Excitability in semiconductor ring lasers: noise triggering vs. triggering by optical pulses

W. Coomans,[1] L. Mashal,[2] S. Beri,[1,2] L. Gelens,[1] G. Van der Sande,[1] G. Verschaffelt,[1] and J. Danckaert[1,2]

[1] Department of Applied Physics and Photonics, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium
[2] Department of Physics, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium

We present an investigation of excitability in a weakly asymmetric semiconductor ring laser (SRL). The response of the SRL to optical trigger pulses is characterized and we show that the phase difference between the fields plays an important role in determining the excitability threshold. A novel mechanism for exciting multiple consecutive pulses using a single trigger pulse will also be discussed. Finally, when excitations are triggered by noise, the variety of pulse shapes is characterized theoretically and compared with experiments.

Introduction

Excitable systems are characterized by a highly nonlinear response to external perturbations. When unperturbed, the system remains quiescent and resides in a resting state. Small perturbations only lead to a small-amplitude linear response. However, if the perturbation is sufficiently large, the system is transferred from the resting state to an excited state (the firing state). After this strong response, the system returns to its initial resting state through a refractory cycle. This large excursion of the system's variables in phase space corresponds to the emission of a large amplitude pulse. During the refractory cycle it is impossible to generate a second pulse—the system does not respond to any external perturbation.

In [1] we proposed a mechanism for excitability in systems with a weakly broken Z_2 symmetry. As optical prototypes of such systems we used semiconductor ring lasers (SRLs), whose active cavity has a circular geometry. As a result SRLs can generate light in two opposite directions referred to as the clockwise (CW) and the counterclockwise (CCW) mode. The convenient device properties of SRLs allow this optical excitable unit to be highly integrable and scalable [2], opening up perspectives for fully integrated all-optical devices and large optical neural networks. It was experimentally shown that short deterministic pulses can be excited by noise in asymmetric SRLs [1] and their origin was explained as a noise-activated escape across a barrier in an asymptotic 2D phase space [3].

From an application point of view it would be desirable to excite pulses in a deterministic way by injection of an external optical trigger in the SRL, which is theoretically investigated in this paper. Although the excitable behavior is predicted by the 2D asymptotic model, we will model the excitable asymmetric SRL by a complete set of rate equations. We do this because including optical injection in the asymptotic model makes it rather cumbersome and its validity can be argued for short pulses (since the model is only valid for time scales slower than the relaxation oscillations [4]). However, projecting these simulation results on the asymptotic phase plane will prove to be useful. The necessary symmetry-breaking to make the SRL excitable is realized by introducing an asymmetry in the magnitude of the linear coupling between the counterpropagating modes.

Model

We use a general rate equation model as proposed in [5] which assumes that the SRL operates in a single transverse and single longitudinal mode and can sustain two counterpropagating directional modes. It consists of two complex mean-field equations for the counterpropagating modes E_{cw} and E_{ccw} and a third equation for the carrier density N.
The linear coupling between the counterpropagating waves, referred to as backscattering, is caused by reflections inside the cavity at the interface with the coupling waveguide and at the cleaved end facets of the output waveguide. The asymmetry in the backscattering amplitude causes the SRL to be excitable [1, 3]. Residence in the mode which experiences the lowest backscattering is favored, allowing for excitable pulses of the counterpropagating mode. The backscattering phase is chosen to be identical for both counterpropagating modes since it has been shown that an asymmetry has no influence on the topology of the phase space structure [3]. The value of the bias current is chosen slightly above the value for which alternate oscillations disappear in a fold of cycles [3, 5].
Although we numerically investigate excitability in asymmetric SRLs using the rate equation model as proposed in [5], the excitability mechanism can be interpreted more easily in a reduced two-dimensional phase space. It has been shown that on time scales slower than the relaxation oscillations the dynamics of the SRL essentially takes place in a two-dimensional phase plane. The dynamical behavior in this phase plane is described by an asymptotically reduced model which has been introduced in [4], characterized by the variables θ and ψ. The resulting asymptotic description of the SRL is valid on time scales slower than the relaxation oscillations and it has been shown that it is able to predict many of the experimentally observed SRL characteristics [1, 3, 6]. The two phase space variables $\theta \in [-\pi/2, \pi/2]$ and $\psi \in [0, 2\pi]$ are defined by

$$\theta \equiv 2\arctan\left(\frac{|E_{ccw}|}{|E_{cw}|}\right) - \frac{\pi}{2}, \quad \text{(1a)}$$

$$\psi \equiv \angle E_{ccw} - \angle E_{cw}. \quad \text{(1b)}$$

Hence θ is a measure for the relative power distribution among the counterpropagating modes ($\theta = \pi/2$ if $|E_{cw}| = 0$, $\theta = -\pi/2$ if $|E_{ccw}| = 0$ and $\theta = 0$ if $|E_{cw}| = |E_{ccw}|$) and ψ is the relative phase difference between the corresponding electric fields. The phase space topology of the asymmetric SRL in the (θ, ψ) phase plane is shown in Figure 1(b). The grey and white regions indicate the basins of attraction of the two stable states in the SRL—the CW and CCW state—which are quasi-unidirectional. In this case, the CCW state is favored. They are separated by the stable manifold of a saddle point indicated by S. We assume that the SRL resides in the CCW state when unperturbed. Injecting an optical pulse with the right phase and amplitude makes the SRL cross both branches of the stable manifold and it will relax back to the CCW state by turning around the CW state, generating a large amplitude pulse. Hence the excitability threshold is defined by

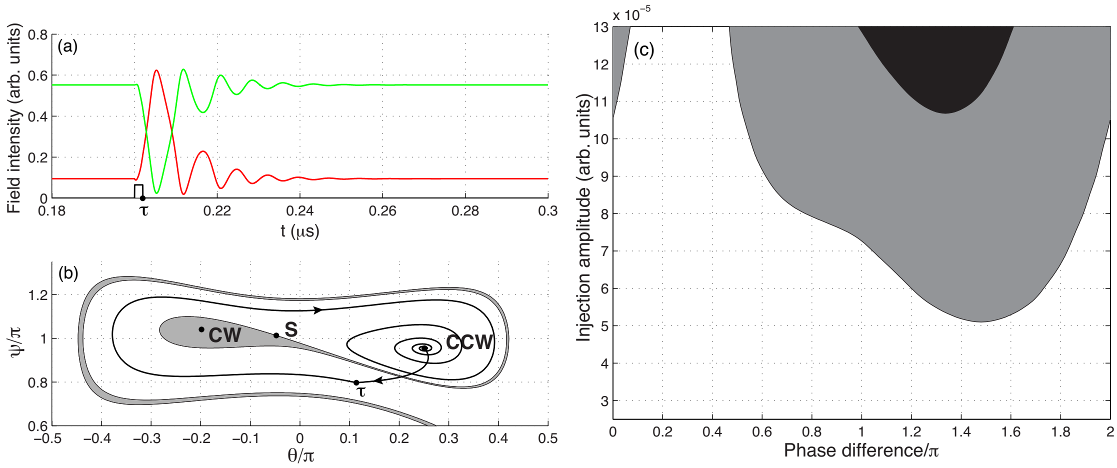

Figure 1: (a,b) Simulation of the rate equations when optically injecting a 2 ns wide square pulse. (a) Time trace of the modal intensities. The CW (CCW) modal intensity is depicted in red (green). The injected pulse is depicted in black. τ indicates the time at which the injected pulse ends. (b) Two-dimensional phase space trajectory corresponding to the time trace. The point τ also corresponds to the moment when the injected pulse ends. The basin of attraction of the CW (CCW) state is depicted in grey (white). S indicates the location of the saddle (the black line which separates the two basins of attraction is its stable manifold, the two branches of the unstable manifold each decay to one of the two stable states). The phase difference between the injected field and the SRL field is taken 1.3π for the purpose of illustration. (c) Influence of the phase difference between the injected field and the SRL field on the excitability threshold of the SRL. Grey indicates excitation of a single pulse while black indicates excitation of two consecutive pulses.

the stable manifold of a saddle point which is folded throughout the phase space and separates the basins of attraction of the counterpropagating states.

Optical triggering

We can show that the initial direction in which the SRL is kicked out from its stable resting state is determined by the optical phase difference between the injected field and the SRL field. Actually, the phase difference corresponds to the angle between the negative θ-axis and the initial (linearized) direction of the trajectory (defined as positive in the clockwise direction). This initial direction has a large influence on the ability to cross the stable manifold, and hence on the excitability threshold. This is confirmed in Figure 1(c), showing the influence of the phase difference on the firing of the SRL by numerical simulation of the rate equation model. We inject a square optical pulse with a fixed width at resonant detuning and monitor the response. In the diagram, both the injection amplitude and the phase difference between the injected field and the SRL fields are varied. For low injection amplitudes, the SRL only fires a pulse when the phase difference is close to $-\pi/2$. In the (θ, ψ) plane this corresponds to an initial direction which goes straight down, which is indeed approximately the shortest way to cross the stable saddle manifold. For higher injection amplitudes, the phase condition is less stringent but nevertheless does not allow firing when the phase difference is close to $\pi/4$, which corresponds to a trajectory being kicked out towards the region just above the saddle and does not cross the stable manifold. We can hence conclude that the phase difference between the fields has an important influence on the excitability threshold.

Conclusion

We theoretically investigated the possibility of triggering an excitable pulse in an asymmetric SRL by using an optically injected trigger pulse. We have used a standard rate equation model for the numerical simulations and an asymptotic two-dimensional phase plane to interpret the results. This two-dimensional phase plane provides a well-defined threshold appearing as two branches of a stable saddle manifold that need to be crossed. Using this approach we have shown a sensitive dependency on the phase difference between the injected field and the SRL fields. An unfavorable phase difference, which we were able to derive from the asymptotic two-dimensional phase plane structure, leads to a failure to excite the SRL.

Acknowledgments

This work has been partially funded by the European Union under project IST-2005-34743 (IOLOS) and by the Research Foundation-Flanders (FWO). This work was supported by the Belgian Science Policy Office under grant IAP-VI10. S.B., L.G. and G. VdS. are Postdoctoral Fellows and W.C. is a PhD Fellow of the FWO.

References

[1] S. Beri, L. Mashall, L. Gelens, G. Van der Sande, G. Mezosi, M. Sorel, J. Danckaert, and G. Verschaffelt. Excitability in optical systems close to Z2-symmetry. *Phys. Lett. A*, 374(5):739–743, 2010.

[2] T. Krauss, P. J. R. Laybourn, and J. Roberts. Cw operation of semiconductor ring lasers. *Electron. Lett.*, 26(25):2095–2097, 1990.

[3] L. Gelens, S. Beri, G. Van der Sande, J. Danckaert, and G. Verschaffelt. Multistable and excitable behavior in semiconductor ring lasers with broken Z2-symmetry. *Eur. Phys. J. D*, 58(2):197–207, 2010.

[4] G. Van der Sande, L. Gelens, P. Tassin, A. Sciré, and J. Danckaert. Two-dimensional phase-space analysis and bifurcation study of the dynamical behaviour of a semiconductor ring laser. *J. Phys. B-At. Mol. Opt.*, 41(9):095402, 2008.

[5] M. Sorel, P. J. R. Laybourn, A. Sciré, S. Balle, G. Giuliani, R. Miglierina, and S. Donati. Alternate oscillations in semiconductor ring lasers. *Opt. Lett.*, 27(22):1992–1994, 2002.

[6] S. Beri, L. Gelens, M. Mestre, G. Van der Sande, G. Verschaffelt, A. Scirè, G. Mezosi, M. Sorel, and J. Danckaert. Topological insight into the non-arrhenius mode hopping of semiconductor ring lasers. *Phys. Rev. Lett.*, 101(9):093903, 2008.