

## Modelling End-Pumped Solid-State Lasers

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*The operation dynamics of end-pumped solid-state lasers are investigated by means of a spatially resolved numerical rate-equation model and a time-dependent analytical thermal model. The rate-equation model allows the optimization of parameters such as the output coupler transmission and gain medium length, with the aim of improving the laser output performance. The time-dependent analytical thermal model is able to predict the temperature and the corresponding induced thermal stresses on the pump face of quasi-continuous wave (qcw) end-pumped laser rods. Both models are found to be in very good agreement with experimental results.*

### Introduction

Diode-end-pumped solid-state lasers are very popular because of their high efficiency, excellent beam quality, compactness and robustness. Due to these good attributes, solid-state lasers have numerous applications in the medical, scientific, military and industrial fields. The ever increasing demand in laser applications ensures that the development and power scaling of high power diode-end-pumped solid-state lasers remains a very active area of research. One of the most valuable tools that assists in the laser design and construction process is the use of mathematical models and simulations that explain and illustrate the operation principles of the laser. In this paper, a spatially resolved quasi-three-level rate-equation model is discussed. This model predicts the laser threshold and efficiency and can be used to optimize the laser output power by determining the optimal output coupler, gain medium length and pump size. The model is verified by applying it to a quasi-three-level qcw Tm:GdVO<sub>4</sub> laser. The second model that is presented is a time-dependent analytical thermal model of the temperature and the corresponding induced thermal stresses on the pump face of quasi-continuous wave (qcw) end-pumped laser rods. To illustrate an application of the model, it is applied to a qcw pumped Tm:YLF rod and found to be in very good agreement with published experimental results.

### Rate-Equation Model

By assuming a two manifold energy level scheme, the rate-equations that describe the transient behavior of population densities in each of the manifolds are given by:

$$\frac{dN_1}{dt} = \frac{\eta_p (I_p^+ + I_p^-)}{h\nu_p} (\sigma_p^{abs} N_0 - \sigma_p^{em} N_1) - \frac{(I_l^+ + I_l^-)}{h\nu_l} (\sigma_l^{em} N_1 - \sigma_l^{abs} N_0) - \frac{N_1}{\tau_1}$$

$$N_0 = N_T - N_1$$

where  $N_0$  is the ground manifold population density,  $N_1$  the upper laser level population density and  $N_T$  the total density of active ions.  $\sigma_p^{em}$  and  $\sigma_p^{abs}$  are the effective emission and absorption cross-sections at the pump wavelength with  $\sigma_l^{em}$  and  $\sigma_l^{abs}$  the cross-sections at the laser wavelength.  $\tau_1$  is the upper laser manifold lifetime,  $\eta_p$  the pump

efficiency while  $I_p$  and  $I_l$  represent the pump and laser intensities respectively with the positive and negative superscripts indicating the forward and backward propagating directions. Under steady-state conditions, analytical solutions to the populations densities can be derived for a given pump and laser intensity. The propagation of the pump and laser intensities in the resonator are described by

$$\frac{dI_p}{dz} = I_p(\sigma_p^{em} N_1 - \sigma_p^{abs} N_0); \quad \frac{dI_l}{dz} = I_l(\sigma_l^{em} N_1 - \sigma_l^{abs} N_0)$$

These differential equations are solved numerically to obtain the field intensity distributions in the resonator while the energy manifold population densities are updated according to intensity at every spatial coordinate.

The rate-equation model is applied to a quasi-three-level Tm:GdVO<sub>4</sub> laser [1] where the model predicts a pump power threshold of 4 W and a maximum output power of 10 W at 38 W of incident pump power, with a slope efficiency of 29% (Fig. 1). These predictions compare extremely well with the experimental results of 5.5 W for threshold and 28% for slope efficiency. Other than the pump efficiency that was estimated as 1.6 due to cross-relaxation process that takes place in Thulium, no parameters were tuned to fit the data. The rate-equation fits the experimental data very well up to about 20 W incident pump power. At larger pump powers, a decrease in the laser's slope efficiency was observed experimentally. It is very likely that the decrease in the slope efficiency at higher pump powers is due to the very strong thermal lens that is induced in Tm:GdVO<sub>4</sub>. Since the rate-equation model doesn't include any thermal effects such as thermal lensing, it is not able to predict this decrease in slope efficiency. While the laser operated with an output coupler reflectivity of 95% and a gain medium length of 3 mm, the model predicts that the laser can be increased from 10.1 W to 13.9 W (increase of 37%) at 38 W incident pump power by optimizing the output coupler reflectivity and gain medium length to 90% and 8 mm respectively (Fig. 2).

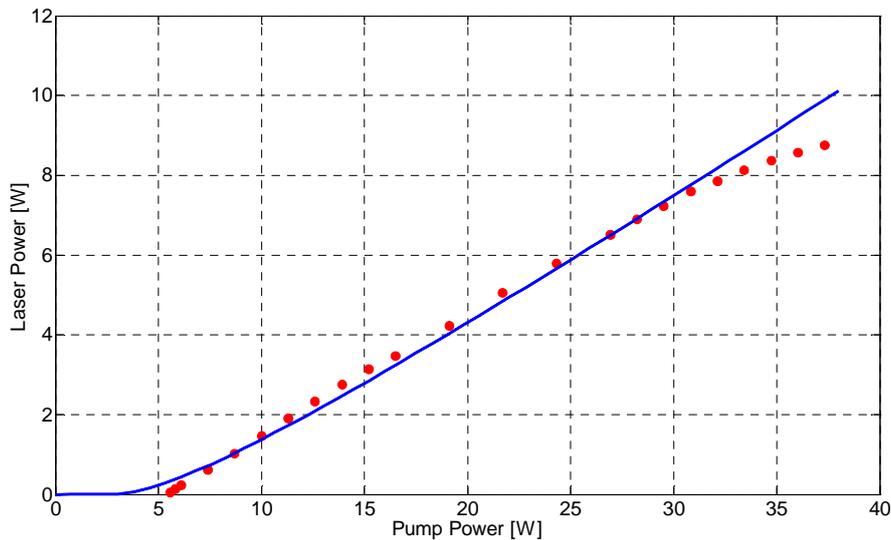


Fig. 1. The Tm:GdVO<sub>4</sub> laser output power as a function of incident pump power. (Experimental data (red dots) along with the rate-equation prediction (blue line)).

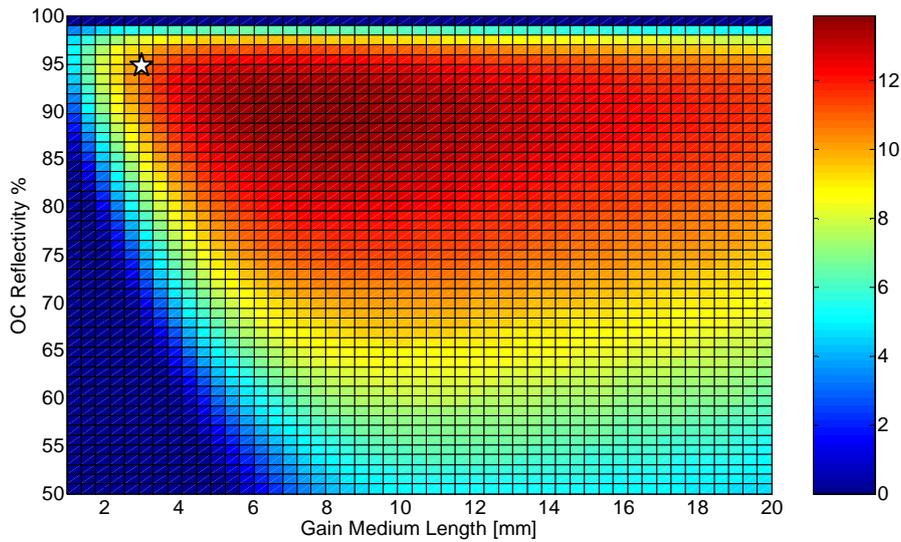


Fig. 2. The simulated laser output power (at 38 W incident pump power) as a function of gain medium length and output coupler reflectivity. The white star indicates the point at which the laser operated.

### Thermal Model

To predict the temperature and thermally induced stresses on the pump face of a cylindrical end-pumped rod, the non-homogeneous heat diffusion equation can be solved by making use of a Green's function approach so that the transient temperature profile on the pump face of a cylindrical rod is given by

$$u(r, pT+t) = \frac{2\alpha\eta ER}{k\pi w\tau_{on}} \sum_{m=1}^{\infty} \frac{J_0(\mu_m r/R) J_1(\mu_m w/R) f(p, t, \mu_m)}{\mu_m^3 J_1^2(\mu_m)},$$

with the time dependence given by  $f(p, t, \mu_m)$  as

$$f(p, t, \mu_m) = \exp\left(-\mu_m^2 \frac{t}{\tau_D}\right) \left\{ \frac{\left[ \exp\left(\mu_m^2 \frac{\tau_{on}}{\tau_D}\right) - 1 \right] \left[ \exp\left(-\mu_m^2 \frac{pT}{\tau_D}\right) - 1 \right]}{1 - \exp\left(\mu_m^2 \frac{T}{\tau_D}\right)} - \left[ 1 - \exp\left(\mu_m^2 \frac{\tau}{\tau_D}\right) \right] \right\},$$

Definitions for all the symbols can be found in [2]. Due to the fact that the stress tensor alone does not provide enough information regarding crystal fracture, we use the maximum shear stress to predict fracture. This is also known as the stress intensity or the Tresca failure criterion, which in the plain-strain approximation reduces to

$$\begin{aligned} \sigma_r(r, pT+t) &= |\sigma_\theta - \sigma_r| \\ &= \left| \frac{2C\alpha\eta ER}{k\pi w\tau_{on}} \sum_{m=1}^{\infty} \frac{J_1(\mu_m w/R)}{\mu_m^3 J_1^2(\mu_m)} J_2(\mu_m r/R) f(p, t, \mu_m) \right|, \end{aligned}$$

As a verification of the analytical thermal model, a time-dependent three dimensional coupled thermal-stress finite element analysis was implemented [3]. The parameters that were used in the numerical and analytical thermal models can be found in [2]. Fig. 3(a) shows the predicted time-dependence of the temperature in the centre of the pump face of the Tm:YLF rod for a 10% pump duty cycle respectively (for a qcw pulse on-time of 10 ms). The upper and lower boundaries of the shaded red region in Fig. 3(a)

indicate the analytical model's predictions of the temperature when the thermal conductivity of the c- and the a-axis of Tm:YLF were used respectively. It is clear from the graphs that there is very good agreement between the analytical and numerical models when the lowest thermal conductivity is used in the calculations. The upper and lower boundaries of the shaded red region in Fig. 3(b) indicate the analytical model's predictions of the maximum stress on the pump face when the two respective linear expansion coefficients of Tm:YLF were used along with the lowest thermal conductivity.

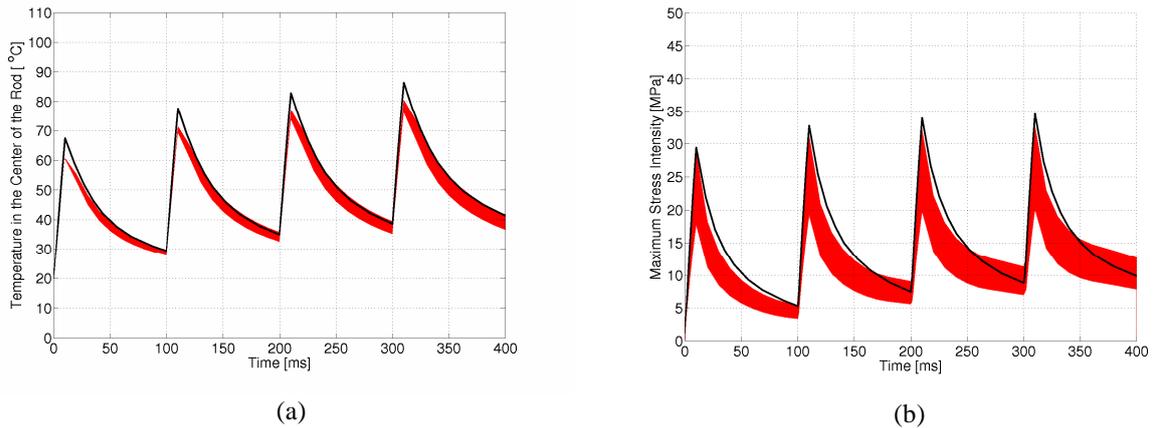


Fig. 3(a). The analytically (red) and numerically (black) predicted temperature in the centre of the Tm:YLF rod and (b) maximum stress on the pump face as a function of time while the rod is subjected to a qcw pump with a peak power of 200 W at 10 Hz (qcw pulse on-time of 10 ms).

## Conclusions

A rate-equation model was developed and applied to a Tm:GdVO<sub>4</sub> laser. The laser efficiency and output power are among the parameters that were investigated. A comparison with experimental values shows that the model is very successful in explaining and predicting the dynamics of the laser during continuous wave operation. The model proved to be a very useful design tool for determining the optimal values of design parameters such as output coupling and crystal length.

A time-dependent analytical thermal model was developed to investigate the transient behaviour of thermally induced stresses in qcw end-pumped laser rods. The analytical model was used to investigate the thermal stress in a Tm:YLF rod and was found to be in very good agreement with that of a time-dependent coupled thermal-stress finite element analysis.

## References

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