Self-consistent theory of locking of coupled semiconductor lasers

D. Lenstra

Eindhoven University of Technology, Dept. of Electrical Engineering, Photonics Integration Group, 5600 MB Eindhoven, The Netherlands

In coupled lasers the effective coupling depends on the individual inversions in each constituent laser. Therefore, locking of coupled lasers defines a nonlinear problem that requires self-consistent treatment. Here we report on a general study of the frequency locking of two Fabry-Pérot lasers using a self-consistent rate-equation approach. Application of the theory to the coupled cavity quantum-well semiconductor lasers of D'Agostino et al. [Optics Letters 40 (2015) 653] yields very good agreement with measurements.

Introduction

A coupled-cavity laser (CCL) based on anti-resonant imaging in a multi-mode interference (MMI) coupler has been demonstrated in 2014 as a photonic integrated circuit [1]. The specially designed MMI anti-phase reflective coupler is based on a 3x3 reflective MMI coupler, described in [1] and provides wavelength-independent coupling with 180 degree phase difference. In this way, a self-stabilizing coupled device was realized.

The successful operation of this widely tunable laser for spectroscopy, sensing and other applications asks for a theoretical investigation into questions as to how and under which conditions the coupling causes individual laser modes to lock in frequency and phase. The present rate-equation theory aims at providing adequate answers to these questions in the context of a model for longitudinally coupled semiconductor lasers with a general type of linear coupler. The theory is self-consistent in the sense that in the strong-coupling regime the effective coupling coefficients depend on the respective inversions in each laser, where the latter in their turn depend on the effective coupling. The self-consistent solution is obtained by an iteration method. Stable steady-state solutions will be presented and discussed.

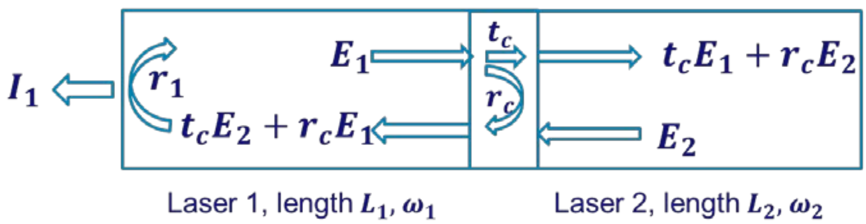

Fig.1 Schematics of coupled-cavity laser. The coupler is characterized by transmission and reflection coefficients t_c and r_c, respectively. In laser j, E_j is the complex electric-field amplitude incident on the coupler and r_j is the reflection coefficient of the mirror opposing the coupler.

Rate-equation model

The rate equations are derived for a general case of longitudinally coupled semiconductor lasers, as sketched in Fig.1. Here, the electric fields near the coupler facets are represented as $E_j(t)e^{i\omega_0 t}$ (j=1,2), where ω_0 is a conveniently chosen reference frequency close to the expected operation frequency in the steady-state locked situation and $E_j(t)$ is the slowly varying complex electric field amplitude. Details of the derivation are given in [2].

The rate equations can be expressed as ($\omega_{j0} \equiv \omega_j - \omega_0$ and ω_j is the frequency at threshold of isolated laser j, i.e. without coupling, or $t_c = 0$)

$$\dot{E}_1(t) = i\omega_{10}E_1(t) + \frac{1}{2}(1 + i\,\alpha_1)\xi_1 N_1 E_1(t) + \kappa_1 E_2(t), \qquad (1)$$

$$\dot{E}_2(t) = i\omega_{20}E_2(t) + \frac{1}{2}(1 + i\,\alpha_2)\xi_2 N_2 E_2(t) + \kappa_2 E_1(t). \qquad (2)$$

Here, κ_j ($j = 1{,}2$) are the (effective) coupling coefficients,

$$\kappa_j = \frac{e^{\frac{1}{2}(1+i\,\alpha_j)\xi_j N_j \tau_j}}{\tau_j}\,\frac{t_c}{r_c}\,e^{i\omega_{j0}\tau_j} \equiv |\kappa_j|e^{i\theta_j}\,. \qquad (3)$$

with

$$\theta_j \equiv \frac{1}{2}\,\alpha_j\xi_j N_j\tau_j + \omega_{j0}\tau_j + \varphi_c\,,\ |\kappa_j| \equiv \frac{e^{\frac{1}{2}\xi_j N_j \tau_j}}{\tau_j}\left|\frac{t_c}{r_c}\right|,\ \varphi_c \equiv \arg(t_c/r_c). \qquad (4)$$

Eqs. (1) and (2) are complemented by the rate equations for the numbers of e.h.-pairs in the respective active regions

$$\dot{N}_1 = \Delta J_1 - \frac{N_1}{T} - \xi N_1 P_1 - \Gamma_1 P_1;\ \dot{N}_2 = \Delta J_2 - \frac{N_2}{T} - \xi N_2 P_2 - \Gamma_2 P_2\,. \qquad (5)$$

where ΔJ_j (in units s^{-1}) is the injection current w.r.t. the threshold current (i.e. when each laser is on its own, that is, without any coupling), T is the spontaneous-recombination lifetime of e.h.-pairs and $\Gamma_j \equiv \frac{2}{\tau_j}(1 - |r_j r_c|)$ is the photon decay rate in isolated laser j.

Steady-state analysis

Introducing the photon number P_j and phase φ_j in laser j, i.e. $E_j = \sqrt{P_j}e^{i\varphi_j}$ and the phase difference $\varphi_{21} \equiv \varphi_2 - \varphi_1$ the following steady-state relations hold:

$$\sqrt{\frac{P_1}{P_2}} = \frac{2|\kappa_1|\cos(\theta_1+\varphi_{21})}{\xi_1 N_1}\,;\quad \sqrt{\frac{P_2}{P_1}} = \frac{2|\kappa_2|\cos(\theta_2-\varphi_{21})}{\xi_2 N_2}\,;\ P_j = \frac{\Delta J_j - \frac{N_j}{T}}{\Gamma_j + \xi_j N_j}. \qquad (6)$$

while the evolution of the phase difference is given by an Adler-type equation

$$\dot{\varphi_{21}} = \omega_{21} + \sqrt{C^2 + D^2}\sin(\varphi_{21} + \Psi)\ , \qquad (7)$$

with $C,\ D,\ \Psi$ functions of N_j, P_j, r_c, t_c as given in [1] and $\omega_{21} \equiv \omega_2 - \omega_1$ the detuning. The stable solution to (7) is given by

$$\varphi_{21} = \mathrm{Asin}\left(\frac{\omega_{21}}{\sqrt{C^2+D^2}}\right) - \Psi + \pi\ , \qquad (8)$$

provided the locking condition $|\omega_{21}| < \sqrt{C^2 + D^2}$ holds. The ability for locking heavily depends on the phase $\varphi_c \equiv \arg(t_c/r_c)$ introduced by the coupler, that is, sometimes no locking occurs for certain value ranges of the coupler phase.

Table 1: Parameter values for the coupled-cavity laser considered

Symbol	**Value**	**Name**
α_j	2.5 to 30	Linewidth enhancement parameter
ξ_j	$1.0 \times 10^3\ s^{-1}$	Linear gain coefficient (at threshold)
τ_1	$2.4 \times 10^{-11}\ s$	Roundtrip time FP cavity 1
τ_2	$2.5 \times 10^{-11}\ s$	Roundtrip time FP cavity 2
T	$1.0\ \times 10^{-9}\ s$	e-h- lifetime for spontaneous recombination
r_c	0.5	Reflection coefficient of coupling element
$\lvert t_c \rvert$	0 to 0.15	Transmission coefficient of coupling element
φ_c	arbitrary,but $\neq \pi/2$	Coupler phase
r_j	0.7	Reflection coefficient of exterior mirrors
$J_{thr,j}$ (j=1,2)	$1.125 \times 10^{17}\ s^{-1}$	Threshold current ($\sim 18\ mA$)

Numerical results

Details of the numerical iteration method are given in [2]. Here we present some results for coupling strengths $t_c = -0.12$ corresponding to strong coupling. It was generally found that a situation with coupling phase $\varphi_c \equiv \arg(t_c/r_c)\ mod(\pi)$ in a small interval around $\pi/2$ is unstable, that is, no locking was obtained. On the other hand, stable locking with sizeable locking range was generally found for $\varphi_c = 0$ or π, except for (very small) values of $|t_c| < \sim 2\ \times 10^{-3}$, where due to a Hopf-instability [3] no locking could be established. It is noted that in this coupling regime entrainment of undamped relaxation oscillations was reported. The parameters used in the numerical evaluation and their values are summarized Table 1. In Figs 2 and 3 the pump strength p_j of laser j is defined as $p_j \equiv \frac{\Delta J_j}{J_{thr,j}}$, with $J_{thr,j}$ the threshold injection current of isolated laser j.

Two typical examples of calculated output intensity of laser 1 and the operation frequency in the locking range are given in Fig.2. The upper curve is the intensity; the lower one the operation frequency relative to ω_1. The left case is for symmetric pumping; the right case for strongly asymmetric coupling. The dependence of the intensity and locking range on α

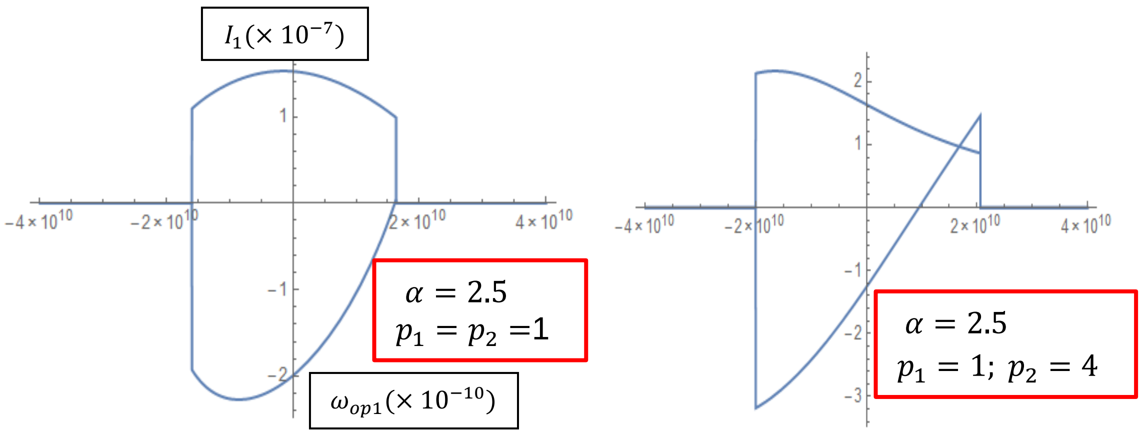

Fig.2. Calculated intensity $I_1(\times 10^{-7})$ and operation-frequency shift $\omega_{op1}(\times 10^{-10})$ versus detuning $\omega_{21} \equiv \omega_2 - \omega_1$ within the locking range for $\alpha = 2.5$. Left: symmetric pumping. Right: strongly asymmetric pumping.

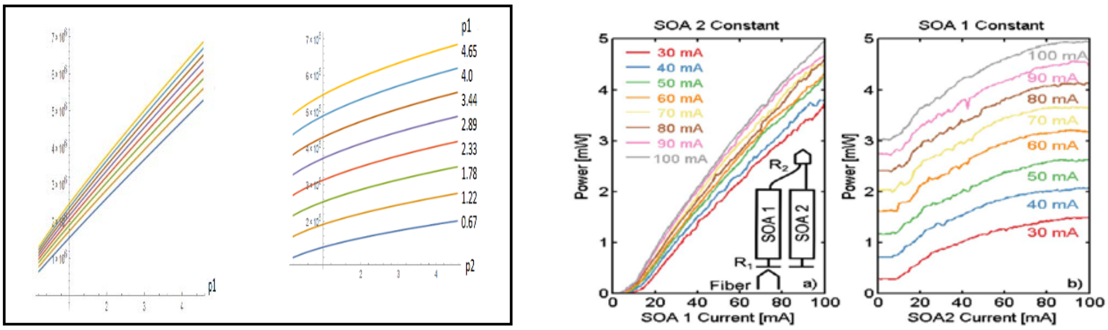

Fig.3. Comparison between calculated output intensity I_1 versus p_1, at fixed p_2 = {0.67, 1.22, 1.78, 2.33, 2.89, 3.44, 4.0, 4.65}; calculated intensity I_1 versus p_2, with fixed p_1 = {same values} and experimental curves taken from [1]; optimized detuning assumed in all cases. The shapes of the numerical curves hardly depend on the precise value of α . The parameter values for the numerical curves, i.e. $r_c = 0.5$, $t_c = -0.12$ and $\alpha = 2.5$, were taken so as to optimize the qualitative and quantitative proportionality agreement with the measured curves in [1].

was seen to be very weak; for α>30 numerical instabilities occurred, which are not believed to correspond to physical instabilities. The operation frequency does strongly depend on α; since in the locked steady state the respective inversions are lower than without coupling, a frequency downshift proportional to α is expected.

In Fig.3 two sets of calculated output-intensity curves are shown in the left window to be compared with the corresponding sets measured in ref.[1]. In fact, the parameter values used in the calculation were taken so as to obtain best agreement (qualitatively and, in proportionality, quantitatively) with the experimental curves.

Conclusions

For two longitudinally coupled semiconductor lasers in the strong coupling regime, we formulated rate equations and analyzed the self-consistent stable steady-state solutions. It was numerically found that in cases of optimized locking, with the coupling phase equal to 0 or π, the coupling can enhance the output intensity substantially (typically ~50 %) with sizeable locking range (typically ~6 GHz), hardly depending on the value of α nor on the pumping asymmetry. For non-optimized coupling, i.e. when the coupling phase $\varphi_c \equiv \arg(t_c/r_c)$ is not equal to 0 or π, but different from $\pi/2$, stable locking can still be achieved, but with lower output intensity and narrower locking width. Due to the coupling, the inversions clamp at lower values than without coupling (as visualized by the lower operation frequency). Very good agreement between the predicted output-intensity curves and the curves measured by [1] has been demonstrated.

References

[1] D. D'Agostino et al, "Coupled cavity laser based on anti-resonant imaging via multimode interference", Optics Letters 40, pp. 653-656, 2015.

[2] D. Lenstra, "Rate-Equation Analysis for a Coupled-Cavity Laser with MMI Anti-Phase Coupler", International Journal of Engineering and Technologies 8, pp. 14-23, 2016

[3] S. Wieczorek et al, "The dynamical complexity of optically injected semiconductor lasers", Physics Reports 416, pp. 1 – 121, 2005.

[4] A. Hohl et al, "Localized synchronization in two coupled nonidentical semiconductor lasers", Phys. Rev. Lett. 78, pp. 4745-4748, 1997.