Modal Modeling Strategies for the Design of Optical Fibers

R. Mussina, B.P. de Hon, R.W. Smink, A.G. Tijhuis

Faculty of Electrical Engineering, University of Technology,
P.O. Box 513, 5600 MB Eindhoven, The Netherlands

In problems of the optimization of refractive index profiles in the design of single-mode optical fibers, the minimization of the cost function heavily depends on the accuracy and speed of the numerical evaluation of fiber characteristics, like dispersion, dispersion slope, bending loss and mode-field diameter. We propose to employ the Galerkin method with Laguerre-Gauss basis functions for the approximate scalar modal problem to generate a suitable initial profile for the local optimization scheme based on our vectorial full-wave mode solver. In this paper we assess the feasibility of this method regarding accuracy and speed.

Introduction

The use of the Galerkin method for the modal analysis of the scalar wave equation is not new. It has been previously used [1-4] for the evaluation of the propagation coefficients and modal fields of optical fibers and slab-waveguides, with the aid of different sets of orthogonal basis functions like Bessel, Laguerre-Gauss and Hermite-Gauss basis functions. The choice of these sets of basis functions is based on the facts that Bessel functions are the eigensolutions of the scalar wave equations for step-index fibers while Laguerre-Gauss and Hermite-Gauss functions represent the modal eigenfunctions for the infinitely extended parabolic profile in circular and rectangular waveguides respectively. The use of the set of Laguerre-Gauss basis functions for the analysis of wave propagation in graded-index fibers has its advantages for its speed, accuracy and computational simplicity [1, 3]. We are developing this idea for the evaluation of fiber characteristics like group slowness, dispersion and dispersion slope for single-mode fibers. This paper provides some numerical results of this approach.
Recently, we have developed an optimization code for the design of single-mode fibers with a vectorial full-wave solver as the computational engine [5]. The rationale of replacing this computational engine by one based on Laguerre-Gauss basis functions in the weak guidance approximation is that this may considerably speed up optimization sessions. We intend to eventually use the resulting "optimized" profile as an initial profile in a vectorial full-wave optimization session.

Formalism

We will follow the procedure for the application of the Galerkin method to radially inhomogeneous fibers used in the work of Meunier, *et al.* [1]. Let us consider the scalar wave equation for a time-harmonic field Ψ, with suppressed time dependence $\exp(-i\omega t)$:

$$\Delta^2\Psi + k_0^2 n^2(r)\Psi = 0 \tag{1}$$

where k_0 is the free-space wavenumber and $n(r)$ is the refractive index profile of the fiber, which only varies in the radial direction. If the direction of propagation is along the z-axis, and β is the propagation coefficient, the field Ψ can be expressed in cylindrical coordinates as

$$\Psi = \psi(r,\theta)\exp(i\beta z).$$

Then Equation (1) takes the form

$$\left[\nabla_t^2 + k_0^2 n^2(r)\right]\psi(r,\theta) = \beta^2\psi(r,\theta)$$

(2)

where ∇_t^2 is the transverse part of the Laplacian operator

$$\nabla_t^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial\theta^2}.$$

As the refractive index profile is independent of θ, we can write $\psi(r,\theta)$ in terms of constituents of the form:

$$\psi(r,\theta) = R_{mn}(r)\exp(-im\theta),$$

where $R_{mn}(r)$ satisfies the radial wave equation:

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + k_0^2 n^2(r) - \frac{m^2}{r^2} - \beta^2\right]R_{mn}(r) = 0 \; .$$

Here *m* and *n* are the azimuthal and radial mode numbers, respectively. Below, we shall consider *m* fixed, and suppress it where possible.

In order to solve Equation (2), we apply the Galerkin method, representing $\psi(r,\theta)$ in the form of an infinite series

$$\psi(r,\theta) = \sum_{i=1}^{\infty} c_i b_i\left(x(r)\right)\exp(-im\theta), \tag{3}$$

where $x(r) = V\left(r/r_0\right)^2$, r_0 is the core radius, V is the normalized frequency, and the basis functions $b_i\left(x(r)\right)$ chosen such that they are finite at $r = 0$ and decay to zero for $r \to \infty$. As the basis functions we may use Laguerre-Gauss polynomials, which form a complete set of orthonormal functions and satisfy the required boundary conditions:

$$b_{mn}\left(x(r)\right) = \left[\frac{V}{\pi r_0^2}\frac{n!}{(n+m)!}\right]^{1/2} \exp\left(-x(r)/2\right)x(r)^{m/2} L_n^m\left(x(r)\right), \quad n = i-1 = 0,1,...$$

$L_n^m(x)$ are the generalized Laguerre polynomials. Then the partial differential equation (2) may be transformed into the system of linear equations

$$\sum_{j=1}^{N}\left[\left\langle b_i,\left(\nabla_t^2 + k_0^2 n^2(r)\right)b_j\right\rangle - \beta^2\delta_{i,j}\right]c_j = 0 \qquad i = 1,2,\ldots,N \tag{4}$$

The matrix of the last system of equations has purely discrete real eigenvalues which provide the propagation coefficients β for the given value of *m* and the components of the corresponding eigenvectors represent the expansion coefficients in (3). The technique allows the determination of the propagation coefficients and field distributions both for single-mode and multimode fibers.

To compute fiber characteristics like group slowness, dispersion and dispersion slope in the case of single-mode fibers we have derived analytical expressions for the first, second and third order derivatives of the propagation coefficient with respect to the frequency. This is a faster and a more accurate approach than the repeated use of finite differences, especially for the dispersion and its slope. This is achieved through the evaluation of the derivatives of the eigenvalues and eigenvectors of the system (4) with respect to the angular frequency.

Numerical Results

We first test the validity of the method for the determination of the propagation coefficients for two types of refractive-index profiles: step-index and truncated parabolic index profiles. As expected, the results of the simulations concur with the results obtained by Meunier, *et al.* In their work a comparison of the results with those

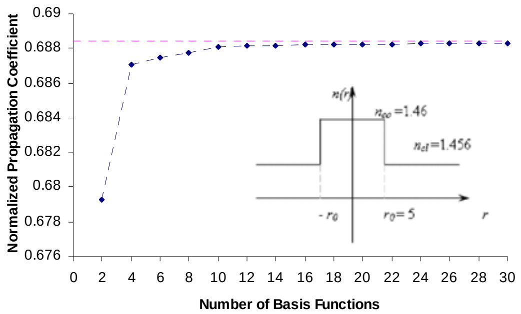

Figure 1. Normalized propagation coefficient vs the number of basis functions for the step-index fiber, λ=1.55μm

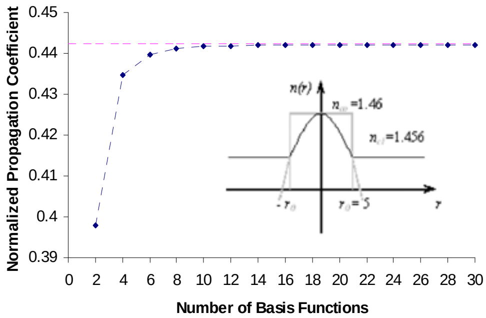

Figure 2. Normalized propagation coefficient vs the number of basis functions for the truncated parabolic fiber, λ=1.55μm

deduced from the exact solution of the scalar wave equation is provided, and excellent agreement except in the vicinity of the V-value near cut-off is noted.

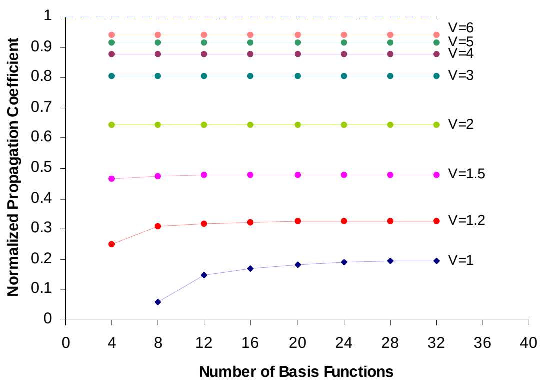

Figure 3. Normalized propagation coefficient vs the number of basis functions for the step-index fiber for different values of V

Figures 1 and 2 give examples for these computations for different values of matrix size N and show very fast convergence of the values of the propagation coefficient. The slightly better convergence in the case of the truncated parabolic profile is expected as we use Laguerre-Gauss polynomials as the basis elements which are the eigenfunctions for the infinitely extended parabolic profile in circular waveguides.

Figure 3 compares the convergence of the values of the propagation coefficient for different values of V. It appears that the higher values of the number of basis functions N do not ensure substantially better accuracy. As a good compromise between accuracy and computation time, values of N in the range 14 to 20 could be chosen except for values of V very near cut-off.

This choice is also valid for the computation of the dispersion. As can be seen from Figure 4, the dispersion values for N=16 are very close to those for N=30 and N=40. The computation time for the series of dispersion values for six values of λ (from 1.2 to 1.7 with the step of 0.1) in the case N=16 is 0.078s. Computations are performed on Pentium(R) 4 CPU 3.20 GHz.

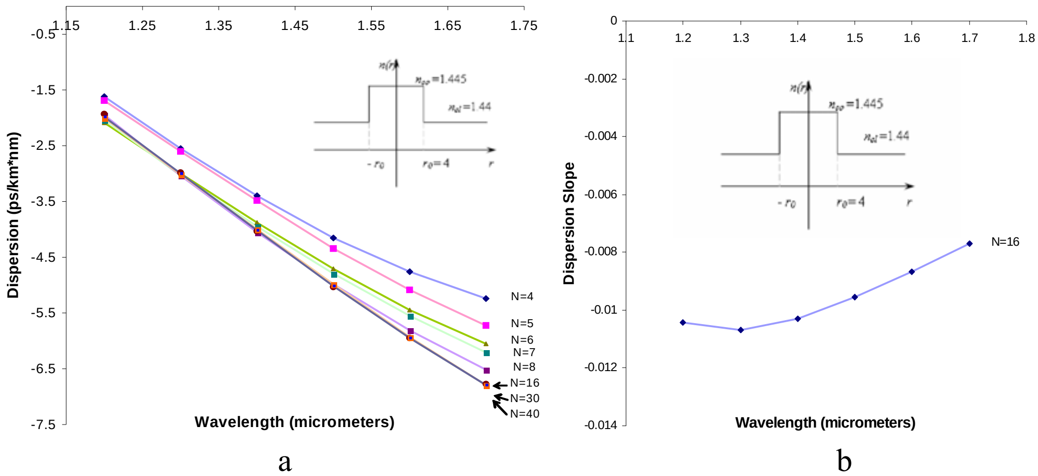

Figure 4. The dispersion (a) and the dispersion slope (b) for the LP_{01} for the step-index fiber as a function of wavelength

Conclusion

The modal modeling strategy for the scalar field in the weak guidance approximation has its advantages of accuracy, speed and simplicity. We have developed a fast and accurate numerical approach for calculating the single-mode fiber characteristics like the group slowness, dispersion and dispersion slope. Future work will include its application for a wide class of refractive index profiles possibly with several dopants and the evaluation of other fiber parameters like mode-field diameter and bending loss. Eventually, we intend to use the modeling code as a computational engine in an optimization code, so as to generate suitable initial profiles in our vectorial full-wave optimization scheme.

References

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