# Sensitivity Analytic Calculation of Rib Waveguides for Sensing Applications

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*To calculate sensitivity of rib waveguides designed for integrated optical sensors, an analytic approach based on well-known Effective Index Method has been developed. To validate the proposed formalism, sensitivity of different rib guiding structures in CMOS compatible technologies has been calculated by developed formulas and by a numerical procedure based on Finite Element Method. Agreement is demonstrated to be good in all considered cases.*

## Introduction

Integrated optical devices are increasingly being used in a great number of sensing applications. This interest is due to their immunity to electromagnetic interference, high sensitivity, good compactness and robustness and high compatibility with fiber networks.

In the last years, different kinds of guided-wave optical sensors have been designed and fabricated, adopting different architectures such as those based on directional couplers [1], Mach–Zehnder interferometers [2-3], grating-coupled waveguides [4], and micro-resonators [5]. A lot of these devices have been fabricated using CMOS compatible technological processes.

In integrated optical sensors the shift in measured chemical/physical quantity produces a change of cover medium refractive index. This change affects the propagating mode effective index. Then, optical field distribution in the guiding structure cross section heavily affects the device sensitivity, so one of the most important design task is the waveguide optimization in order to maximize this parameter.

In a number of integrated optical sensors, a rib waveguide having a width *w*, a total height $h_1$ and an etch depth $h_1$-$h_2$, is adopted, as shown in Fig. 1(a).

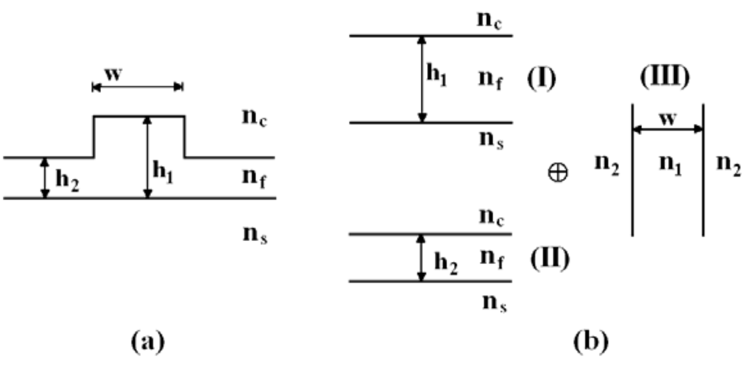

Fig. 1. (a) Rib waveguide structure; (b) Effective-index method applied to the rib waveguide.

Waveguide sensitivity is usually expressed as:

$$S_w = \frac{\partial N}{\partial n_c} \tag{1}$$

where $N$ is the effective index of a mode propagating in the rib waveguide and $n_c$ is the refractive index of cover medium. In [6], a normalized analytic formalism has been introduced to optimize the sensitivity of slab waveguides.

In this paper we propose an analytic procedure to calculate the sensitivity of a rib waveguide, to be included in an integrated optical sensor. Our approach is based on the application of Effective Index Method (EIM) to the guiding structure (see Fig. 1(b)). Results are compared with those derived by a numerical rigorous procedure carried out by full-vectorial Finite Element Method (FEM) [7].

## Analytic estimation of sensitivity

Two analytic expressions for rib waveguide sensitivity can be obtained by applying EIM [8] for quasi-TE and quasi-TM polarization, respectively:

$$S_w^{TE} = \frac{1}{N}\frac{Q + k_0 w n_2^3 A_1^{TE}\left(n_1 a_2\sqrt{a_2} + n_2 a_1\sqrt{a_2}\right)}{k_0 w n_2^3\sqrt{a_2}\left(n_2 a_1 + n_1 a_2\right) + 2a_1^2 a_2^2\left(a_1 + a_2\right)} \tag{2}$$

$$S_w^{TM} = \frac{1}{N}\left[A_1^{TM} + \frac{2a_1\left(A_2^{TM} - A_1^{TM}\right)}{\left(a_1 + a_2\right)\left(2 + k_0 w\sqrt{a_2}\right)}\right] \tag{3}$$

and

$$Q = 4a_1 a_2 A_2^{TE} n_1^2 - 4a_1 a_2 A_1^{TE} n_2^2 + 2a_2 A_1^{TE} a_1^2 a_2^2 + 2a_1 A_2^{TE} a_1^2 a_2^2 \tag{4}$$

$$a_1 = n_1^2 - N^2 \tag{5}$$

$$a_2 = N^2 - n_2^2 \tag{6}$$

$$A_i^{TE} = \frac{n_c}{\sqrt{c_i}\left(1 + \frac{c_i}{f_i}\right)\left(h_i k_0 + \frac{1}{\sqrt{c_i}} + \frac{1}{\sqrt{s_i}}\right)} \qquad \text{i=1,2} \tag{7}$$

$$A_i^{TM} = \frac{\left(2c_i + n_c^2\right)\left(\frac{c_i n_f^4 + f_i n_c^4}{f_i n_c^4}\right)^{-1}\frac{1}{n_c^3\sqrt{c_i}}}{\left[\frac{h_i k_0}{n_f^2} + \frac{\left(c_i + f_i\right)n_c^2}{\left(c_i n_f^4 + f_i n_c^4\right)\sqrt{c_i}} + \frac{\left(s_i + f_i\right)n_s^2}{\left(s_i n_f^4 + f_i n_s^4\right)\sqrt{s_i}}\right]} \qquad \text{i=1,2} \tag{8}$$

$$c_i = n_i^2 - n_c^2 \qquad \mathrm{i} = 1,2 \tag{9}$$

$$s_i = n_i^2 - n_s^2 \qquad \mathrm{i} = 1,2 \tag{10}$$

$$f_i = n_f^2 - n_i^2 \qquad \mathrm{i} = 1,2 \tag{11}$$

where $k_0$ is the vacuum wave-number, $n_1$ is the effective index of a mode propagating in the slab waveguide with height $h_1$ (waveguide I in Fig. 1(b)), $n_2$ is the effective index of a mode propagating in the slab waveguide with height $h_2$ (waveguide II in Fig. 1(b)), $n_c$, $n_f$ and $n_s$ are rib waveguide cover, core and substrate refractive indices, respectively.

## Analytic method validation

Rib waveguide sensitivity, as defined in (1), can be also numerically estimated by a Finite Element Method (FEM) based approach. This approach is based on rigorous calculation of cover medium refractive index $n_c$ change in a narrow range and the relevant shift of effective index $N$. To estimate the effective index $N$, the full-vectorial FEM has been used.

To validate the proposed analytical method, sensitivity of two waveguides adopting silicon oxide ($SiO_2$) as substrate and silicon oxynitride (SiON) and silicon nitride ($Si_3N_4$) as guiding film, has been investigated. As cover medium water (whose refractive index can be changed by mixing it with ethanol [3]) has been considered. An operating wavelength $\lambda = 633$ nm has been assumed in both cases.

For quasi-TE and quasi-TM modes, waveguide sensitivity dependence on etch depth ($ED = h_1$-$h_2$) has been investigated by proposed analytic formula and by FEM-based numerical approach. Comparisons are sketched in Fig. 2 and Fig. 3.

Waveguide sensitivity calculated by both analytical and numerical approaches exhibits similar dependence as a function of etch depth. Moreover, the moderate difference between results obtained by both methods remains quite constant. For silicon nitride waveguides, we obtain a maximum difference of 15% for quasi-TE and 10% for quasi-TM.

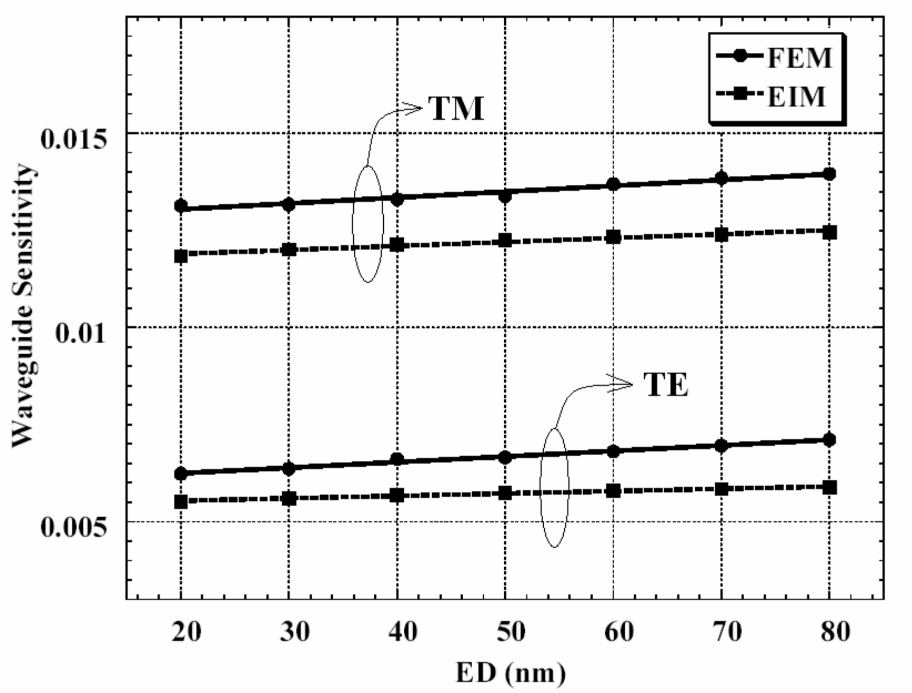

Fig. 2. Silicon nitride waveguide sensitivity dependence on *ED*, estimated by proposed analytic approach and by FEM-based numerical method, for quasi-TE and quasi-TM modes. Other parameters are $w$= 1000 nm, $h_1$= 600 nm, $n_s$= 1.46, $n_f$= 2 and $n_c$= 1.33.

In case of silicon oxynitride waveguides, we have a maximum difference of 11% for quasi-TE and 10% for quasi-TM. It is clear that the behavior with polarization still remains the same for smaller index contrast structures (as in Fig. 3).

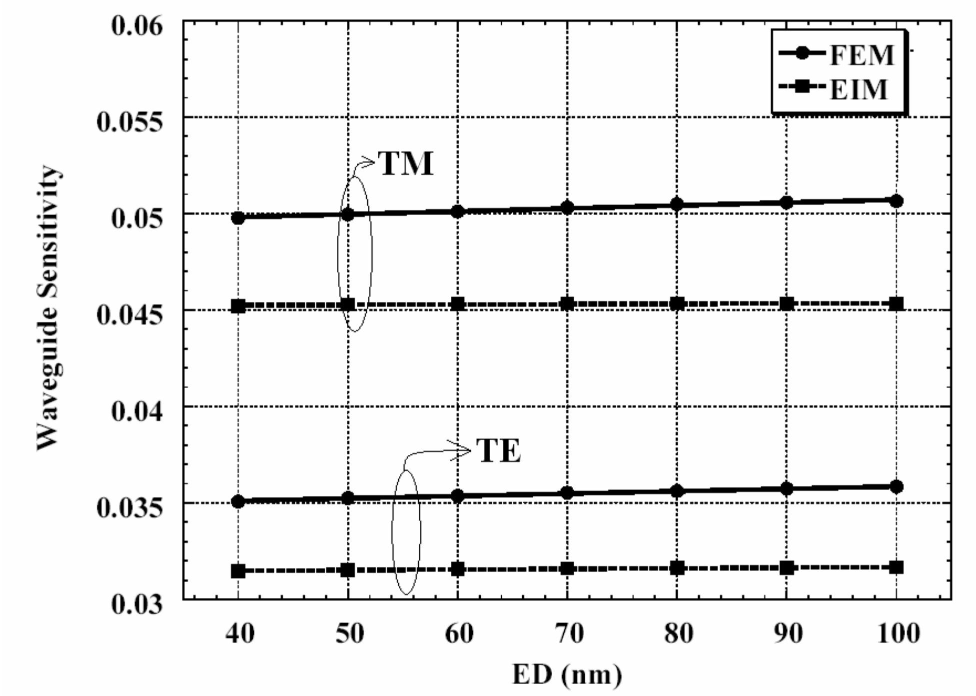

Fig. 3. Silicon oxynitride waveguide sensitivity dependence on *ED*, estimated by proposed analytic approach and by FEM-based numerical method, for quasi-TE and quasi-TM modes. Other parameters are $w$ = 2000 nm, $h_1$ = 500 nm, $n_s$ = 1.46, $n_f$ = 1.55 and $n_c$ = 1.33.

## Conclusion

In this paper an analytic approach based on effective index method is applied for the estimation of sensitivity to cover refractive index change in rib waveguides for sensing applications. The results are in good agreement with those obtained by a rigorous numerical approach based on full-vectorial finite element method.

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