Phase noise analysis of an RF local oscillator signal generated by optical heterodyning of two lasers

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We investigate a way of generating an RF local oscillator (LO) signal by means of optical heterodyning of two lasers. This LO signal will be used to down-convert a Ku-Band (10.7 GHz ~12.75 GHz) RF signal from a phased array antenna (PAA) to an intermediate frequency (950 MHz to 3 GHz) at the mixer. In this paper we have modeled the laser spectrum perturbed by phase noise. In addition, we analyze the impact of the lasers phase noise on the quality of the generated LO by optical heterodyning.

Introduction

Optical generation of LO signals is of great interest especially in large scale distribution, for example in phased array antenna systems for radio astronomy. Among the available generation techniques, the potential of optical generation is considered more advantageous because it offers unique advantage in immunity from electromagnetic interference, fast response and low loss [1]. The optical heterodyning system is a way to generate an LO signal where its frequency can be tuned over a wide range. This system is based on two lasers tuned to slightly different frequencies. The two laser outputs are combined, and the frequency difference results in a beat signal at the output of a photo-detector (PD) [2]. An optical heterodyning for PAA system is presented in Fig. 1.

Ideally the spectrum of a laser is a delta function centered at its carrier frequency. When laser is perturbed by phase noise, its spectrum broadens where the -3dB width is called its linewidth. As we will show later, broadening of the laser linewidth ultimately broadens the RF signal from the heterodyning [3]. To deal with this phase noise analytically, it must be accurately modeled. In this paper, we derive a mathematical model of a laser perturbed by phase noise, which is presented in the next section. In the third section, the impact of laser phase noise on the generation of the RF signal by optical heterodyning is analyzed.

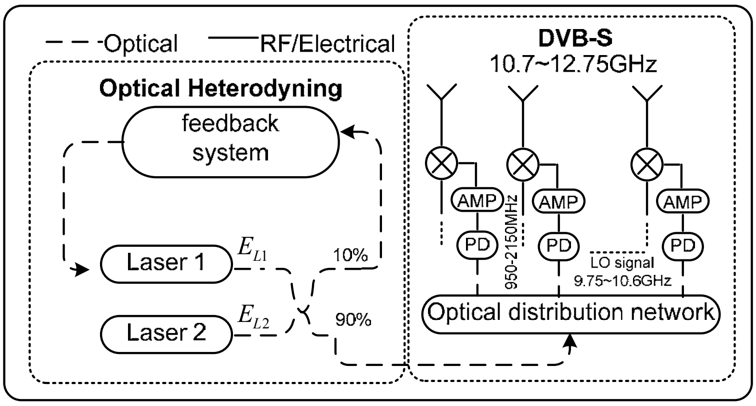

Figure 1: Phased array antenna receiver. The LO frequency for down conversion of the RF signal is generated optically

Laser Spectrum Analysis Considering the Laser Phase Noise

Let us consider laser field that is perturbed by a phase noise process $\phi(t)$. The complex field of the laser can be expressed as

$$E(t) = E_0 e^{j[2\pi f_0 t + \phi(t) + \theta]} \quad (1)$$

where E_0, f_0 and $\phi(t)$ are the amplitude, central frequency and phase noise of the laser, respectively. The random variable θ is uniform over [0, 2π], and independent of $\phi(t)$. It is introduced to make $E(t)$ wide-sense stationary (WSS) [3].

The phase noise is usually described in terms of a frequency noise process. The frequency noise $f(t)$ is white with a two sided power spectral density (PSD) of

$$S_f(f) = N_0/2 \quad (2)$$

Since the phase is the integral of the frequency, we can write

$$\phi(t) = 2\pi \int_0^t f(\tau)d\tau \quad (3)$$

Where its differential form can be written as

$$\dot{\phi}(t) = 2\pi f(t) \quad (4)$$

And its PSD becomes,

$$S_{\dot{\phi}}(f) = 2\pi S_f(f) = \pi N_0 \quad (5)$$

Using the Wiener–Khinchin theorem the autocorrelation function is

$$R_{\dot{\phi}}(\tau) = \pi N_0 \delta(\tau) \quad (6)$$

where δ(τ) is the Dirac delta function.

To find the PSD of $E(t)$, we first derive its autocorrelation function. Using Eq. (1),

$$R_E(\tau) = \mathrm{E}[E(t)E(t+\tau)] = E_0^{\,2} e^{j2\pi f_0 \tau} \mathrm{E}[e^{j\Phi(\tau)}] \quad (7)$$

where $\Phi(\tau)$ is defined as

$$\Phi(\tau) \triangleq \phi(t+\tau) - \phi(t) = \int_t^{t+\tau} \dot{\phi}(\tau)d\tau \quad (8)$$

$\Phi(\tau)$ is a zero-mean Gaussian random variable, using Eq. (6) its variance

$$\begin{aligned}
\sigma_\Phi^2 = \mathrm{E}\left[\Phi^2(\tau)\right] &= \mathrm{E}\left[\int_t^{t+\tau}\int_t^{t+\tau} \dot{\phi}(u)\dot{\phi}(v)dudv\right] \\
&= \int_t^{t+\tau}\int_t^{t+\tau} \mathrm{E}\left[\dot{\phi}(u)\dot{\phi}(v)\right]dudv \\
&= \int_t^{t+\tau}\int_t^{t+\tau} R_{\dot{\phi}}(u-v)dudv \\
&= \pi N_0 \int_t^{t+\tau}\int_t^{t+\tau} \delta(u-v)dudv \\
&= \pi N_0 \tau
\end{aligned} \quad (9)$$

It is well known that for a zero-mean Gaussian random variable Φ with variance σ_Φ^2, the relation below holds [4]

$$\mathrm{E}[e^{j\Phi}] = e^{-\sigma_\Phi^2/2} \tag{10}$$

From Eq. (7), we then can write,

$$R_E(\tau) = E_0^{\,2} e^{j2\pi f_0 \tau} e^{-\pi\tau N_0/2} \tag{11}$$

Again using the Wiener–Khinchin theorem, the PSD of $E(t)$ is:

$$S_E(f) = E_0^{\,2} \frac{1/\pi^2 N_0}{1 + \left(\dfrac{(f - f_0)}{\pi N_0}\right)^2} \tag{12}$$

This function is plotted in Fig 2. The spectral shape is referred to as Lorentzian. Solving Eq. (12) for the frequency where the power spectral density has dropped by 3 dB relative to the maximum, yields

$$\pi N_0 = f_{3dB} - f_0 = \Delta\upsilon / 2 \tag{13}$$

$$\text{So, } N_0 = \Delta\upsilon / 2\pi \tag{14}$$

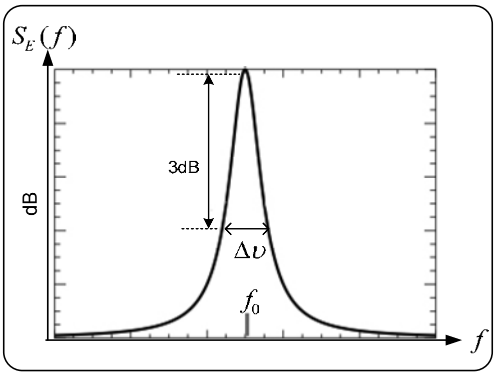

Figure 2: The Lorentzian spectral shape of a laser spectrum effected by phase noise process.

where $\Delta\upsilon$ is called the linewidth of the laser.

Eq. (12) becomes,

$$S_E(f) = 2E_0^{\,2} \frac{1/\pi\Delta\upsilon}{1 + \left(\dfrac{2(f - f_0)}{\Delta\upsilon}\right)^2} \tag{15}$$

Optical Heterodyning of Two Lasers Containing Phase Noise

Let us consider two lasers L_1 and L_2 at frequencies f_1 and f_2 respectively. We combine the light of the laser with the same polarization. A beat signal is created at the output of the photo-detector (PD). This system is shown in Fig 3.

In the absence of frequency noise the photo-detector (PD) output current $I(t)$ is a sinusoid having a frequency equal to the frequency difference of the two lasers $f_{\mathrm{LO}} = f_1 - f_2$ [5]. From Eq. (1), taking only the real part and using the relation of the power as $E_{0n} = \sqrt{2P_{Ln}}$, let the electric field from the laser be given by:

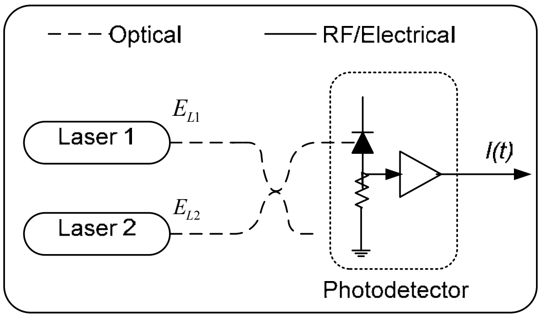

Figure 3. Optical Heterodyning

$$E_{\mathrm{L}n}(t) = \sqrt{2P_{\mathrm{L}n}} \cos\left[2\pi f_{\mathrm{L}n} t + \phi_n(t)\right] \tag{16}$$

where the subscript n=1,2 represents the first and second laser. $P_{\mathrm{L}n}$ is the received optical power while $\phi_n(t)$ represents the phase of the laser light. After combining these two lasers, the resulting signal is inserted into the PD; the power incident on the detector can then be expressed by [6]:

$$P_{opt}(t)=\frac{1}{2}\Big[\sqrt{2P_{L1}}\cos\big[2\pi f_{L1}t+\phi_1(t)\big]+\sqrt{2P_{L2}}\cos\big[2\pi f_{L2}t+\phi_2(t)\big]\Big]^2$$
$$=\frac{P_{L1}}{2}\Big[1+\cos 2\{2\pi f_{L1}t+\phi_1(t)\}\Big]+\frac{P_{L2}}{2}\Big[1+\cos 2\{2\pi f_{L2}t+\phi_2(t)\}\Big]+\sqrt{P_{L1}P_{L2}}$$
$$\Big[\cos\big(2\pi f_{L1}t+2\pi f_{L2}t+\phi_1(t)+\phi_2(t)\big)+\cos\big(2\pi f_{L1}t-2\pi f_{L2}t+\phi_1(t)-\phi_2(t)\big)\Big] \quad (17)$$

The output current from the detector can be expressed as:

$$I(t)=r_{pd}P_{opt}(t) \quad (18)$$

where r_{pd} is the PD responsivity. The phase variation $\phi_{LO}(t)\triangleq\phi_1(t)-\phi_2(t)$ is the difference of the independent phase variations $\phi_1(t)$ and $\phi_2(t)$ of the lasers. From Eq. (17) and (18) and neglecting the optical frequency terms, it follows that:

$$I(t)=\frac{r_{pd}}{2}\big(P_{L1}+P_{L2}\big)+r_{pd}\sqrt{P_{L1}P_{L2}}\cos\big[2\pi f_{LO}t+\phi_{LO}(t)\big] \quad (19)$$

where $r_{pd}\sqrt{P_{L1}P_{L2}}$, is the signal amplitude. The PSD of $I(t)$ is given by the Fourier transform of its autocorrelation function $R_I(t,\tau)$, adhering the same steps as in Eq. (20) and (12), it follows that [7]

$$S_I(f)=K\frac{1/\pi\Delta\upsilon}{1+\left(\dfrac{2(f-f_{LO})}{\Delta\upsilon_1+\Delta\upsilon_2}\right)^2} \quad (20)$$

where $K=2r_{pd}^2P_{L1}P_{L2}$ is a constant that depends on the responsivity of the PD and the power of the lasers. The beat linewidth $\Delta\upsilon=\Delta\upsilon_1+\Delta\upsilon_2$ is equal to the sum of the two linewidths of the lasers. In the presence of white frequency noise, the PSD of RF is a Lorentzian centered at f_{LO}.

Conclusion

A comprehensive analysis of laser phase noise on its spectrum has been presented and shown that due to the phase noise, the spectral linewidth of the laser broadens into a Lorentzian shape. An approach to derive the RF signal from the optical heterodyning is also presented where it is shown that the RF spectrum is also Lorentzian having the linewidth equal to the sum of the linewidths of the two lasers.

References

[1] A. J. Seeds, *et al.*, "Microwave photonics," J. Lightw. Technol., vol 24, 4628-4641, 2006.
[2] J. Yao, "Microwave photonics," J. Lightw. Technol., vol 27, No 3, 2009.
[3] J. R. Barry, *et al.*, “Performance of coherent optical receivers” Pro. of the IEEE, Vol. 78, 1990.
[4] A. Papoulis, “Probability, Random Variables and Stochastic Processes”, New York: McGraw-Hill, 1984.
[5] Ezra Ip, *et al.*, “Linewidth Measurements of MEMS-Based Tunable Lasers for Phase-Locking Applications”, IEEE photonics technology letters, Vol. 17, No. 10, October 2005.
[6] J. F. Holmes *et al.*, “Optimum optical local-oscillator power levels for coherent detection with photodiodes”, Applied Optics, Vol. 34, No. 6, 1995.
[7] W. Zhou, *et. al.* “Linewidth measurement of Littrow structure semiconductor laser with improved methods”, Physics Letters A 372, pp 4327–4332, 2008.