Robust Optimization of 2x2 Multimode Interference Coupler Affected by Parametric Uncertainties

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Robust optimization of an integrated photonic device affected by parametric uncertainties is exhibited. The optimization is performed on an approximate, cheap model of the expensive integrated photonic device simulation. This model is constructed using an interpolation technique known as Kriging. To illustrate the method, we robustly minimize the imbalance of an MMI coupler affected by an etching uncertainty that causes dilation or shrinkage of the fabricated device geometry. The robust optimum is found by minimizing the maximum realizable value of the imbalance with respect to the uncertainty set. The method can be applied to other photonic devices affected by uncertainties.

Introduction

The feature size of integrated photonic devices is steadily becoming smaller. With this reduction in size, sensitivity to fabrication variations becomes acute. If the designer does not take the limitations of the fabrication process into account, the designed devices either can't be fabricated or have low yield.

These uncertainties may arise due to variations in the material properties, fabricated design geometry, temperature variations, *etc*. Uncertainties can be categorized into two types. Parametric uncertainties affect the problem data or parameters, while implementation error affects the design variables of a problem. The design variables are variables that are controlled by the designer to direct the optimization process.

It is assumed in this work that only the bounds within which each uncertainty varies are known while the probability distribution of the uncertainty set is not available. In this situation, the robust optimization of a device is performed by finding the best worst-case solution for the objective with respect to the uncertainty set. We apply the optimization on a cheap model of the computationally expensive integrated photonics simulation. The metamodel, based on Kriging [1], is constructed by sampling the expensive integrated photonics simulation using a space-filling technique known as Latin hypercube sampling (LHS) [2]. We showcase the application of robust optimization on an integrated photonic device by minimizing the imbalance of a 2x2 multimode interference coupler affected by variations in its fabricated design geometry.

This paper is related to the work by Rehman *et al* [3], in that metamodelling is used to apply robust optimization on an MMI coupler in both works. The research by Rehman *et al.* [3], however, was focused on applying robust optimization on an integrated photonic device affected by implementation error. In this work, we apply robust optimization on an integrated photonic device under the more general supposition that the device is affected by parametric uncertainty.

Robust optimization of problems affected by parametric uncertainties

A deterministic unconstrained optimization problem is an optimization problem that does not involve uncertainties. Such a problem is defined as

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad (1)$$

where, $f(\mathbf{x})$ is the objective and $\mathbf{x}$ is the set of design variables. We now assume that our problem is affected by a parametric uncertainty. Parametric uncertainties reside in a dimension that is separate from the design domain. Let us denote the set of parametric uncertainties as $\mathbf{\Delta p}$, where $\mathbf{\Delta p}$ belongs to the uncertainty set $\mathcal{U}$. In order to find the robust optimum we must minimize the maximum possible realization of the objective $f(\mathbf{x}, \mathbf{\Delta p})$ with respect to the uncertainty set $\mathcal{U}$. This can be defined as

$$\min_{\mathbf{x}} g(\mathbf{x}, \mathbf{\Delta p}) \quad (2)$$

where

$$g(\mathbf{x}, \mathbf{\Delta p}) = \max_{\mathbf{\Delta p} \in \mathcal{U}} f(\mathbf{x}, \mathbf{\Delta p}). \quad (3)$$

$g(\mathbf{x}, \mathbf{\Delta p})$ represents the worst-case cost of the objective with respect to $\mathcal{U}$. We seek to find the best worst-case cost, which means that we must minimize $g(\mathbf{x}, \mathbf{\Delta p})$.

Numerical Modelling

Robust optimization is applied on an approximate Kriging [1] model of the expensive integrated photonic device simulation. We construct the cheap model by sampling the expensive simulation using design of experiments. We use a space-filling technique known as Latin hypercube sampling to provide these sampling locations.

Kriging is an interpolation technique that uses a parameterized Gaussian basis function. The values for the unknown parameters are chosen by the Kriging fitting process such that the likelihood of the observed data is maximized. Based on these parameters, the Kriging interpolation is performed by maximizing the combined likelihood of the observed data and the Kriging prediction.

Robust optimization of integrated photonic devices: MMI coupler

A multimode interference coupler is a well-known integrated photonic device which is used to split, combine or couple light [4]. In this work, the focus is on designing an MMI coupler that performs 3dB splitting of the input light. The device is simulated in PhoeniX software [5]. The coupler is designed in Silicon-on-Insulator (SOI) and operates at a wavelength of $1.55\mu m$. A mode solver is used to compute the length at which we should ideally observe 3dB splitting, given a nominal width of $W = 1.5\mu m$.

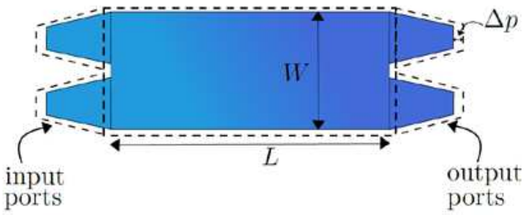

Fig. 1: 2x2 MMI coupler top view. The dashed line shows the dilation due to etching uncertainty Δp

The corresponding length for 3dB splitting was $L = 8.23\mu m$. Ideally, this should be the length for 3dB splitting when no uncertainty is involved. Bidirectional eigenmode propagation (BEP) is used as the propagation simulation. The propagation simulation is needed to analyze the performance of the MMI coupler in the presence of uncertainties.

Optimization problem definition

It is assumed that during fabrication, the etch step is only accurate up to a range of $\mathcal{U} = [-30nm, 30nm]$. This etching uncertainty Δp causes a dilation or shrinkage of the whole geometry, meaning the principal MMI waveguide as well as the input/output ports. The length and width of the MMI coupler are used as design variables.

Since our focus is on achieving 3dB splitting, the imbalance between the two output ports, in the presence of the etching uncertainty Δp, needs to be minimized. The imbalance is defined as

$$\text{Imb} = 10\log_{10}(P_1/P_2) \quad (4)$$

where P_1 and P_2 are the powers at the upper and lower output port respectively. The robust optimization problem, in the presence of the uncertainty Δp, can be written as:

$$\min_{L,W} g(L,W,\Delta p) \quad (5)$$

where

$$g(L,W,\Delta p) = \max_{\Delta\in\mathcal{U}} \left\{ (\text{Imb}(L,W,\Delta p))^2 \right\} \quad (6)$$

The design domain is limited to a $\pm 4\%$ variation around the nominal values for L and W. We take the square of the imbalance so that the response remains positive and differentiable.

Results

In this work, we compare the relative effectiveness of two sampling strategies to find the robust optimum on the MMI coupler. The first strategy is a 1-stage approach where the design space is sampled across the three design variables, using LHS, at $N = 40$ locations and the response is computed on the expensive MMI simulation. A Kriging metamodel is constructed based on the samples and robust optimization is applied on the resulting design landscape.

Tab. 1: Robust optimization of imbalance of a 2x2 MMI coupler under parametric uncertainty using 1-stage and 2-stage approach

	1-stage approach	2-stage approach
W (μm)	1.507	1.506
L (μm)	8.281	7.905
Δp (μm)	0.006	0.006
Kriging worst-case Imb(dB)	0.432	1.440
True worst-case Imb(dB)	2.179	1.755

The second strategy is a 2-stage approach where, at the first stage, the design space is sampled at $n_1 = 25$ locations using LHS and the robust optimum is found based on the Kriging metamodel of the response at these locations. We then sample the design space again at $n_2 = 15$ locations in a local region centered around the robust optimum found at the first stage. A Kriging metamodel is built based on the n_2 samples and robust optimization is performed in this local region.

The robust optima found using the two approaches are compared. Tab. 1 shows the results. The respective robust width and length values found through the two methods are shown in Tab. 1. The table also shows both the true best worst-case imbalance found on the expensive simulation of the MMI, for the respective length and width values, as well as the values for Δp at which this best worst-case result is found.

Comparing the worst-case imbalance on the expensive MMI simulation for the two methods, we note that the 2-stage approach provides a result that is 0.4 dB better, i.e. lower, than the 1-stage approach. In both cases, this worst-case result is not found at the boundaries of the uncertainty set Δp. This shows that the response is multimodal even within the uncertainty set and an approximate design landscape is difficult to model with a small number of samples. Interestingly, the worst-case imbalance predicted on Kriging by the 2-stage approach much better matches the corresponding result on the expensive simulation than the 1-stage approach. This indicates that in the local area around the robust optimum, the 2-stage approach predicts the true behavior of the MMI more accurately. The results exhibit that an adaptive sampling strategy such as *expected improvement* [1] may be better suited to applying robust optimization on a metamodel of an MMI coupler than a space-filling technique such as Latin hypercube sampling.

We observe that the width found for the robust optimum for both approaches is almost the same and this value is also quite similar to the nominal value of $W = 1.5\mu m$. However, the length at which the robust optimum is found for the 2-stage approach is significantly lower than the length found for the 1-stage approach as well as the nominal value of $L = 8.23\mu m$. This suggests that the coupler is much more sensitive to changes in length as opposed to the width.

Conclusion

In this work, we have applied robust optimization on the imbalance of a 2x2 MMI coupler in the presence of an etching uncertainty that causes a dilation or shrinkage of the fabricated device geometry. It was shown that the fabrication variation can be treated as a parametric uncertainty. Under this framework, robust optimization was applied on a cheap approximate model of the device using two different sampling strategies. We observed that robust optimization based on a 2-stage adaptive sampling strategy provided better results on the MMI coupler than a 1-stage space-filling sampling strategy. This indicates that an adaptive sampling strategy such as *expected improvement* may be better suited for this application. The method shown in this work is generic and can easily be applied to other photonic devices affected by parametric uncertainties.

References

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