The Asymptotic Properties of Electromagnetic Pulse Transmission in Photonic Crystals by Saddle-point Analysis

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We calculate the initial electromagnetic field that has been transmitted through a finite one dimensional photonic crystal in terms of the parameters of the crystal. The shape of the Sommerfeld precursor merely depends on the spatial average of the refractive indices of the homogeneous constituents of the crystal, whereas the shape of the Brillouin precursor depends in a complicated manner on the index contrast.

Introduction

A photonic crystal (PC) [1] is a spatially repeated geometrical structure (crystal) of various material compounds that each individually have in general a different interaction strength with an applied electromagnetic (EM) field. Therefore this field experiences a periodically varying potential. Exactly as in the case of electrons in interaction with a periodic atomic lattice where there are band-gaps in the electron dispersion relation due to Bragg scattering, the lattice of material compounds in PCs gives rise to photonic band-gaps (PBGs). Therefore, with a PC, it is in principle possible to manipulate the propagation of an EM field at the very small scale of its own wavelength.

In order to quantify the effect of a PC on a transmitted EM-pulse, the dependence of the shape (instantaneous amplitude and period) of the transmitted pulse in terms of the parameters of the PC must be calculated. However, since the frequency integral equation for the transmitted field is not analytically solvable, one is directed to search for approximate solutions which usually give good results for the transmitted field only in a restricted range of the time domain.

In the case of pulse propagation in a *homogeneous* medium [2], the saddle-point method is a successful technique to calculate approximately the so-called asymptotic field after a certain propagation distance in the medium. Here with asymptotic field is meant the initial field, immediately behind the wavefront: this is the restriction in the time domain that results from the choice to consider only those frequency components that interact weakly with the system. The saddle-point technique can be extended to *inhomogeneous* media with the concept of equivalent homogeneous index of refraction [3]. Thus we derive an expression of the asymptotic transmitted field in terms of the parameters of the PC.

A substantial drawback of our approach is that it gives only accurate results for the initial transmitted field; we did not obtain results for the more interesting 'main part' of the pulse, where usually the strong interaction with the crystal takes place.

Model

Our model for a one dimensional PC is the rectangular N-layer dielectric, nonmagnetic medium (NLM), depicted in Fig. 1. It is surrounded by another dielectric nonmagnetic medium.

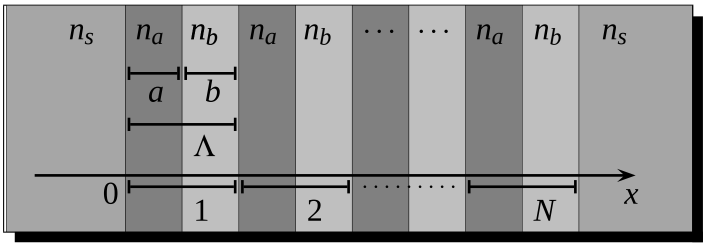

Figure 1: The N-layer medium and its surroundings.

The parameters of this system can be divided into two types. There are *geometrical* parameters that scale the interaction due to scattering of the EM-field at interfaces: these are the slab widths a and b, that add up to the layer width, $a+b=\Lambda$, and N, the total number of layers. The other type of parameters are *material* parameters that describe the (macroscopic) interaction of the EM-field with the atoms: these are the permittivity ε for the electric component and the permeability μ for the magnetic component, or, equivalently, the refractive index $n=\sqrt{\varepsilon\mu}$ and the impedance $Z=\sqrt{\frac{\mu}{\varepsilon}}$. However, for nonmagnetic media, $\mu=1$ and the material interaction can be stated in terms of the refractive index only. The homogeneous refractive indices of the slabs are n_a and n_b and of the surrounding medium is n_s.

In information technology, often mentioned as a significant application area for PCs, information is usually encoded in *optical* EM-pulses. We will therefore consider optical applied pulses and PCs that have a PBG in the optical range of the frequency spectrum. In order to obtain a band-gap in the visible part of the EM-spectrum, where the frequency $\omega \sim 10^{15}$rad/s, Λ should be of the order of 10^{-7}m, assuming that all refractive indices are of order 10^0.

The two restrictions that we impose on our applied EM-pulse, coming in from the left-hand-side of Fig. 1, are that it lasts a finite amount of time T (measured at the entrance plane $x=0$) and that its amplitude is finite. An example pulse is depicted in Fig. 2.

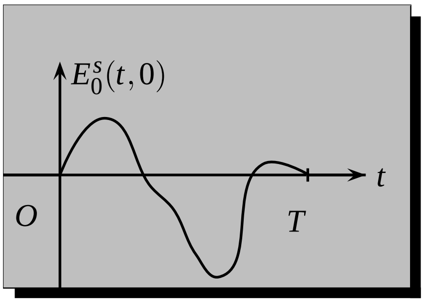

Figure 2: Time dependence of electric field component, evaluated at entrance plane of NLM, of an example applied pulse

The electric component of the EM pulse is in a plane wave decomposition

$$E_0^s(t,x)=\int \mathrm{d}\omega \tilde{E}_0^s(\omega)\mathrm{e}^{-\mathrm{i}\omega(t-n_s(\omega)x/c)}, \qquad (1)$$

where the spectral density is

$$\tilde{E}_0^s(\omega)=\frac{\sum_m}{2\pi}\frac{\omega_m E_{0m}}{\omega^2-\omega_m^2}\left((-1)^m\mathrm{e}^{\mathrm{i}\omega T}-1\right). \qquad (2)$$

Here, E_{0m} denotes the amplitude at carrier frequency

$$\omega_m=\frac{m\pi}{T}. \qquad (3)$$

For purely optical pulses, the E_{0m} takes on nonzero values only at carrier frequencies that lie in the optical range.

Theory

The exact expression for the transmitted field, evaluated at the plane $x=N\Lambda$, is, in mono-exponential form,

$$E_N^s(t,N\Lambda)=\int \mathrm{d}\omega \tilde{E}_0(\omega)\mathrm{e}^{\frac{N\Lambda}{c}v_N(\omega)}, \qquad (4)$$

in which the propagation is described by the function

$$v_N = -\mathrm{i}\omega\left(\Theta_N - n_N\right) \equiv X_N + \mathrm{i}Y_N \tag{5}$$

with X_N, Y_N real. Here, $\Theta_N = \frac{ct}{N\Lambda}$ is the time in natural units and

$$n_N = \frac{-\mathrm{i}c}{N\Lambda}\frac{1}{\omega}\log t_N \tag{6}$$

is the equivalent homogeneous index of refraction for the NLM, with t_N the transmission coefficient.

The integration path in Eq. (4) is deformed such that X_N is minimal. Denote the deformed path as S. This path S has been sketched in Fig. 3 in a small environment (circle) around the SP, taken on at frequency ω_s. On S, the SP of X_N are crossed along lines of steepest descent (SD). Dominant contributions in Eq. (4) then come from the peaks of X_N around SP. These peaks are sharp when the scaling parameter $\frac{N\Lambda}{c}$ is 'large'. Denote the locations of SPs in the complex frequency plane as ω_s. When ω_s is not too close to the optical ω_m where E_{0m} is nonzero, then the spectral density $\tilde{E}_0^s$ is nearly constant around this SP (that is, for $\omega \simeq \omega_s$) and satisfies $\tilde{E}_0^s(\omega) \simeq \tilde{E}_0^s(\omega_s)$. In that case, Eq. (4) is approximately equal to a Gaussian integral,

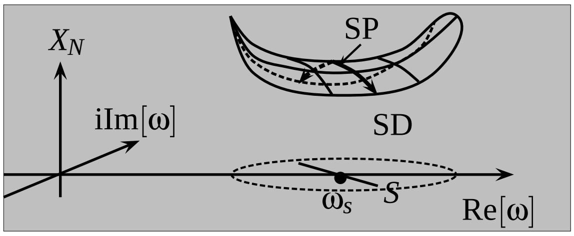

Figure 3: The integration path S near a saddle-point

$$E_N^s(t, N\Lambda) \simeq \tilde{E}_0(\omega_s)\mathrm{e}^{\frac{N\Lambda}{c}v_N(\omega_s)}\int_S \mathrm{d}\omega \mathrm{e}^{\frac{N\Lambda}{c}v_N''(\omega_s)(\omega-\omega_s)^2}. \tag{7}$$

This approximation does therefore not give contributions from frequency components close to, or inside, the optical PBG.

Results

For the very large-frequency components (near SP with $|\omega_s| >>$optical frequencies), the effective index may be expanded about infinite frequency. Up to second order, this series is

$$n_N(\omega) = n_N(\infty) + \left(-\omega^2 n_N'\right)|_{|\omega|=\infty}\omega^{-1} + \frac{1}{2!}\left(\omega^3\left(\omega n_N'' + 2n_N'\right)\right)|_{|\omega|=\infty}\omega^{-2}. \tag{8}$$

For Lorentz type slab refractive indices, we have calculated,

$$n_N(\infty) = 1, \quad \left(-\omega^2 n_N'\right)|_{|\omega|=\infty} = 0, \quad \left(\omega^3\left(\omega n_N'' + 2n_N'\right)\right)|_{|\omega|=\infty} = \omega_{p\Lambda}^2, \tag{9}$$

where $\omega_{p\Lambda}^2 \equiv \frac{a\omega_{pa}^2 + b\omega_{pb}^2}{\Lambda}$ is the average plasma frequency squared. These components give rise to the Sommerfeld precursor, starting at times $\delta_N^S \equiv \Theta_N - n_N(\infty) \geq 0$. Its amplitude is found as

$$E_N^s(t, N\Lambda) = \sum_m \frac{\omega_m}{\omega_{p\Lambda}} E_{0m}\sqrt{2\delta_N^S}J_1\left(\omega_{p\Lambda}\frac{N\Lambda}{c}\sqrt{2\delta_N^S}\right), \tag{10}$$

where J_1 denotes the first order Bessel function.

For the very small-frequency components (near SP with $|\omega_s| <<$optical frequencies), n_N may be expanded about zero frequency,

$$n_N(\omega) = n_N(0) + n'_N(0)\omega + \frac{1}{2!}n''_N(0)\omega^2, \quad |\omega| << \text{optical frequencies} \tag{11}$$

where we have calculated the coefficients as

$$n_N(0) = \left(\frac{n_\Lambda^2 + n_s^2}{2n_s}\right)\Big|_{\omega=0}, \qquad \left(n_\Lambda^2 \equiv \frac{an_a^2 + bn_b^2}{\Lambda}\right) \tag{12}$$

$$n'_N(0) = \left(\frac{n_\Lambda^2 + n_s^2}{2n_s}\right)'\Big|_{\omega=0} + \frac{\mathrm{i}N\Lambda}{2c}\left(\frac{n_\Lambda^2 - n_s^2}{2n_s}\right)^2\Big|_{\omega=0}, \tag{13}$$

$$n''_N(0) = \left(\frac{n_\Lambda^2 + n_s^2}{2n_s}\right)''\Big|_{\omega=0} + \frac{\mathrm{i}N\Lambda}{c}\left(\left(\frac{n_\Lambda^2 - n_s^2}{2n_s}\right)^2\right)'\Big|_{\omega=0} \tag{14}$$

$$-\frac{2}{3}\frac{N^2\Lambda^2}{c^2}\left(\frac{n_\Lambda^2 + n_s^2}{2n_s}\right)\left(\frac{n_\Lambda^2 - n_s^2}{2n_s}\right)^2\Big|_{\omega=0} + \frac{ab}{3c^2}\left(n_a^2 - n_b^2\right)\left(\frac{\frac{an_a^2 - bn_b^2}{\Lambda} - \frac{a-b}{\Lambda}n_s^2}{2n_s}\right)\Big|_{\omega=0}. \tag{15}$$

Note that the slab index difference is only in term $n''_N(0)$. The small-$|\omega|$-components give rise to the Brillouin precursor at times $\delta_N^B \equiv \delta_N^S - n_N(0) \geq 0$,

$$E_N^s(t, N\Lambda) \simeq \sqrt{\frac{2c}{\pi N\Lambda\sqrt{D_N}}}\sum_m \frac{E_{0m}}{\omega_m}\mathrm{e}^{\frac{N\Lambda}{c}X_N(\omega_s)}\cos\left(\frac{N\Lambda}{c}Y_N(\omega_s) + \pi/4\right), \tag{16}$$

where the SP-location $\omega_s = \mathrm{i}\left(\frac{2\mathrm{i}n'_N(0)}{3n''_N(0)}\right) + \frac{\sqrt{D_N}}{3n''_N(0)}$ with $D_N = 4n'_N(0)^2 + 6n''_N(0)\delta_N^B$.

Conclusions

We have calculated the initially transmitted field through an NLM, expressed in the parameters of the NLM. As in the case of a homogeneous medium, the wave-front propagates at the speed of light c, immediately followed by the Sommerfeld precursor. This precursor experiences the spatial average of the slab indices: it is not influenced by the slab contrast. The arrival time of the Brillouin precursor is also not influenced by the slab contrast, but its shape is: roughly, its amplitude and period decrease with contrast.

References

[1] J.D. Joannopoulos, R.D. Meade and J.N. Winn, Photonic Crystals, Princeton University Press, Princeton, 1995.

[2] L. Brillouin, Wave propagation and group velocity, Academic Press, New York and London, 1960

[3] M. Centini, C. Sibilia, M. Scalora, G. D'Aguanno, M. Bertolotti, M.J. Bloemer, C.M. Bowden and I. Nefedov, *Phys. Rev. E.* Vol. 60, Nr. 4-B, 1999.