# Manipulating frequency entangled photons

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*We present an original method for manipulating frequency entangled photons. Photon pairs are produced in the C-band by parametric down conversion. Each photon is manipulated directly in the frequency domain with electro-optic phase modulators and is then selected by narrow filtering. This leads to an interference pattern which proves that we are manipulating high dimensional entanglement. Our experiment also allows violation of Bell inequalities, thereby showing that this interference pattern could not be reproduced by a classical theory. In future this method could be exploited to achieve quantum communication tasks in standard telecommunication fibers.*

## Introduction

Since the birth of Quantum Mechanics (QM), entanglement appeared as one of its most fascinating aspects. It implies that two particles that have previously interacted can stay strongly correlated no matter how distant they are. As described by Einstein, Podolsky and Rosen in a famous paper [1], the non local correlations obtained when local measurement are performed on the particles seem paradoxal. So called Local Hidden Variable (LHV) theories were introduced to explain these phenomena from a local and deterministic point of view. In 1964 Bell derived a mathematical expression able to test if LHV theories are the correct description of the world [2]. He showed that the predictions of QM cannot be reproduced by LHV theories. Since the first experimental violation of a Bell inequality [3], and thus the recognition of QM predictions being correct, entanglement has become a useful tool both for fundamental tests of physical principles and for applications such as Quantum Key Distribution (QKD). Photons being privileged carriers of quantum information, many different kinds of photonic entanglement have been produced, including entanglement in polarization [4], momentum [5], angular momentum [6], time-energy [7] and its discretized version called *time bins* [8]. In the present work we consider a new kind of entanglement. We show how energy – or equivalently frequency – entangled photons can be manipulated directly in the frequency domain.

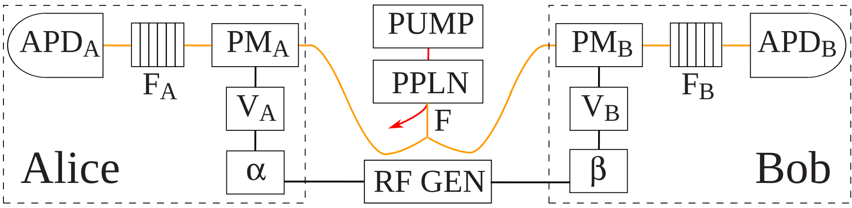


Figure 1: Experimental setup. The PUMP creates photon pairs in the PPLN waveguide and is removed by a filter (F). The photons are manipulated by phase modulators ($PM_{A,B}$), selected by narrowband filters ($F_{A,B}$), and detected by avalanche photo-diodes ($APD_{A,B}$). The phase modulators are driven by a 12.5 GHz radio frequency generator (RF GEN) whose output is controlled by variable attenuators ($V_{A,B}$) and phase shifters ($\alpha, \beta$).

## Experiment

Our experimental setup is depicted in Figure 1. A quasi-monochromatic pump laser ($\lambda = 773.865\,\text{nm}$, $P \approx 4\,\text{mW}$) produces photon pairs at telecommunication wavelengths (around $\lambda_0 = 2\pi c/\omega_0 = 1547.73\,\text{nm}$) by parametric down conversion in a periodically poled lithium niobate waveguide, and is then removed with a drop filter. Photon pairs are collected by a polarization maintaining fiber and sent through a 3dB-coupler. Interesting cases occur when the photon pair is split: one photon is sent to Alice (A) and the other to Bob (B). The (idealized) entangled state can be written as

$$|\Psi\rangle = \int d\omega |\omega_0 + \omega\rangle_A |\omega_0 - \omega\rangle_B. \tag{1}$$

The photons pass through electro-optic phase modulators to which are applied sinusoidally varying voltages at frequency $\Omega/2\pi = 12.5\,\text{GHz}$, with amplitudes $V_{A,B}$ and phases $\alpha, \beta$ which can be controlled. The induced time dependent optical phases $\phi_A(t) = a\cos(\Omega t - \alpha)$ and $\phi_B(t) = b\cos(\Omega t - \beta)$, where $a = \pi V_A/V_\pi$, $b = \pi V_B/V_\pi$, and $V_\pi$ is the half-wave voltage of the modulators, lead to the unitary transformations

$$|\omega\rangle \to \sum_{p\in\mathbb{Z}} |\omega + p\Omega\rangle U_p(c,\theta), \tag{2}$$

where $p = p_A$ or $p_B$, $U_p(c,\theta) = J_p(c)e^{ip(\theta-\pi/2)}$, $c = a$ or $b$, $\theta = \alpha$ or $\beta$, and $J_p$ is the $p$th-order Bessel function of the first kind. Using Equations (1,2), one can readily compute the entangled state after the phase modulation:

$$|\Psi\rangle \to \int d\omega' \sum_{d\in\mathbb{Z}} |\omega_0 + \omega'\rangle_A |\omega_0 - \omega' + d\Omega\rangle_B \, c_d(a,b,\alpha,\beta), \tag{3}$$

with $\omega' = \omega + p_A\Omega$, $d = p_A + p_B$, and $c_d(a,b,\alpha,\beta) = \sum_{p_A} U_{p_A}(a,\alpha)U_{d-p_A}(b,\beta)$. The probability of detecting Alice's photon at frequency $\omega_A = \omega_0 + \omega'$ and Bob's photon at frequency $\omega_B = \omega_0 - \omega' + d\Omega$ is given by

$$P(\omega_0 + \omega', \omega_0 - \omega' + d\Omega | a,b,\alpha,\beta) = |c_d(a,b,\alpha,\beta)|^2 = P(d|a,b,\Delta). \tag{4}$$

It depends on the index $d$ (but not on $\omega_0$ and $\omega'$), on the RF phase difference $\Delta = \alpha - \beta$ (but not on $\alpha$ and $\beta$) and on the RF amplitudes $a$ and $b$. Narrow filtering gives access to these probabilities: the transmission frequency of Alice's ultra narrow filter is kept fixed on $\omega_A = \omega_0$ while Bob's filter is aligned on $\omega_B = \omega_0 + d\Omega$ with $d$ an integer. The photons are then detected by avalanche photo-diodes and a coincidence measurement is realized.

## Results

In Figure 2 we plot the predictions of Eq. (4) and compare them to our experimental results. The amount of entanglement that is manipulated by the phase modulators is approximately given by the number of values of $d$ for which $P$ takes a significant value. At larges $a$, $b$, we are manipulating at least eleven-dimensional entanglement: there are contributions from $d = 0$ to $d = 5$, and by symmetry also from $d = -1$ to $d = -5$. By scanning the phase at a given value of $a$, $b$, one can clearly see the interference – induced by the phase modulation – between frequencies separated by integer multiples of $\Omega$. We obtained a visibility of interferences approximately equal to 98%.

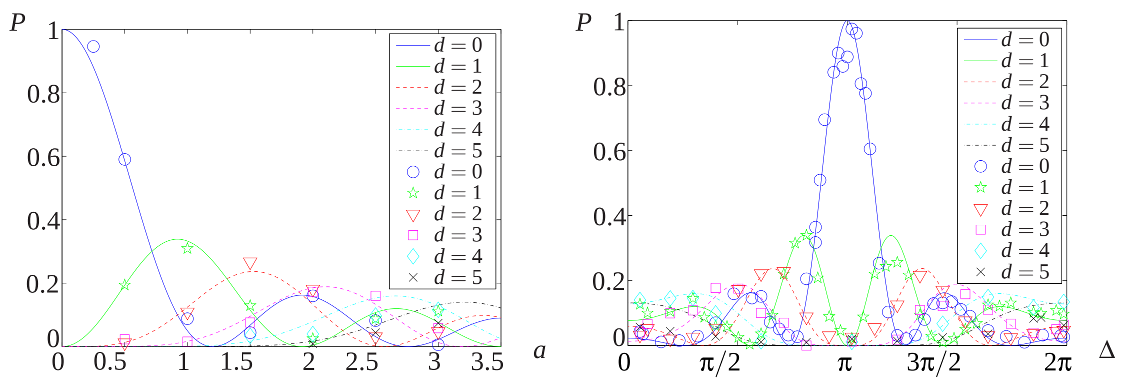


Figure 2: Theoretical predictions (curves) and experimental measurements (symbols) of the coincidence probability $P(d|a,b,\Delta)$ (see Eq. (4)) for $d = 0$, 1, 2, 3, 4, 5. Left: the amplitude $a$ is scanned and the phase $\Delta$ is kept fixed: $P(d|a,b,\Delta) = P(d|a,a,0)$. Right: the phase $\Delta$ is scanned and the amplitude $a$ is kept fixed: $P(d|a,b,\Delta) = P(d|3,3,\Delta)$. Experimentally $P$ is a normalized coincidence rate, i.e. the ratio of true coincidences to accidental coincidences divided by this ratio when the modulation is off.

Now we briefly show that this interference pattern could not be reproduced by a classical theory. Consider the Bell expression

$$S = C(A_1B_1) + C(A_1B_2) + C(A_2B_1) - C(A_2B_2), \tag{5}$$

where

$$C(A_iB_j) = P(d = 0|A_iB_j) - P(d \neq 0|A_iB_j) = 2P(d = 0|A_iB_j) - 1 \tag{6}$$

are the *generalized correlators* when Alice and Bob choose measurement settings $A_i = (a_i, \alpha_i)$ and $B_j = (b_j, \beta_j)$, respectively. It follows from the properties of the correlators that all LHV theories satisfy $S_{LHV} \leq 2$. However, choosing for simplicity $a_{1,2} = b_{1,2} = a$, a numerical optimization of the phases $\alpha_{1,2}$, $\beta_{1,2}$ leads to values of $S > 2$. When $a$ increases, the violation becomes bigger and bigger. Experimentally, we obtained a value as high as $S = 2.58 \pm 0.11$, therefore violating the LHV bound of 2 by more than 5 standard deviations.

## Conclusion

In summary, we have demonstrated the manipulation of high dimensional entanglement. Though other experiments – following the original proposal of Franson [9] – have been realized on time-energy entanglement, the method presented here allows to manipulate the entanglement in an original way: our interferometer works in the frequency domain and not in the time domain. This method is inspired by QKD systems in which the quantum information is encoded in frequency sidebands of an attenuated coherent state [10]. In view of the proven success of these systems, this seems a promising technique for quantum communication tasks. Future work will focus on studying other Bell inequalities, such as the CHSH [11] and CGLMP [12] inequalities, and demonstrating entanglement based QKD.

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