Sub-Diffraction Light Structures in Optical Parametric Oscillators

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Previously, we have proposed a method of diffraction management in optical resonators with left-handed materials. This method is now applied to parametric oscillators. We show that it is possible to engineer the sign and strength of diffraction for either the pump or signal beam – not both. We study the influence of the altered diffraction on the periodic structures and phase solitons emerging from the resonator's output mirror. We also show that our scheme of diffraction management allows for sub-diffraction-limited structures, even if only the signal diffraction can be reduced. This effect is attributed to the homogeneity of the pump field.

Introduction

Left-handed materials, i.e., materials with negative permittivity and permeability, have received a lot of attention recently. Although it was shown as early as 1968 that these materials exhibit interesting properties including negative refraction [1], it was only in 2001 that the first left-handed material for microwaves was demonstrated experimentally [2]. Their extraordinary electromagnetic properties are typically derived from their artificial sub-wavelength structure containing various electric circuits to provide the electric and magnetic dipole moments leading to negative permittivity and permeability. During the last few years, researchers have scaled down metamaterials to optical wavelengths and have proposed them for various applications, such as electromagnetic cloaking [3] and sub-wavelength imaging [4]. See Ref. [5] for a more detailed review of left-handed materials.

Previously, we have investigated the use of left-handed materials in the cavity of a resonator with dispersive Kerr nonlinearity, for which we have shown that the compensation in optical path length due to the left-handed material allows engineering the magnitude as well as the sign of the diffraction coefficient [6]. By decreasing the diffraction coefficient to small values, the size of dissipative structures and cavity solitons in such resonators can become sub-diffraction-limited, although nonlocal effects will impose a new limit on their size [7].

Here, we want to investigate if this method of diffraction management can be generalised to other nonlinear components such as a degenerate optical parametric oscillator. More specifically, we consider the structure of Fig. 1. A ring cavity containing a material with second-order nonlinearity and a left-handed material is illuminated with a pump beam of frequency ω. The nonlinear crystal converts energy from the pump beam to a signal beam at frequency ω/2, whereas the left-handed material modifies the optical path length of the signal beam as explained below.

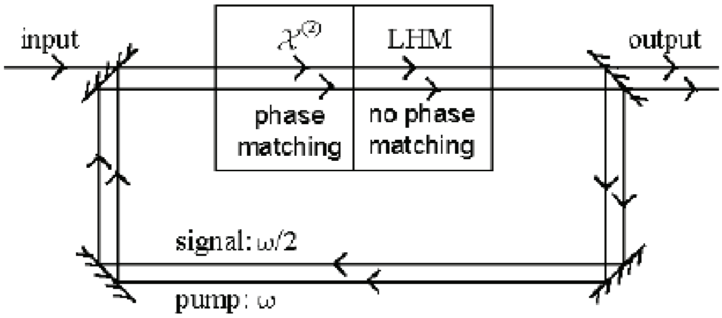


Fig. 1: A degenerate optical parametric oscillator in which a crystal with $\chi^{(2)}$ nonlinearity converts energy from a pump beam to a signal beam. The left-handed material (LHM) modifies the linear propagation properties of the signal beam.

Diffraction Management in Optical Parametric Oscillators

Starting from a set of nonlinear Schrödinger equations to describe the propagation of light in the $\chi^{(2)}$ crystal and in the left-handed material, we have derived a mean-field model for this optical parametric oscillator. This model is based on reasonable assumptions such as the slowly varying envelope approximation, impedance matching between the crystal and the left-handed material, small evolution of the envelope during one roundtrip in the cavity and high finesse of the cavity. The model describes the time evolution of the envelopes of the signal field A_s and the pump field A_p with the following equations:

$$\begin{aligned}\frac{\partial A_\mathrm{s}}{\partial t} &= -\left(1+\mathrm{i}\Delta_\mathrm{s}\right)A_\mathrm{s} + A_\mathrm{p}A_\mathrm{s}^* + \mathrm{i}\mathcal{D}_\mathrm{s}\nabla_\perp^2 A_\mathrm{s},\\ \frac{\partial A_\mathrm{p}}{\partial t} &= E-\left(1+\mathrm{i}\Delta_\mathrm{p}\right)A_\mathrm{p} - A_\mathrm{s}^2 + \mathrm{i}\mathcal{D}_\mathrm{p}\nabla_\perp^2 A_\mathrm{p},\end{aligned} \tag{1}$$

where E is the amplitude of the input beam, and $\Delta_{\mathrm{s,p}}$ is proportional to the detuning between the frequency of the cavity mode and the frequency of the signal and pump beams, respectively.

The coefficient $\mathcal{D}_\mathrm{s}$ of the signal diffraction term is given by

$$\mathcal{D}_s = \frac{\lambda_s \mathcal{F}}{2\pi^2}\left(\frac{l_\mathrm{QC}}{n_\mathrm{QC}} + \frac{l_\mathrm{L}}{n_{\mathrm{LHM},s}}\right). \tag{2}$$

The first term in Eq. (2) is positive, whereas the second term is negative due to the negative index of refraction of the left-handed material. Careful tuning of the thickness of the quadratic crystal l_QC and of the thickness of the left-handed material l_L thus allows engineering the value and sign of the diffraction coefficient $\mathcal{D}_\mathrm{s}$.

However, today's left-handed materials have typically a very small bandwidth, which means that they do not have negative index of refraction at both the signal and pump frequencies. Therefore, our method of diffraction management by insertion of a left-handed material will only work for the signal beam. Note also that there is no longer a fixed relationship between $\mathcal{D}_\mathrm{s}$ and $\mathcal{D}_\mathrm{p}$ in the system under study, since there is no phase matching between signal and pump beams in the left-handed material.

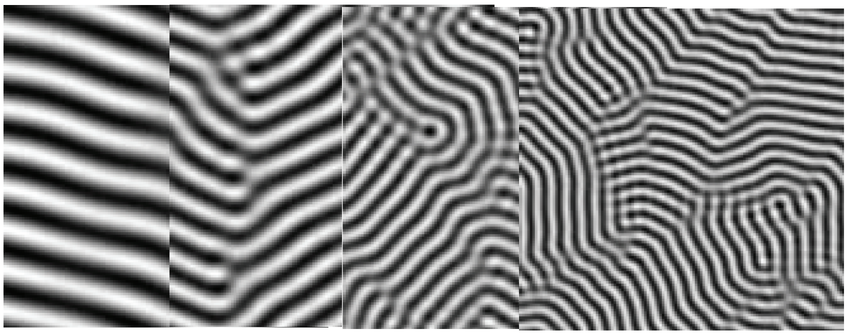


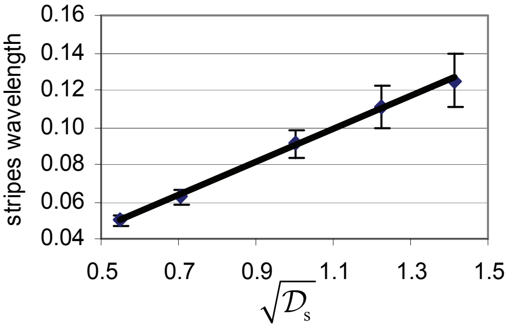
Fig. 2: Dissipative patterns emerging from the output mirror for inputs below the lasing threshold. Signal diffraction strength is decreased from left to right. Pump diffraction is constant.

Size Scaling of Dissipative Structures

The degenerate optical parametric oscillator under study is a dissipative system far from equilibrium, in which the interaction between diffraction and nonlinearity can result in modulational – or Turing – instability. For certain values of the resonator's parameters (input amplitude, detunings and diffraction coefficients), the homogeneous output state can be destabilised, resulting in the formation of different type of structures in the emitted field of the resonator, such as stripe-like patterns and phase domains.

When the input amplitude is below the lasing threshold of the oscillator, we find that stripe-like patterns are formed for positive detunings (see Fig. 2 for some numerically obtained patterns). The stability analysis of these structures predicts that their wavelength is proportional to the square root of the signal diffraction coefficient. This result is confirmed by our numerical simulations [see Fig. 3(a)]. Note that the value of the pump diffraction has almost no influence on the size of the stripes. This is attributed to the relative homogeneity of the pump beam, which can be established from a perturbation analysis (the modulational effect appears only as a second-order term in the pump field).

For inputs above the lasing threshold, two homogeneous solutions with nonzero signal amplitude exist. Both solutions have the same amplitude, but a different phase. An interesting parameter region is where these homogeneous states are unstable, and phase domains or solitons are formed instead. Such structures consist of regions that are locally composed of the two homogeneous solutions. See Fig. 4 for examples obtained from our numerical calculations. We again see that the typical size of the structures decreases for smaller signal diffraction.



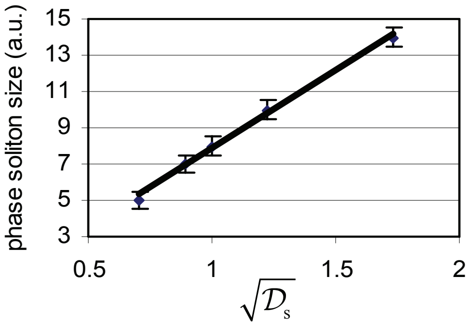


Fig. 3: Scaling of the typical size of dissipative structures with the signal diffraction coefficient. (left) Wavelength of stripe-like solutions below the lasing threshold. (right) Size of phase solitons above the lasing threshold.

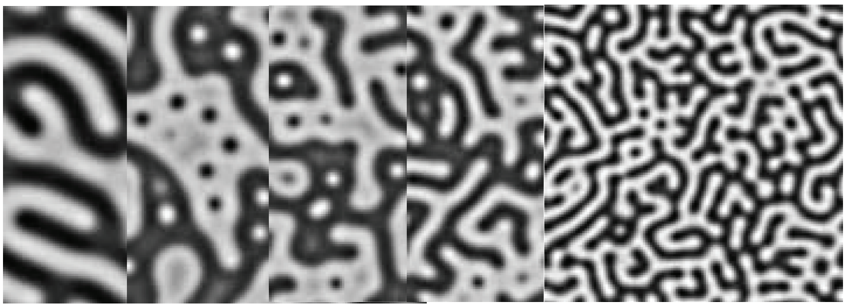


Fig. 4: Phase domains/solitons emerging from the output mirror above the lasing threshold. Signal diffraction strength is decreased from left to right. Pump diffraction is constant.

Based on the numerically obtained patterns, we have calculated the average size of the phase solitons and we have plotted the results in Fig. 3(b). We observe that the average size is still proportional to the square root of the signal diffraction coefficient. From Eqs. (1), one would expect such scaling only if both diffraction coefficients are decreased together, but we rather find that the scaling is independent from the pump diffraction.

Conclusions

We have applied our method of diffraction management with left-handed materials to optical parametric oscillators. We derived a mean-field model for this system and showed that the diffraction coefficient of the signal beam can indeed be engineered due to the presence of the left-handed materials. However, the method of diffraction management cannot be simultaneously used for the pump beam due to the small bandwidth in which left-handed materials operate. Nevertheless, dissipative structures emerging in the output beam can still be scaled down, since we find that their size is almost independent from the pump diffraction.

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