

Bifurcation analysis of the dynamical behaviour of semiconductor ring lasers

L. Gelens¹, G. Van der Sande^{1,2}, P. Tassin¹, A. Sciré² and J. Danckaert¹

¹Department of Applied Physics and Photonics, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussel, Belgium

²Instituto de Física Interdisciplinar y Sistemas Complejos (IFISC, CSIC-UIB), Campus Universitat Illes Balears, E-07122 Palma de Mallorca, Spain

A basic rate equation model for the counter-propagating fields in semiconductor ring lasers, accounting for saturation and backscattering effects, consists of five real equations. By applying asymptotic techniques, we are able to reduce these original rate equations to two real ones. A bifurcation study of this reduced system is performed, which greatly enlarges our physical understanding of the dynamical behaviour of these ring lasers.

Introduction

Semiconductor ring lasers (SRLs) are promising candidates for applications in photonic integrated circuits [1]. They do not require cleaved facets or gratings for optical feedback and are thus particularly suited for monolithic integration. Some monolithic SRLs exhibit unidirectional operation making them highly desirable in applications, because of their wavelength stability. Examples of possible applications are wavelength filtering, unidirectional travelling-wave operation, and multiplexing/demultiplexing applications. This operation regime can also lead to bistability. This opens the possibility of using SRLs in systems for all-optical switching, gating, wavelength-conversion functions, and optical memories.

Different theoretical models have been proposed for the analysis of generalized rings and two-mode laser systems. These theories focus on the interplay between the two counter-propagating modes and their interaction with the active medium. Recently, a comparison between experimental results in SRLs and a suitable theoretical explanation has been carried out systematically in Ref. [2] by Sorel *et al.* They have experimentally observed bidirectional and unidirectional regimes of continuous-wave mode operation. Moreover, a bidirectional regime where the two counter-propagating modes experience harmonic alternate oscillations has been discovered. The theoretical framework used in Ref. [2] is based on two mean-field equations for the counter-propagating modes, and a third rate equation for the carriers. The model accounts for self- and cross-gain saturation effects and backscattering contributions from the coupling to an output waveguide.

In this paper, we apply a singular perturbation technique to reduce the original laser equations to two equations. The analysis takes advantage from the different timescales present in the system. These reduced equations simplify the bifurcation analysis of the possible steady-state solutions considerably, and they allow for a two-dimensional phase-space description of the different dynamical regimes.

General rate equation model

The rate equation model presented in Ref. [2] considers an electric field as a sum of two counter-propagating waves:

$$E(z, t) = E_1(t) \exp[i(\omega_0 t - k_0 z)] + E_2(t) \exp[i(\omega_0 t + k_0 z)] + c.c. \quad (1)$$

and is formulated mathematically in terms of two rate equations for the slowly varying amplitudes $E_{1,2}$ of forward and backward propagating waves and one rate equation for the carrier number N . The equations read as follows:

$$\dot{E}_1 = \kappa(1 + i\alpha) [N(1 - s|E_1|^2 - c|E_2|^2) - 1] E_1 - (k_d + ik_c)E_2, \quad (2)$$

$$\dot{E}_2 = \kappa(1 + i\alpha) [N(1 - s|E_2|^2 - c|E_1|^2) - 1] E_2 - (k_d + ik_c)E_1, \quad (3)$$

$$\dot{N} = \gamma[\mu - N - N(1 - s|E_1|^2 - c|E_2|^2)|E_1|^2 - N(1 - s|E_2|^2 - c|E_1|^2)|E_2|^2], \quad (4)$$

where dot represents differentiation with respect to t . In Eqs. (2)-(4), κ is the field decay rate, and γ is the decay rate of the carrier population. α is the linewidth enhancement factor, μ is the renormalized injection current ($\mu \approx 0$ at transparency, $\mu \approx 1$ at lasing threshold). Self- and cross-saturation effects are modeled by s and c . Backscattering results in a linear coupling between the two fields via the dissipative (k_d) and conservative (k_c) scattering parameters.

Asymptotic analysis

Our numerical investigations of Eqs. (2)-(4) show that (for the given parameter set) the quantity $N - 1$ remains small. The parameter values suggest to investigate the limit $\kappa/\gamma \rightarrow \infty$, assuming $N - 1$, s and c smaller than 1, and k_d and k_c smaller than κ . To be able to define the order of magnitude of all parameters and to be able to determine the leading order approximation to Eqs. (2)-(4), we introduce a dimensionless time $\tau = \gamma t$, and a smallness parameter ρ as $\rho = \gamma/\kappa$.

In a next step, we define a new parameters and a new carrier variable, taking into account the typical scale of the different parameters:

$$N - 1 = \rho n, \quad (5)$$

$$s = \rho S, \quad (6)$$

$$c = \rho C, \quad (7)$$

$$k_d/\kappa = \rho K_d, \quad (8)$$

$$k_c/\kappa = \rho K_c, \quad (9)$$

where n , S , C , K_d and K_c are assumed to be $O(1)$.

After this multiple-scales analysis and an amplitude/phase decomposition:

$$E_{1,2} = Q_{1,2} e^{i\phi_{1,2}}, \quad (10)$$

we find a conservation law:

$$Q_1^2 + Q_2^2 = \mu - 1. \quad (11)$$

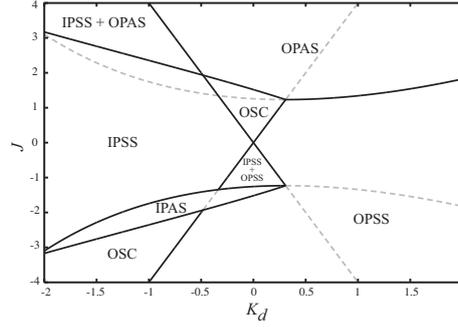


Figure 1: Stability diagram in the plane (K_d, J) for fixed values of $K_c = 2.2$ and $\alpha = 3.5$ showing the different stable operation regimes.

In a last step of the analysis, we define two new dynamical 'angular' variables: θ as a measure for the relative modal intensity, and ψ the phase difference between the two modes:

$$Q_1 = \sqrt{\mu - 1} \cos\left(\frac{\theta + \pi/2}{2}\right), \quad (12)$$

$$Q_2 = \sqrt{\mu - 1} \sin\left(\frac{\theta + \pi/2}{2}\right), \quad (13)$$

$$\psi = \phi_2 - \phi_1. \quad (14)$$

with $\theta \in [-\pi/2, \pi/2]$, and $\psi \in [0, 2\pi]$. This coordinate transformation then leads us to a two-dimensional system, describing the evolution of θ and ψ . Defining a new current $J = (C - S)(\mu - 1)$, these reduced equations read

$$\theta' = -2K_c \sin \psi + 2K_d \cos \psi \sin \theta + J \sin \theta \cos \theta, \quad (15)$$

$$\cos \theta \psi' = \alpha J \sin \theta \cos \theta + 2K_d \sin \psi + 2K_c \cos \psi \sin \theta. \quad (16)$$

Stability and bifurcation diagram

In Figure 1, we have plotted the bifurcation currents as a function of the dissipative backscattering parameter K_d for a fixed value of the conservative backscattering parameter $K_c = 2.2$ and linewidth enhancement factor $\alpha = 3.5$. These bifurcation points have been obtained analytically by performing a linear stability analysis of the reduced system [Eqs. 15-16]. No less than seven different operating regimes can be distinguished in this parameter range: IPSS, OPSS, bistable OPAS, bistable IPAS, oscillatory behavior (OSC), tristability between the IPSS and the two OPAS (IPSS + OPAS), bistability between IPSS and OPSS (IPSS + OPSS). To gain insight in the mechanisms leading to the appearance of these dynamical regimes, we will now consider one particular current path at $K_d = 0.1635$ for positive values of J . We construct the bifurcation diagram of θ with J as the bifurcation parameter [see Figure 2(a)]. The two-dimensional phase-space structure in Figure 2(b) illustrates the dynamical behavior in the different operating regimes. At $J = 0$, the OPSS is selected. When increasing the current, the OPSS is destabilized at the Hopf bifurcation point J_{OPSS}^H . At this Hopf point, a stable limit cycle centered around the OPSS

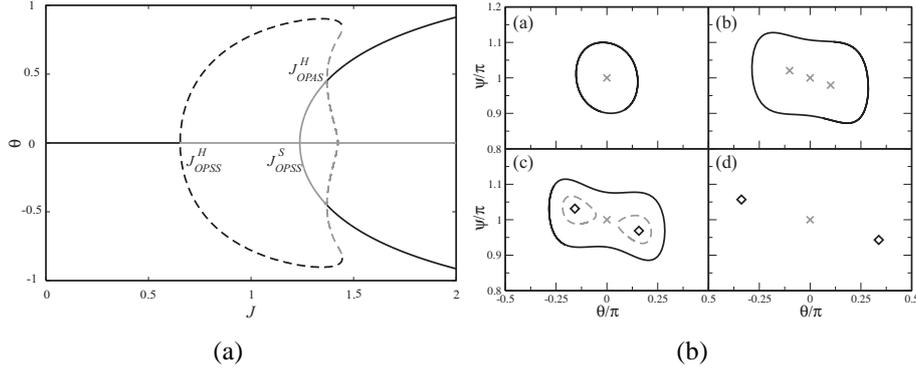


Figure 2: (a) Bifurcation diagram depicting the extremes of θ vs. injection current J . The steady-state values of θ are denoted by full lines, while the extrema of periodically oscillating θ are indicated with dashed lines. (b) Phase-space structure for different values of the injection current J . (I) $J = 0.75$ (II) $J = 1.3$ (III) $J = 1.4$ (IV) $J = 2.0$. Black indicates stable operation, and unstable operation is depicted in grey. $K_d = 0.1635$, $\alpha = 3.5$.

emerges (I). The amplitude of these time-periodic oscillations continues to grow with J . At the pitchfork bifurcation J_{OPSS}^S , two OPAS fixed points appear (II). These fixed points are unstable until the current exceeds the Hopf point J_{OPAS}^H . From this point, small unstable limit cycles grow from both OPAS (III). The stable limit cycle centered around the unstable OPSS, connects with the unstable limit cycles centered around the OPAS, and these time-periodic structures disappear (IV).

Conclusion

In this contribution, we have used asymptotic methods to derive a two-variable reduced model for the dynamical behavior of semiconductor ring laser. The model accounts for both backscattering processes and gain saturation effects. Thanks to this analysis, we have been able to perform a systematic and largely analytical bifurcation study of all the steady states and time-periodic states of the model. Our analysis has shown that many different regions of operation might be expected depending on the parameter values.

Acknowledgments

The authors thank the Research Foundation - Flanders (FWO-Vlaanderen) for their fellowships, as well as for project support. Also support of the EU-STREP IOLOS ("Integrated optical logic and memory using ultra-fast micro-ring bistable semiconductor lasers"-FP6) project is gratefully acknowledged.

References

- [1] M. T. Hill *et al*, *Nature*, vol. 432, pp. 206, 2004.
- [2] M. Sorel *et al*, *IEEE J. Quantum Electron.*, vol. 39, pp. 1187, 2003.