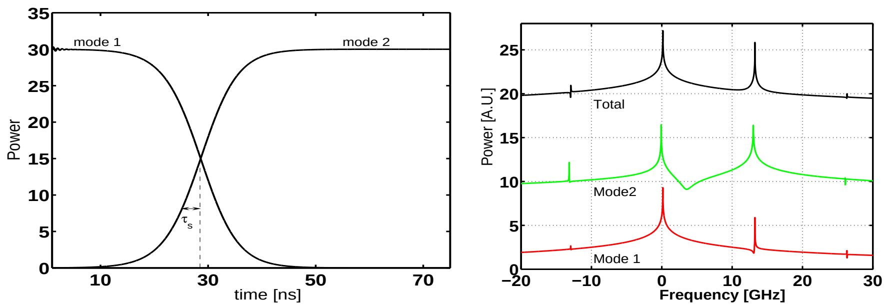

Figure 1: (Left) Switch of lasing mode due to modal interaction: the frequency detuning between the two modes is chosen to be 80GHz. The diffusion time for the grating is $T_1 = 100$ps. Mode 1 correspond to the shortest wavelength. The switching time τ_s is indicated by the double arrow. (Right) Mode resolved spectra and total optical spectrum.

wavelength. In Fig. (1) (right) we show the optical spectrum of the laser when two modes are contemporary lasing in the cavity. As expected, two peaks are present in the total spectrum, however, despite the gain being the same for both modes, the spectral amplitudes are different and the laser favours the longer wavelength over the shorter one. This result is consistent with the experimental work of Bogatov et al. [4]. When the spectral contents of the cavity modes are considered, it appears that both eigenfrequencies are present in the dynamics of each of the modal profiles, in accordance with Eq. (5).

Conclusions

We presented a set of rate equations for the description of a generic class B laser operating in multimode regime. The intrafield interactions and the field-inversion interaction take into account the interference terms between different modes. We predict a different stability for different wavelengths, an asymmetric optical spectrum where the longer frequencies are enhanced compared to the short ones, and a non-trivial distribution of optical power among the different frequencies. The diffusion coefficient regulates the magnitude of these effects.

Previous rate equations models can be obtained from Eq. (4,5) by taking the limit of very strong diffusion. In this case, only the inversion component N_0 survives in Eq. (4) and the effects of parametric interaction, spatial hole burning and diffusion disappear.

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