Spatial Effects in the Emitted Field of Zero Diffraction Resonators

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In a previous work, we have devised a way to eliminate diffraction from optical resonators by introducing left-handed materials – or negative index materials – in an optical cavity. Such zero diffraction resonators are promising devices, because they can potentially generate sub-diffraction-limited light beams. However, when diffraction vanishes completely, other physical effects will determine the spatial structure of the emitted field. In this work, we derive models to describe these effects and we study the dynamical behaviour of zero diffraction resonators. We address two different contributions: the inherent nonlocality of the left-handed metamaterial and higher order cavity effects.

Introduction

Metamaterials, i.e. materials with an artificial internal structure, promise to provide previously unexplored electromagnetic behaviour. In recent years, researchers have been able to fabricate metamaterials with negative permittivity and permeability (see Ref. [1] for an overview of these so-called left-handed materials). Exotic electromagnetic wave propagation such as negative refraction in left-handed materials has been predicted [2] and demonstrated [3]. Moreover, metamaterials offer possibilities for new applications, such as sub-wavelength imaging [4] and electromagnetic cloaking of objects [5].

Left-handed materials can also be used to improve optical devices. In previous work [6], we have proposed a double layered nonlinear Fabry-Perot resonator containing both right-handed and left-handed materials (see also Fig. 1). We have derived a mean-field model for this resonator, according to which the time evolution of the envelope A of the electric field of the output field is given by

$$\frac{\partial A}{\partial t} = A_{\text{in}} - \left(1 + \mathrm{i}\Delta\right) A + \mathrm{i}\left|A\right|^2 A + \mathrm{i}\alpha\nabla_\perp^2 A. \qquad (1)$$

In this equation, A_{in} is the amplitude of the incident wave and Δ is the detuning between the frequency of the optical carrier and the nearest cavity mode. We have shown that diffraction acts in the left-handed material layer with the opposite sign as in the right-handed layer. Therefore, it is possible to tune the value of the diffraction coefficient α to either positive or negative values. By careful adjustment of the layer thicknesses l_R and l_L, it should also be possible to achieve complete diffraction compensation between the layers, thus eliminating diffraction from the resonator. In this case, the diffraction coefficient α in Eq. (1) becomes zero.

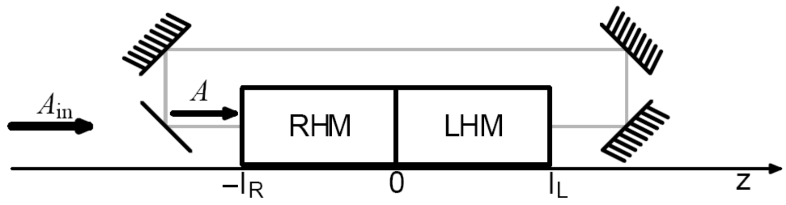


Fig. 1: Scheme of an optical resonator with diffraction compensation between the layer of left-handed material (LHM) and the layer of right-handed material (RHM).

Sub-Diffraction-Limited Light Beams

A nonlinear optical resonator, such as the ring cavity depicted in Fig. 1, is a dissipative system far from equilibrium, in which the interaction between diffraction and nonlinearity can result in modulational – or Turing – instability. This means that small fluctuations can lead to the formation of dissipative structures in the emitted field of the resonator [7]-[8]. In such systems, the typical feature size of the emerging structures is not determined by the boundary conditions, but rather by the strength of diffraction. From the scaling of the space coordinates in Eq. (1), it can be seen that the larger the diffraction coefficient α, the larger the size of the patterns. By tuning the diffraction coefficient to arbitrarily small values as discussed above, one could generate dissipative structures with ever smaller feature sizes.

However, the Lugiato-Lefever equation [Eq. (1)] is no longer appropriate for small values of α. Indeed, the derivation of this equation relies on two consecutive perturbation expansions [5]. In a first step, the fast optical oscillations of the light field are separated from the slower variations of the field envelope A. In a second step, the time evolution of this envelope is expanded with respect to the cavity length under the assumption of small changes during one roundtrip. In both expansions, all but the first-order terms are neglected. When diffraction – which is the only spatially dependent first-order term – is eliminated, higher order terms should be taken into account. In particular, we have identified two physical effects contributing to these higher terms: the inherent nonlocality of the left-handed material and higher-order cavity effects.

Nonlocal Interactions in the Left-Handed Material

Most left-handed materials contain a large number of very small resonators interacting with the electromagnetic fields, e.g. tiny wires and split-ring resonators in Smith's first metamaterial [3]. These interactions contribute to the value of the permittivity and permeability and can make them negative. Typically, the resonators are spaced extremely closely, resulting in interactions between neighbouring resonators. Consequently, metamaterials should be modelled with nonlocal constitutive relations:

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0 \iiint \varepsilon_r(\mathbf{r}-\mathbf{r}')\mathbf{E}(\mathbf{r}')\,\mathrm{d}\mathbf{r}', \quad \mathbf{B}(\mathbf{r}) = \mu_0 \iiint \mu_r(\mathbf{r}-\mathbf{r}')\mathbf{H}(\mathbf{r}')\,\mathrm{d}\mathbf{r}'. \tag{2}$$

When these relations are introduced in Maxwell's equations and the optical carrier is removed from the model with a multiple scales analysis, one finds the classical nonlinear Schrödinger equation with a number of additional terms. However, symmetry considerations reveal that many of these terms vanish. Applying the normal mean-field approximations, we have finally found the following generalisation of Eq. (1):

$$\frac{\partial A}{\partial t} = A_{\text{in}} - (1 + \text{i}\Delta) A + \text{i}|A|^2 A + \text{i}\alpha\nabla_\perp^2 A + \text{i}\beta\nabla_\perp^4 A. \quad (3)$$

The nonlocality thus gives rise to a bilaplacian with real coefficient. Gelens *et al.* have shown that this equation has localised solutions and have investigated the size limits on these solutions imposed by the new term [9].

Higher Order Cavity Effects

When the typical space scales of nonlinearity and diffraction are larger than the cavity length, the nonlinear Schrödinger equation can be integrated approximately by considering its right-hand side as constant. The propagation operator for the field envelope obtained in this way can then be expanded as a Taylor series in the layer widths l_R and l_L:

$$e^{i(\text{D}+\text{N})l_{R,L}} = 1 + \text{i}\,l_{R,L}\,\text{D} + \text{i}\,l_{R,L}\,\text{N} - \frac{1}{2} l_{R,L}^2\,\text{D}^2 + \ldots, \quad (4)$$

where D and N are operators associated to diffraction and nonlinear effects (which we assume here to be Kerr-like), respectively:

$$D = \frac{\text{i}}{2k_{R,L}}\nabla_\perp^2, \qquad N(A) = |A|^2. \quad (5)$$

In the mean-field approximation, only the first three terms of Eq. (4) are kept in the analysis. However, when the second term vanishes due to elimination of diffraction, the next higher order spatially dependent term should also be taken into account. This leads to the following generalised mean-field equation:

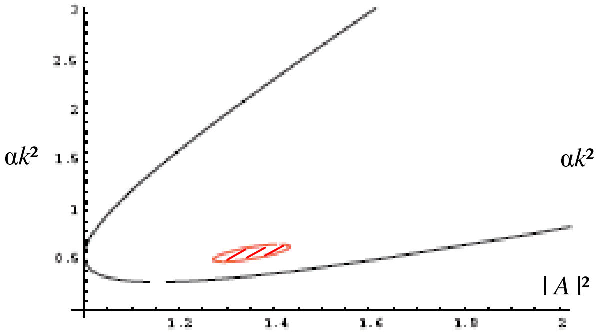
$$\frac{\partial A}{\partial t} = A_{\text{in}} - (1 + \text{i}\Delta) A + \text{i}|A| A + \text{i}\alpha\nabla_\perp^2 A - \frac{1}{2}\alpha^2\nabla_\perp^4 A. \quad (6)$$

The correction due to higher cavity effects thus leads to a bilaplacian term in the mean-field equation too, be it in this case with a real coefficient. It is important to note that the coefficient of the bilaplacian is proportional to α^2. This means that this effect will not change the traditional scaling law of dissipative structures: their typical width is proportional to the square root of the diffraction coefficient α. However, the stability will of course be altered.

Stability Analysis

We have carried out the linear stability analysis of the mean-field equation with either of both corrections. The stability of Eq. (3) with an imaginary bilaplacian term has been studied in detail in Ref. [9], where also the size limitation on localised solutions of this equation are investigated.

We have also analysed the linear stability of Eq. (6), which has a real bilaplacian term. The effect of this term is mainly to stabilise the homogeneous solution. For $\Delta < 1.44$, the homogeneous solution is now completely stable. For higher values of the detuning, the marginal stability curve (see Fig. 2) closes, reducing the range of unstable modes and stabilising the homogeneous solution for high intensities.



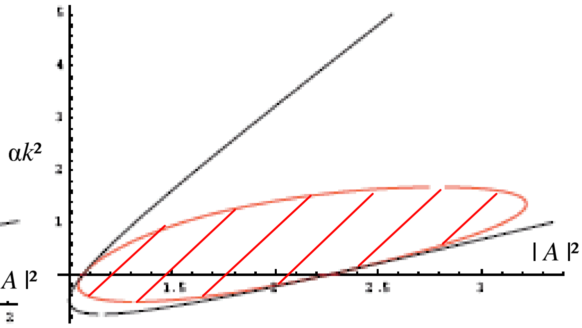


Fig. 2: Marginal stability of the mean-field equation. The open curves correspond to the original equation [Eq. (1)], whereas the closed curves are for the equation with the real bilaplacian term [Eq. (6)]. The detuning is taken equal to $\Delta = 1.45$ (left graph) and $\Delta = 2.5$ (right graph). Unstable regions are hatched.

Conclusions

We have studied the zero diffraction regime of a nonlinear optical resonator containing right- and left-handed materials. When the strength of diffraction in this system is tuned to very small values, the Lugiato-Lefever equation breaks down, and other spatial effects come into play. We identified two contributions, coming from the nonlocal interaction in the left-handed material between the polarisation field and the electric field of light, and from higher order cavity effects. Only the first effect changes the size scaling law of optical patterns and localised structures explicitly, but both influence their formation by changing the stability of the homogeneous solution.

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